Counterparty and Non-Random Risk

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Counterparty and Non-Random Risk

- *Counterparty*: other side of ongoing financial agreement.
  - Friend pays you $5 every time someone says "Edgeworth."
  - A bank enters into a swap with you on the S&P 500.
- Counterparty Risk
  - Risk resulting from default/bankruptcy of a counterparty.
  - Strictly: Risk to you from one of your counterparties.
  - Broadly: Includes effects on overall market (our concern).
- Non-random Risk
  - What if randomness ceases to dominate risk? Goes away?
  - Just a few thoughts; more philosophical in nature.

Counterparty Risk: Why We Care

- Affects overall market when large bankruptcy looms/occurs:
  - Near-bankruptcy of Bear Stearns (May 2008)
  - Bankruptcy of Lehman Brothers (Sep 2008)
  - Bankruptcy of Refco Inc? (Oct 2005, owned #1 CME broker)
- Outstanding notional at CME before ceasing trading:

Bear	Lehman	Refco LLC
\$700 BB	\$1,300 BB	\$130 BB

  - N.B. No defaults or trade halts at CME for these events.
- Other bankruptcies: Askin (1994), LTCM (1998, why I care)
  - Counterparty risk: concern. . . and contributor?

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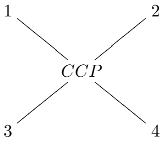
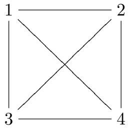
Network Topologies

- Investigate two extremes of *n*-counterparty networks.



Star network

(Futures market w/CCP[1])



Fully-connected network

(Bilateral OTC market)

- Each node is a counterparty (capital K, risk aversion λ).
- Each edge is a contract[2] linking counterparties i and j
- Contract exposure: $q_{ij} = -q_{ji}$; $q_{i<j} \overset{iid}{\sim} N(0, \eta^2)$
- Counterparty i's net exposure: $Q_i = \sum_{j \neq i} q_{ij}$.
- Same net exposures (Q_i's) in both networks.

[1] Centralized counterparty.

[2] A swap or forward on a risky asset.

Event Timing

To study counterparty risk, events occur at discrete times.

$t = 0$: Bankruptcy of counterparty n occurs.

- All contracts with counterparty n are invalidated.
- Pushes unwanted exposure onto other $n - 1$ counterparties.

$t = 1$: Living counterparties trade in response to bankruptcy.

$t = 2$: Living counterparties close out bankruptcy-induced exposure.

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Price Impact of Trading

- Use price impact model of Almgren and Chriss (2001).
    - Impact has permanent and temporary components.
    - Permanent component impacts prices for later traders.
    - Temporary components only affect price for that trade.
    - Impact is linear in trade size plus fixed cost.
- Each counterparty trades x_i shares at time $t = 1$.
- Price of trade for counterparty i at $t = 1$:

$$p_{i,1} = p_0 + \underbrace{\pi x_i}_{\text{permanent}} + \underbrace{\tau x_i + \phi \, \text{sgn}(x_i)}_{\text{temporary}} \tag{1}$$

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Price Evolution

- Trading occurs during periods 1 and 2:
  - The order of trading is random, not strategic; and,
  - Ordering and price impact create low and high prices.
- Time periods are very short; two simplifying assumptions:
  1. Prices have no drift other than price impact due to trading.
  2. Price diffusion is Gaussian (not log-normal).
- Thus the price at the end of period 1 is:

$$p_1 = p_0 + \sigma Z_1 + \pi \sum_{j=1}^{n-1} x_j \tag{2}$$

where $Z_{t \in \{1,2\}} \overset{iid}{\sim} N(0,1)$.

Effects of Invalidated Contracts

- Bankruptcy invalidates each contract with exposure q_{in}.
- Star network: only contract with CCP is invalidated.
- Fully-connected network:
  - Each counterparty has unwanted exposure of $-q_{in}$
  - Net unwanted exposure: $\sum_{i \neq n}(-q_{in}) = \sum_{i \neq n} q_{ni} = Q_n$.
- Full rehedge (in either network) implies net trade of $-Q_n$.
- However, counterparties trade in own interest.
  - Do they rehedge immediately? Push market further?

