

# ELECTROMIGRATION AND THE BACK FLOW POTENTIAL IN THIN FILMS AND LINES

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## ABSTRACT

Electromigration (EM) in a metal line is the phenomenon of flow of the metal atoms along the line. The flow is driven by the current of electrons and is also affected by the variation of chemical potential along the line. The replacement of atoms leads to a change in eigenstrain, which, in turn, alters the chemical potential. Since the chemical potential is very often dominated by a stress term, this back flow is very often attributed to the presence of a stress gradient. In microelectronic applications metal lines are usually subjected to the actions of large thermally induced eigenstrain even before the current is turned on. This large eigenstrain, together with the em-induced eigenstrain, may lead to a nonlinear back flow, which is the subject of investigation of this paper.

**Keywords:** Electromigration, chemical potential, and eigentransformation.

## INTRODUCTION

The configurational deformation of a single-component solid from a uniform reference state to a nonuniform state is characterized by an *eigentransformation* that may be defined in terms of the underlying crystal structure and the experimentally measured molar volume of the solid. If the uniform reference state is defined in a fixed reference coordinate system  $\mathbf{X}$ , then the effect of an eigentransformation,  $\mathbf{F}^*$ , is to convert a differential element  $d\mathbf{X}$  into a new differential element  $d\mathbf{X}^{\text{SF}} = \mathbf{F}^* d\mathbf{X}$ . For convenience,  $\mathbf{X}^{\text{SF}}$  will be referred to as a *stress-free coordinate system*. An eigentransformation field is in general incompatible and, as a result, must be accompanied by an elastic transformation,  $\mathbf{F}^e$ , so that the combined effect is an integrable deformation-gradient field,  $\mathbf{F} = \mathbf{F}^e \mathbf{F}^*$ , and  $d\mathbf{x} = \mathbf{F} d\mathbf{X}$  is the differential element in the spatial coordinate system  $\mathbf{x}$ . In general, neither  $d\mathbf{X}^{\text{SF}} = \mathbf{F}^* d\mathbf{X}$  nor  $d\mathbf{X}^{\text{SF}} = \mathbf{F}^* \mathbf{F}^{-1} d\mathbf{x}$  is integrable but  $\mathbf{F}^e = \mathbf{F} \mathbf{F}^{*-1}$  ties the stress-free element  $d\mathbf{X}^{\text{SF}}$  to the spatial element  $d\mathbf{x}$ . We exploit this last observation by insisting that the Helmholtz free energy involved in  $d\mathbf{X} \rightarrow d\mathbf{x}$  may be obtained in terms of the enthalpy of mixing in  $d\mathbf{X} \rightarrow d\mathbf{X}^{\text{SF}}$  and the elastic strain energy in  $d\mathbf{X}^{\text{SF}} \rightarrow d\mathbf{x}$ . The derivative of this free energy with respect to the eigentransformation turns out to be the energy momentum tensor that may be directly linked to the chemical potential (Wu, 2001). In this paper the single component solid is taken to be a perfect conductor and the effect of the electron "gas" is coupled to the aforementioned chemical potential.

The eigentransformation in a single-component solid with vacancies as well as the three-way transformations among the three sets of coordinates are explicitly defined in Section 2. The thermodynamics of the single-component solid is presented in Section 3 where the desired

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Helmholtz free energy is firmly established in terms of the molar Gibbs free energy and the isothermal and isocomposition strain energy per unit stress-free volume. This concept was first conceived in a recent paper by (Wu, 2001). If the chemical potential is simply taken to be the hydrostatic stress, researchers have known the desired coupling and some of the results may be found in (Sandler, 1999; Eshelby, 1970; Larche and Cahn, 1978).

