PHYS 461: Midterm Exam
October 8, 2001

1 One mole of a van der Waals gas obeys the equation of state

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT,$$

where P, v and T denote, respectively, the pressure, molar volume, and temperature of the gas. a, b and R are positive constants.
(i) Show that the molar specific heat

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v,$$

where s is the molar entropy, does not depend upon v.
(ii) Show that

$$du = c_v dT + a\frac{dv}{v^2},$$

where u is the molar internal energy.
(iii) Let v_i and T_i be, respectively, the initial molar volume and temperature of the gas. Determine, the final temperature T_f in terms of T_i, v_i, and the final molar volume v_f when the gas undergoes a free expansion. Assume that c_v is constant between T_f and T_i. Note that during a free expansion, the internal energy of a gas remains constant. What is the sign of $\Delta T = T_f - T_i$?

2 An ideal gas is slowly heated and allowed to do work quasistatically at constant pressure going from the initial state (V_1, P) to the final state (V_2, P).
(i) Show that the ratio $\left|\frac{\Delta Q}{\Delta W}\right|$ of the quantity of heat ΔQ and work ΔW received by the system is a constant. What is the

value of this constant?
(ii) Determine the entropy change $S_2 - S_1$ as a function of the work ΔW and temperature T_1 of the system in the initial state.
Hint. An ideal gas obeys the two equations of state:

$$PV = NRT \quad \text{and} \quad U = cNRT,$$

where P is the pressure, V the volume, N the number of moles, T the temperature. R and c are constant.