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Small Bankruptcy

- First consider bankruptcy of a small financial firm.
- Cause of bankruptcy may be market factors or idiosyncratic.
- What do we know about net exposure to the bankrupted?
    - Net exposure is likely to be small;
    - Possible non-market causes; cannot estimate net exposure.
- Each counterparty maximizes mean-variance utility:

$$U_i(x) = \underbrace{-\pi x^2 - \tau x^2 - \phi|x|}_{\text{impact of period 1 trading}} \underbrace{-\lambda \frac{\sigma^2}{2}[q_{in}^2 - (x - q_{in})^2]}_{\text{variance penalty}} \underbrace{-\pi q_{in}(x - q_{in}) - \tau(x - q_{in})^2 - \phi|q_{in} - x|}_{\text{impact of period 2 trading}} \quad (3)$$

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Small Bankruptcy: Optimal Trade

- The optimal trade size is then given by[3]:

$$x_i = \frac{(\pi + 2\tau + \lambda\sigma^2)q_{in} - \phi\,\mathrm{sgn}(x) - \phi\,\mathrm{sgn}(q_{in} - x)}{2\pi + 4\tau + \lambda\sigma^2}. \qquad (4)$$

- Higher impact splits trades: $\pi \uparrow \infty$ or $\tau \uparrow \infty \Rightarrow x \to q_{in}/2$;
- Higher volatility, hedge faster: $\sigma \uparrow \infty \Rightarrow x \to q_{in}$; and,
- High spreads reduce trade splitting: $\phi \uparrow \infty$, $x \to q_{in}$.

[3] This is, admittedly, in improper form due to the signum functions.

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Small Bankruptcy: Added Volatility

- How much volatility does this trading add?
- Recall that $q_{i<j} \overset{iid}{\sim} N(0, \eta^2)$.
- Assume $(\pi + 2\tau + \lambda\sigma^2) q_{in} > \phi$ (trade in both periods).
- Variance added to prices in period 1 due to exposures q_{in}:

$$\text{Var}(p_1) = \sigma^2 + \overbrace{(\pi + \tau)^2 (n-1) \left(\frac{\pi + 2\tau + \lambda\sigma^2}{2\pi + 4\tau + \lambda\sigma^2} \right)^2 \eta^2 + \phi^2}^{\text{added variance}}. \quad (5)$$

- This result applies only to fully-connected network.
- Ignore variance in period 2; may have setup-related artifacts.

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Small Bankruptcy: Results

- Use sensible parameters[4] and $n = 10$ counterparties:

$$p_0 = \$50.00 \qquad \sigma = \$0.95 \text{ (30\% annual)}$$
$$\lambda = 1 \times 10^{-6} \qquad \eta = 100,000$$
$$\phi = \$0.01 \qquad \text{volume} = 5 \text{ MM shares/day}$$

- Period 1 price impact: \$0.20 permanent, \$0.03 temporary.
- Period 1 volatility: $\$1.36 = 1.43 \times \$0.95$
- On an annualized basis, volatility went from 30% to 43%.
- In this model, higher volatility only lasts two periods.

[4] Impact parameters are as derived in Almgren and Chriss (2001).

Large Bankruptcy

- Next consider the bankruptcy of a large financial firm.
- Assume large market move at $t = 0$ induces bankruptcy.
- Net exposure likely to be large; can estimate Q_n.
- Maximize mean-variance utility for expected trading by others:

$$U_i(x) = \underbrace{-\pi(E\sum_{j\neq i} x_j + x)x - \tau x^2 - \phi|x|}_{\text{impact of period 1 trading}}$$
$$\underbrace{-\lambda\frac{\sigma^2}{2}[q_{in}^2 - (x - q_{in})^2]}_{\text{variance penalty}} \qquad (6)$$
$$\underbrace{-\pi E Q_n(q_{in} - x) - \tau(x - q_{in})^2 - \phi|q_{in} - x|}_{\text{impact of period 2 trading}}$$

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Network Differences

- Large bankruptcies are where we see network differences.
- For a star network, only the central counterparty trades.
  - Eliminates expectations of net exposure, trading.
  - Matches real world: CCP can penalize predatory traders.
  - Star network solution is same as for small bankruptcy.
- For fully-connected network, all counterparties may trade.
  - All must anticipate trading by others.
  - Since each is selfish, trouble can arise.

Large Bankruptcy: Optimal Trade?