## THE EIGENTRANSFORMATION IN A SINGLE-COMPONENT SOLID WITH VACANCIES

Let a single-component solid be defined by a molar concentration [mol/m<sup>3</sup>]  $C_a$  and a vacancy concentration  $C_v$ , so that the total molar concentration  $C$ , which is the number of available lattice sites per unit volume, and the associated mole fractions  $x_a = C_a / C$  and  $x_v = C_v / C$  satisfy

$$C = C_a + C_v, \quad 1 = x_a + x_v. \quad (1)$$

Since the two mole fractions are not independent, we express them in terms of a single mole-fraction value  $x$  as follows

$$x_a = x, \quad x_v = 1 - x. \quad (2)$$

The use of  $x$  as the kernel letter for mole fractions is a historical one and should not cause any confusion with the three components of the spatial coordinate system.

Let  $\underline{V}(P, T, x)$  be the molar volume [m<sup>3</sup> / mol] of the solid at pressure  $P$ , temperature  $T$ , and mole fraction  $x$ . It is an important material property that may be experimentally determined. The molar volume  $\underline{V}(0, T_0, x_0)$  may be used to define a uniform state for a solid body occupying a region  $V$  in a chosen reference coordinate system  $\mathbf{X}$ . When  $T = T(\mathbf{X}, t)$ , and hence  $x = x(\mathbf{X}, t)$ , becomes nonuniform, the associated  $\underline{V}(0, T, x)$  may be used to compute the Jacobian  $J^*$  of an eigentransformation  $\mathbf{F}^*$  defined by

$$J^*(\mathbf{X}, t) = \underline{V}(0, T(\mathbf{X}, t), x(\mathbf{X}, t)) / \underline{V}(0, T_0, x_0), \quad \mathbf{F}^* = (J^*)^{1/3} \mathbf{K}, \quad \det \mathbf{K} = 1 \quad (3)$$

where  $\mathbf{K}$  is a constant transformation that may be related to the underlying crystal structure. For convenience, we use

$$d\mathbf{X}^{\text{SF}} = \mathbf{F}^* d\mathbf{X}, \quad dV^{\text{SF}} = J^* dV \quad (4)$$

to stress the existence of a stress-free state. The molar volume  $\underline{V}(P, T, x)$  may be written as

$$\underline{V}(P, T, x) = x_a \bar{V}_a(P, T, x) + x_v \bar{V}_v(P, T, x), \quad \bar{V}_i(P, T, x) = \left. \frac{\partial [C \underline{V}(P, T, x)]}{\partial C_i} \right|_{P, T, C_{j \neq i}} \quad (5)$$

where the subscript  $i$  is either  $a$  or  $v$ , and  $\bar{V}_a(P, T, x)$  and  $\bar{V}_v(P, T, x)$  are the partial molar volumes.

The transformation  $\mathbf{F}^*$  is in general incompatible in the sense that the first of (4) can not be integrated to obtain  $\mathbf{X}^{\text{SF}}$  as single-valued functions of  $\mathbf{X}$ . The solid body, which occupies  $V$  in  $\mathbf{X}$ , is therefore forced to deform into a new configuration that occupies  $\nu$  in a spatial reference coordinate system  $\mathbf{x}$ . The associated transformation is the deformation gradient  $\mathbf{F} = \partial \mathbf{x} / \partial \mathbf{X}$ . Finally, the combination of  $\mathbf{F}$  and  $\mathbf{F}^*$  is termed the elastic transformation  $\mathbf{F}^e = \mathbf{F}\mathbf{F}^{*-1}$ . We have

$$d\mathbf{x} = \mathbf{F}d\mathbf{X}, \quad d\nu = JdV, \quad J = \det\mathbf{F} \quad (6)$$

$$d\mathbf{x} = \mathbf{F}^e d\mathbf{X}^{\text{SF}}, \quad d\nu = J^e dV^{\text{SF}}, \quad J^e = \det\mathbf{F}^e = J / J^* \quad (7)$$

which, together with (4), complete the needed three-frame kinematics. Let us now denote the pressure in the spatial reference by  $p(\mathbf{x}, t)$ , then  $\underline{v}(p, T, \underline{x}) \equiv \underline{V}(p(\mathbf{x}, t), T(\mathbf{x}, t), \mathbf{x}(\mathbf{x}, t))$  is the partial molar volume of the mixture associated with  $(p, T, \mathbf{x})$  which happens to be present at  $(\mathbf{x}, t)$ . Thus,

