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Functional Analysis: An Introduction for Physicists. by Nino Boccara

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bative treatment. The author goes beyond the usual textbook treatments in deriving conditions on the external potential that guarantee the existence and unitarity of the S -matrix. He also shows how a unique phase for the S -operator follows from a causality requirement of the type introduced by N. N. Bogolubov and D. V. Shirkov [1]. The general results are applied to obtain expressions for electron scattering, pair production, and vacuum polarization to lowest nontrivial order in an expansion in the fine structure constant.

The final chapter is, according to the author, the *raison d'être* of the book: he wanted an honest derivation of the renormalized perturbation series of QED without manipulations of ill-defined integrals. H. Epstein and V. Glaser gave such a method two decades ago. This is the first textbook account of it, and it seems to this reviewer to be a considerable success. The author manages to cover both the general theory and its application to such nontrivial applications as the electron self energy and the lowest-order radiative correction to the magnetic moment of the electron.

To pack all this into a volume of 220 pages required skill and discipline. This is not a book one consults to find detailed comparisons of the many alternative but equivalent versions of renormalization theory, but that is perhaps one of its attractions.

REFERENCES

- [1] N. N. BOGOLINBOV AND D. V. SHIRKOV, *Introduction to the Theory of Quantized Fields*, Interscience, New York, 1959.
- [2] H. EPSTEIN AND V. GLASER, *The role of locality in perturbation theory*, Ann. Inst. Henri Poincaré, 19 (1973), pp. 211–255.

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Functional Analysis: An Introduction for Physicists. *By Nino Boccara.* Academic Press, San Diego, 1990. xiii + 327 pp. \$44.50. ISBN 0-12-108810-3.

It seems clear that physicists could profit by knowing more functional analysis. They do use sophisticated mathematical concepts such as distributions, unbounded linear operators, integration on infinite-dimensional spaces, and

so forth. However, they use them in an unsophisticated way, without much idea as to when a technique is appropriate or reliable. Often this does not matter and the physics comes out anyway, but perhaps just as often the defective technique leads to erroneous or meaningless results and the physics is obscured.

The remedy is better books (and training) for physicists. It is not much good to refer them to the standard mathematical books on functional analysis, for they are immediately lost in a sea of unfamiliar abstract concepts. There are some nicely written books on mathematical physics which cover this ground, for example, Reed and Simon [1], or Choquet-Bruhat, DeWitt-Morette, and Dillard-Bleick [2]. But even these are probably somewhat daunting for a nonmathematician. For example, distributions are not introduced until the theory of topological linear vector spaces is first developed.

There is then a need for an elementary book on functional analysis aimed at physicists and with minimal prerequisites. This is the need to which Boccara's book is addressed, and by and large it does the job. The author assumes only a first course in analysis. Concepts are introduced precisely but economically. Each topic is reinforced by numerous examples, often in the form of problems with solutions. The examples are generally of a type occurring in physics problems.

The choice of topics is good. The first chapter is a standard treatment of measure theory, without existence theorems, but enlivened by Koch curves and Sierpinski carpets. Then there is a chapter on L^p spaces and the Fourier and Laplace transform, followed by a chapter on Hilbert space, especially orthogonal expansions. Next is a very concrete chapter on distributions, loaded with examples. The book closes with a chapter on linear operators on a Hilbert space. This includes the spectral theorem for compact self-adjoint operators, but stops short of the full spectral theorem.

One could quibble about some things here and there. Some definitions are not as clear as they might be (for example, the delta function on a surface and the adjoint of an unbounded linear operator). The definition of the Fourier transform uses an unusual convention: there is a 2π in the exponential. Also, physicists might appreciate more examples from quantum mechanics.

But these are minor points in an otherwise successful book. It is certainly to be recommended for a graduate level physics course.

REFERENCES

- [1] M. REED AND B. SIMON, *Methods of Modern Mathematical Physics*, Vols. I-IV, Academic Press, New York, 1972.
- [2] Y. CHOQUET-BRUHAT, C. DEWITT-MORETTE, AND M. DILLARD-BLEICK, *Analysis, Manifolds and Physics*, North-Holland, New York, 1977.

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Mathematical Foundations of Classical Statistical Mechanics: Continuous Systems. By D. Ya. Petrina, V. I. Gerasimenko, and P. V. Malyshev (translated from the Russian by P. V. Malyshev and D. V. Malyshev). Gordon and Breach, New York, 1989. xviii + 338 pp. \$198.00 (SAS Price \$95.00). ISBN 2-88124-681-8. Advanced Studies in Contemporary Mathematics, 6.

One of the main features of this book is the study of particles interacting by means of a completely elastic hard-core plus an arbitrary, bounded potential of finite range beyond the core. The first third of the book is concerned with the existence and uniqueness of the Hamiltonian equations of motion for systems of this sort, and for the equations of motion of various distribution functions. The second part discusses existence and uniqueness in ensemble theory, and the existence of solutions to the Kirkwood-Salsburg equations. The last part concerns the thermodynamic limit for nonequilibrium systems.

In my opinion there is a problem with the presentation in this book. For example, on page 12, the authors state, "According to Theorem 1.1, a single trajectory $X(t, x)$ passes through almost every point in phase space. . . ." By this statement is meant, as it turns out, that for almost every point in phase space there exists a system trajectory which passes through it, and not as might be supposed, that a single trajectory passes (eventually) through almost every point of phase space. Even so, the statement about "almost every point in phase space" is literally false, and so is Theorem 1.1 as written because, as the authors made clear in a previous section, they are ignoring what

they call "forbidden configurations" where the hard-cores overlap. Another example appears on page 27, where it is asserted that an (infinite) direct sum of strongly continuous operators is strongly continuous. I think that this conclusion is correct for their case because the evolution operator that is under discussion is also isometric, but the statement as it appears is not true generally and needs to be interpreted in an uncertain context.

These points may seem picky, but the distinctions are necessary in a book devoted to rigorous mathematical foundations. On pages 35 and 36, there is a notational conflict between (3.10) and (3.15) by a factor of $N!$ in the definition of the probability density function, which leads to confusion in trying to follow the subsequent computations. On page 40, an inequality is given between the expected number of particles in a system and the time-dependent total probability (unnormalized) which states that $\langle N \rangle \leq D(t)$. It is, however, easy to give counterexamples to this inequality in general. Presumably, the authors intended, but did not state explicitly, some necessary restrictions. I could go on, but I hope these examples will be sufficient to make the point that the presentation in this book confronts the reader with unnecessary obstacles.

There is a valuable and worthwhile story to be told on this book's subject, and it can, I think, be extracted by a careful reading. Unfortunately, the mode of presentation is less salutary than could be desired. The bibliographic notes that appear at the end of each chapter are good, and are very helpful guides to the literature. They help to put the chapter topic in perspective.

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Topics in Boundary Element Research. Vol. 6: Electromagnetic Applications. Edited by C. A. Brebbia. Springer-Verlag, Berlin, 1989, xiv + 234 pp. \$117.90. ISBN 0-387-50607-1 (US Only).

An initial "quick read" of this eye-catching, slim, red textbook might lead one to believe as does the editor that

This volume makes an important contribution to the series and is essential reading