- Optimal trade for fully-connected network is then:

$$x_i = \frac{-\pi(EQ_n + E\sum_{j\neq i} x_j) + (2\tau + \lambda\sigma^2)q_{in}}{2\pi + 4\tau + \lambda\sigma^2} - \frac{\phi\,\text{sgn}(x) + \phi\,\text{sgn}(x - q_{in})}{2\pi + 4\tau + \lambda\sigma^2}. \tag{7}$$

- Note that this gives the partial equilibrium answer.
- Full equilibrium tends toward $q_{in}/2$ each period.
- $Q_n, \sum_{j\neq i} x_j$ uncertainty add more volatility than star network.
- But, is this really the optimal trade?

Large Bankruptcies

- For large Q_n, trading at $t = 1, 2$ will move market a lot.
- Move will be further in direction that caused bankruptcy.
- This raises two distressing possibilities:
  - Move might greatly weaken other counterparties; or even,
  - A counterparty's hedging might bankrupt itself[5].
- Counterparties anticipate this in their utility function.
- Does it matter? Does it get worse? (Yes.)

[5] Checkmate.

Checkmate

Let B_k be the event $\{$bankruptcy of counterparty $k\}$.

Proposition (Likeliness of Checkmate)

In a fully-connected network, for all $\alpha \in (0, 1)$ there is a $Q_n \in (0, \infty)$ such that $P(B_k) > \alpha$ for some $k < n$ and any x_k.

For a large initial bankruptcy, follow-on bankruptcy may be very likely — despite the best hedging actions in periods 1 and 2.

Hunting

Recall B_k is the event $\{\text{bankruptcy of counterparty } k\}$.

Proposition (Profitability of Hunting)

In a fully-connected network, there exists a $Q_n \in (0, \infty)$ such that trades $x_i, E\sum_{j \neq i} x_j$ yields $P(B_k)E(profit|B_k) > 0$ for some $k < n$.

For a large initial bankruptcy, counterparties may expect profit by forcing the next-weakest counterparty into bankruptcy.

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Large Bankruptcies: Results

- Consider large bankruptcy for $n = 10$ counterparties:

$$p_0 = \$50.00 \qquad \sigma = \$0.95 \text{ (30\% annual)}$$
$$Q_n = 3,000,000 \qquad \eta = 1,000,000$$

- For star network, optimal trade is $0.54Q_n$ at $t = 1$.
  - Expected market impact: \$3.25 permanent, \$0.34 temporary.
- Assume fully-connected network hunts, trades $2EQ_n$ at $t = 1$.
  - Expected market impact: \$12.00 permanent, \$1.21 temporary.
  - Period 1 volatility: $\$24.91 = 26.22 \times \$0.95$
  - On an annualized basis, volatility went from 30% to 790%.

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Large Bankruptcies: Not So Random

- Fully-connected networks admit two destabilizing events:
  - Checkmate: weak counterparty may have no beneficial trade.
  - Hunting: counterparties force others into bankruptcy.
- Worse, hunting is a full equilibrium behavior.
  - Market may be pushed far beyond one follow-on bankruptcy.
- Are counterparties selfishly amoral/evil? Maybe not.
  - Trade amount may pre-hedge expected follow-on bankruptcies.
  - This reduces surprise need for trading in period 2.
- Star networks do not admit these destabilizing events.
  - Suggests central clearing reduces OTC market volatility.

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Non-Random Risk: Some Thoughts

- In large bankruptcies, hunting may dwarf randomness.
- Hunting is not random, but it is a serious risk.
- In these situations, market volatility increases greatly.
  - Does anybody then know what σ is — even roughly?
  - Would one trade or wait for volatility to decrease?
- Recalls another distress phenomenon: volume decreases.
- In markets, does randomness (σZ diffusion) sometimes:
  - Decrease, so that non-random actions (like hunting) dominate?
  - Go away entirely, as when markets cease to trade?
- Is this the right way to think about some risks?

Conclusion

From a simple OTC market with price impact, we've seen that:

- Even small bankruptcies temporarily increase volatility.
- Large bankruptcy effects depend on network structure.
- For a large bankruptcy in a fully-connected network:
  - Counterparties may be unable to save themselves (checkmate).
  - Counterparties may hunt their weakest peers for profit.
- A large bankruptcy in a star network only increases volatility.
  - That increase may be less than in a fully-connected network.
- Suggests benefits to centralized clearing in OTC markets.

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