$$J = \frac{\underline{v}(p, T, \mathbf{x})}{\underline{V}(0, T_0, \mathbf{x}_0)}, \quad J^e = \frac{\underline{v}(p, T, \mathbf{x})}{\underline{V}(0, T, \mathbf{x})}, \quad (8)$$

which compliments (3). We now identify  $C = 1 / \underline{V}(0, T_0, \mathbf{x}_0)$ , so that  $C$  is the constant number of lattice sites per unit volume of the solid in  $\mathbf{X}$ . Molar densities may now be defined in terms of unit volumes in  $\mathbf{x}$  and  $\mathbf{X}^{\text{SF}}$ . The three sets of densities are:

$$C = C_a(\mathbf{X}, t) + C_v(\mathbf{X}, t), \quad C^{\text{SF}} = C_a^{\text{SF}}(\mathbf{X}, t) + C_v^{\text{SF}}(\mathbf{X}, t), \quad c = c_a(\mathbf{x}, t) + c_v(\mathbf{x}, t). \quad (9)$$

There are also the identities:

$$J^* = dV^{\text{SF}} / dV = C / C^{\text{SF}}(\mathbf{X}, t) = C_a(\mathbf{X}, t) / C_a^{\text{SF}}(\mathbf{X}, t) = C_v(\mathbf{X}, t) / C_v^{\text{SF}}(\mathbf{X}, t), \quad (10)$$

$$J = d\nu / dV = C / c(\mathbf{x}, t) = C_a(\mathbf{X}, t) / c_a(\mathbf{x}, t) = C_v(\mathbf{X}, t) / c_v(\mathbf{x}, t), \quad (11)$$

$$J^e = d\nu / dV^{\text{SF}} = C^{\text{SF}}(\mathbf{X}, t) / c(\mathbf{x}, t) = C_a^{\text{SF}}(\mathbf{X}, t) / c_a(\mathbf{x}, t) = C_v^{\text{SF}}(\mathbf{X}, t) / c_v(\mathbf{x}, t), \quad (12)$$

$$x_i = c_i / c = C_i / C = C_i^{\text{SF}} / C^{\text{SF}} \quad (i = a \text{ or } v). \quad (13)$$

In terms of the partial molar thermodynamic description of (5), we have

$$\underline{v}(p, T, \underline{x}) = x_a \bar{v}_a(p, T, \underline{x}) + x_v \bar{v}_v(p, T, \underline{x}), \quad \bar{v}_i(p, T, \underline{x}) = \left. \frac{\partial c \underline{v}(p, T, \underline{x})}{\partial c_i} \right|_{p, T, c_{j \neq i}}, \quad (14)$$

$$\underline{V}(0, T_0, \mathbf{x}_0) = x_a \bar{V}_a(0, T_0, \mathbf{x}_0) + x_v \bar{V}_v(0, T_0, \mathbf{x}_0), \quad \bar{V}_i(0, T_0, \mathbf{x}_0) = \left. \frac{\partial C \underline{V}(0, T_0, \mathbf{x}_0)}{\partial C_i} \right|_{p, T, C_{j \neq i}} \quad (15)$$

where, again, the subscript  $i$  is either  $a$  or  $v$ . It follows from the first of (8) and the above that

$$\bar{v}_a(p, T, x) = J\bar{V}_a(0, T_0, x_0), \quad \bar{v}_v(p, T, x) = J\bar{V}_v(0, T_0, x_0). \quad (16)$$

We conclude this section by defining the following convenient symbols for a number of inverse quantities:

$$\mathbf{f} \equiv \mathbf{F}^{-1}, \quad j \equiv 1/J; \quad \mathbf{f}^* \equiv \mathbf{F}^{*-1}, \quad j^* \equiv 1/J^*; \quad \mathbf{f}^e \equiv \mathbf{F}^{e-1}, \quad j^e \equiv 1/J^e. \quad (17)$$

## THE THERMODYNAMICS OF A SINGLE-COMPONENT SOLID WITH VACANCIES

### *Mass Balance*

The mass balance equations are:

$$c\dot{x}_a = -\text{div}\mathbf{j}_a, \quad c\dot{x}_v = -\text{div}\mathbf{j}_v, \quad (18)$$

where  $\mathbf{j}_a$  and  $\mathbf{j}_v$  are the mass diffusion fluxes [ $\text{mol} / \text{m}^2\text{s}$ ] defined with respect to the spatial coordinate system  $\mathbf{x}$  and over dots indicate material time derivatives. The result of expressing the above in  $\mathbf{X}$  is

$$\dot{C}_a = \frac{\partial C}{\partial t} = -\text{Div}\mathbf{J}_a, \quad \dot{C}_v = \frac{\partial C}{\partial t} = -\text{Div}\mathbf{J}_v, \quad (19)$$

where

$$\mathbf{J}_a = J\mathbf{f}\mathbf{j}_a, \quad \mathbf{J}_v = J\mathbf{f}\mathbf{j}_v. \quad (20)$$

### *Energy Balance*

We balance the energy associated with an arbitrary volume  $v(t)$ :

$$\frac{d}{dt} \int_v c\mathbf{U} dv = \int_{\partial v} [\dot{\mathbf{x}} \cdot \mathbf{t}_n - \mathbf{j}_{(q)} \cdot \mathbf{n}] da + \int_v [\dot{\mathbf{x}} \cdot \mathbf{f}^{(em)} + \mathcal{F}z_a\mathbf{j}_a \cdot \mathbf{E} - \mathcal{F}z_a\mathbf{j}_{(e)} \cdot \mathbf{E}] dv, \quad (21)$$

where  $\mathbf{U}$  is the molar internal energy,  $\dot{\mathbf{x}}$  is the velocity,  $\mathbf{j}_{(q)}$  is the heat flux vector, and  $\mathbf{t}_n$  is the Cauchy stress vector. The very last integral comes from the electromagnetic contribution with

$$\mathbf{f}^{(em)} = \mathcal{F}z_a\mathbf{j}_a \times \mu_0\mathbf{H} - \mathcal{F}z_a\mathbf{j}_{(e)} \times \mu_0\mathbf{H} \quad (22)$$

being the net Lorentz force,  $\mathbf{H}$  the magnetic field,  $\mathbf{E}$  the electric field,  $z_a$  the valence of the element,  $\mathbf{j}_{(e)}$  the flux of the electron "gas", and  $\mathcal{F}$  the Faraday (Avogadro's number x charge of electron = 96352 C/mol). The local form of the above is

$$c\dot{\underline{U}} = c\dot{\underline{q}} + \dot{\mathbf{x}}_{,i} \cdot \mathbf{t}_i - \mathcal{F} z_a \mathbf{j}_a \cdot \text{grad } \phi + \mathcal{F} z_a \mathbf{j}_{(e)} \cdot \text{grad } \phi \quad (23)$$

where  $\text{grad } \phi = -\mathbf{E}$  and the following balance conditions have been used:

$$c\dot{\underline{q}} = -\text{div } \mathbf{j}_{(q)}, \quad \mathbf{t}_{i,i} + \mathbf{f}^{(em)} = \mathbf{0}. \quad (24)$$

The result of converting the above into the  $\mathbf{X}$ -representation is

$$\dot{U} = C\dot{\underline{U}} = \dot{Q} + \dot{\mathbf{x}}_{,i} \cdot \mathbf{T}_i - \mathcal{F} z_a \mathbf{J}_a \cdot \text{Grad } \phi + \mathcal{F} z_a \mathbf{J}_{(e)} \cdot \text{Grad } \phi \quad (25)$$

where  $U = CU$  is the internal energy per unit volume in  $\mathbf{X}$ ,  $\dot{Q} = C\dot{\underline{q}}$ , and  $\mathbf{T}_i$  and  $\mathbf{t}_i$  are related to the Piola stress tensor  $\mathbf{P}$  and Cauchy stress tensor  $\boldsymbol{\sigma}$  by

$$\mathbf{t}_n da = \mathbf{t}_i n_i da = \boldsymbol{\sigma}_{ij} n_i \mathbf{e}_j da = \mathbf{T}_i N_i dA = \mathbf{P}_{ij} N_i \mathbf{e}_j dA, \quad (26)$$

$$\mathbf{P} = \mathbf{J}\mathbf{f}\boldsymbol{\sigma} \quad \text{i.e.} \quad \mathbf{P}_{ij} = \mathbf{J}f_{,i} \boldsymbol{\sigma}_{ij}. \quad (27)$$

#### *Entropy Balance and The Second Law*

The statements is:

$$\frac{d}{dt} \int_v c\mathcal{S} dv + \int_{\partial v} [\mathbf{j}_{(q)} - \mu_a \mathbf{j}_a - \mu_v \mathbf{j}_v] \cdot \frac{\mathbf{n}}{T} da \geq 0, \quad (26)$$

where  $\mathcal{S}$  is the molar entropy and  $\mu_a$  and  $\mu_v$  are the chemical potentials. The associated local forms are:

$$cT\dot{\mathcal{S}} - c\dot{\underline{q}} - \frac{1}{T} \mathbf{j}_{(q)} \cdot \text{grad } T + c\mu_a \dot{x}_a - T\mathbf{j}_a \cdot \text{grad} \left( \frac{\mu_a}{T} \right) + c\mu_v \dot{x}_v - T\mathbf{j}_v \cdot \text{grad} \left( \frac{\mu_v}{T} \right) \geq 0 \quad (27)$$

$$T\dot{\mathcal{S}} - \dot{Q} - \frac{1}{T} \mathbf{J}_{(q)} \cdot \text{Grad } T + \mu_a \dot{C}_a - T\mathbf{J}_a \cdot \text{Grad} \left( \frac{\mu_a}{T} \right) + \mu_v \dot{C}_v - T\mathbf{J}_v \cdot \text{Grad} \left( \frac{\mu_v}{T} \right) \geq 0 \quad (28)$$

where  $S = C\mathcal{S}$  is the entropy per unit volume in  $\mathbf{X}$ . Introducing Helmholtz free energy per unit volume in  $\mathbf{X}$  by  $A = U - TS$ , and combining (25) and (28), we obtain the Clausius-Duhem inequality

$$\begin{aligned} & -[\dot{A} + S\dot{T} - \mathbf{P} \cdot \dot{\mathbf{F}} - \mu_a \dot{C}_a - \mu_v \dot{C}_v] - \frac{1}{T} \mathbf{J}_{(q)} \cdot \text{Grad } T - T\mathbf{J}_v \cdot \text{Grad} \frac{\mu_v}{T} \\ & - T\mathbf{J}_a \cdot \text{Grad} \frac{\mu_a}{T} - \mathcal{F} z_a \mathbf{J}_a \cdot \text{Grad } \phi + \mathcal{F} z_a \mathbf{J}_{(e)} \cdot \text{Grad } \phi \geq 0 \end{aligned} \quad (29)$$

where  $\mathbf{P} \cdot \dot{\mathbf{F}} = \dot{\mathbf{x}}_1 \cdot \mathbf{T}_1$ . It follows that the above constitutive constraint may be met by the use of  $A(\mathbf{F}, T, C_a, C_v) = CA(\mathbf{F}, T, \mathbf{x})$ , where  $\underline{A}$  is the molar Helmholtz energy, as a potential such that

$$\mathbf{S} = -\frac{\partial A}{\partial T}, \quad \mathbf{P} = \left( \frac{\partial A}{\partial \mathbf{F}} \right)^T, \quad \mu_a = \frac{\partial A}{\partial C_a}, \quad \mu_v = \frac{\partial A}{\partial C_v} \quad (30)$$

and

$$-\frac{1}{T} \mathbf{J}_{(q)} \cdot \nabla T - T \mathbf{J}_v \cdot \nabla \frac{\mu_v}{T} - T \mathbf{J}_a \cdot \nabla \frac{\mu_a}{T} - \mathcal{F} z_a \mathbf{J}_a \cdot \nabla \phi + \mathcal{F} z_a \mathbf{J}_{(e)} \cdot \nabla \phi \geq 0 \quad (31)$$

where  $\nabla \equiv \text{Grad}$  and the dissipation inequality may be satisfied by the usual argument. The main purpose of this paper is to link the chemical potentials to an appropriately defined energy momentum tensor, so that the electron gas induced eigentransformation can be tied to the deformation of the solid. This is accomplished by the Helmholtz energy introduced in the following section.

## THE HELMHOLTZ FREE ENERGY

We begin with the assumption that the stress-free molar Gibbs energy  $\underline{G}^{\text{SF}}(T, \mathbf{x}) = \underline{G}(0, T, \mathbf{x})$  and the isothermal and isocomposition strain energy per unit stress-free volume  $\underline{W}^{\text{SF}}(\mathbf{F}^e, T, \mathbf{x})$  are known, as they are the mixture properties that can be experimentally measured. The desired Helmholtz free energy per unit volume of  $V$  in  $\mathbf{X}$  is

$$A(\mathbf{F}, T, C_a, C_v) = CA(\mathbf{F}, T, \mathbf{x}) \quad \text{and} \quad \mathbf{F} = \mathbf{F}^e \mathbf{F}^* \quad (32)$$

where the molar Helmholtz energy  $\underline{A}(\mathbf{F}, T, \underline{\mathbf{x}}) = \underline{G}(\mathbf{P}, T, \underline{\mathbf{x}}) + \mathbf{P} \cdot \mathbf{F} / C$  has the properties

$$\underline{A}(\mathbf{I}, T_0, \mathbf{x}_0) = 0, \quad (33)$$

$$\underline{A}(\mathbf{F}^*, T, \mathbf{x}) = \underline{G}^{\text{SF}}(T, \mathbf{x}) \equiv \underline{G}(\mathbf{P}, T, \mathbf{x})|_{\mathbf{P}=0}, \quad (34)$$

$$\begin{aligned} \underline{A}(\mathbf{F}^e \mathbf{F}^*, T, \mathbf{x}) &= \underline{A}(\mathbf{F}^*, T, \mathbf{x}) + \underline{A}^{\text{SF}}(\mathbf{F}^e, T, \mathbf{x}), \\ \underline{A}^{\text{SF}}(\mathbf{F}^e, T, \mathbf{x}) &= [\underline{A}(\mathbf{F}^e \mathbf{F}^*, T, \mathbf{x}) - \underline{A}(\mathbf{F}^*, T, \underline{\mathbf{x}})] \end{aligned} \quad (35)$$

The condition (33) sets the uniform state as a reference, (34) follows from the fact that  $\mathbf{F}^*$  is the stress-free eigentransformation at temperature  $T$  and composition  $\mathbf{x}$ , and the definition of  $\underline{A}^{\text{SF}}$  requires that  $\underline{A}^{\text{SF}}(\mathbf{I}, T, \underline{\mathbf{x}}) = 0$ . Using the above properties, we obtain from (32)

$$A(\mathbf{F}, T, C_a, C_v) = CA(\mathbf{F}^e \mathbf{F}^*, T, \mathbf{x}) = C[\underline{A}(\mathbf{F}^*, T, \mathbf{x}) + \underline{A}^{\text{SF}}(\mathbf{F}^e, T, \mathbf{x})], \quad \text{or} \quad (36)$$

$$A = CG(0, T, x) + \frac{C}{C^{SF}} C^{SF} \underline{A}^{SF}(\mathbf{F}^e, T, x) = CG^{SF}(T, x) + J^* W^{SF}(\mathbf{F}^e, T, C_a^{SF}, C_v^{SF}), \quad (37)$$

where (10) has been used and  $W^{SF}(\mathbf{F}^e, T, C_a^{SF}, C_v^{SF}) = C^{SF} \underline{A}^{SF}(\mathbf{F}^e, T, x)$  is the strain energy per unit stress-free volume at temperature  $T$  and molar concentrations  $C_a^{SF}$  and  $C_v^{SF}$ . Substituting (37) into (30), we get (for  $i = a$  or  $v$ )

$$\mu_i = \mu_i^{SF} + \partial(J^* W^{SF}) / \partial C_i, \quad \mu_i^{SF} = \bar{G}_i^{SF}, \quad \text{and} \quad (38)$$

$$\mu_i = \mu_i^{SF} + \frac{\partial J^*}{\partial C_i} W^{SF} + \left(1 - \frac{C_i}{J^*} \frac{\partial J^*}{\partial C_i}\right) \frac{\partial W^{SF}}{\partial C_i^{SF}} + J^* \frac{\partial W^{SF}}{\partial \mathbf{F}^*} \frac{\partial \mathbf{F}^*}{\partial C_i}, \quad (39)$$

where the last term has the energy momentum tensor connection:

$$J^* \frac{\partial W^{SF}}{\partial \mathbf{F}^*} \frac{\partial \mathbf{F}^*}{\partial C_i} = J^* \mathbf{f}^* \Sigma^e \cdot \frac{\partial \mathbf{F}^*}{\partial C_i} = \Sigma \mathbf{f}^* \cdot \frac{\partial \mathbf{F}^*}{\partial C_i} \quad (40)$$

in which

$$\Sigma^e = W^{SF} \mathbf{I} - \mathbf{P}^e \mathbf{F}^e, \quad \mathbf{P}^e = [\partial W^{SF} / \partial \mathbf{F}^e]^T, \quad (41)$$

$$\Sigma = J^* W^{SF} \mathbf{I} - \mathbf{P} \mathbf{F}, \quad \mathbf{P} = [\partial(J^* W^{SF}) / \partial \mathbf{F}]^T, \quad (42)$$

which are the energy momentum tensors that have been reported in (Wu, 2001; Epstein and Maugin, 1990; Maugin, 1993; Eshelby, 1970). The chemical potential (39) is completely general and nonlinear, as opposed to the special nature of several published results (Larche and Cahn, 1978a,b, 1982; Stephenson, 1988; Bartholomeusz, 1995).

## ELECTROMIGRATION, EIGENTRANSFORMATION AND ELASTIC DEFORMATION

Electromigration describes the atomic motion in a metal under the influence of applied electrical field. The resulting nonuniform molar concentration, however, leads to a state of eigentransformation and the incompatible nature of which forces the metal to deform further. This nonlinear, three-way interplay is briefly summarized in this section. Under isothermal condition and when the fluxes of the element and vacancy are conserved  $\mathbf{J}_a + \mathbf{J}_v = 0$ , (31) becomes

$$-\mathbf{J}_a \cdot \nabla (\mu_a - \mu_v + \mathcal{F} z_a \phi) + \mathcal{F} z_a \mathbf{J}_{(e)} \cdot \nabla \phi \geq 0. \quad (43)$$

The associated pair of flux equations may be written as

$$\mathbf{J}_a = -L_{aa} \nabla (\mu_a - \mu_v + \mathcal{F} z_a \phi) + L_{ac} \mathcal{F} z_a \nabla \phi \quad (44)$$

$$\mathbf{J}_{(e)} = -L_{ea} \nabla(\mu_a - \mu_v + \mathcal{F} z_a \phi) + L_{ee} \mathcal{F} z_a \nabla \phi \quad (45)$$

where  $L_{aa}$ ,  $L_{ae}$ ,  $L_{ea}$  and  $L_{ee}$  are the phenomenological coefficients (Groot & Mazur, 1962; Prigogine, 1967; Huntington, 1975). The diffusion associated with (44) describes the atomic motion, but the atomic motion induced deformation is now absorbed in the chemical potential via (39).

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