Submitted to
manuscript EJOR-D-10-02163R1

Consignment contracts with retail competition

Elodie Adida*
Mechanical and Industrial Engineering, University of Illinois at Chicago, elodie@uic.edu

Nantaporn Ratisoontorn
Mechanical and Industrial Engineering, University of Illinois at Chicago, nratis2@uic.edu

Consignment contracts have been widely employed in many industries. Under such contracts, items are sold at a retailer's but the supplier retains the full ownership of the inventory until purchased by consumers; the supplier collects payment from the retailer based on actual units sold. We investigate how competition among retailers influences the supply chain decisions and profits under different consignment arrangements, namely a consignment price contract and a consignment contract with revenue share. First, we investigate how these two consignment contracts and a price only contract compare from the perspective of each supply chain partner. We find that the retailers benefit more from a consignment price contract than from a consignment contract with revenue share or a price only contract, regardless of the level of retailer differentiation. The supplier's most beneficial contact, however, critically depends upon the level of retailer differentiation: a consignment contract with revenue share is preferable for the supplier if retailer differentiation is strong; otherwise a consignment price contract is preferable. Second, we study how retailer differentiation affects the profits of all supply chain partners. We find that less retailer differentiation improves the supplier's profit for both types of consignment contract. Moreover, less retailer differentiation improves profits of the retailers in a consignment price contract, but not necessarily in a consignment contract with revenue share.

Key words: Supply chain, consignment, retail competition.

History: Submitted November 2010, revised May 2011

---

1. Introduction

In the Spring of 2005, The Home Depot, a leading home improvement retailer, launched the Pay-By-Scan (PBS) program across the live plant industry. The goal was to help reduce risk for seasonal plant products, while providing a venue for growers to sell their production. The PBS program allows Home Depot to decide the retail price for each of the products, and the growers decide how much inventory is allocated into designated Home Depot retail sites. Ownership of the products does not transfer until they are sold to customers. Once products are sold, Home Depot makes a payment to the suppliers and retains the difference between this payment and the retail price

* Corresponding author. 842 W. Taylor St. (MC 251) Room 3025 ERF, Chicago IL 60607, `elodie@uic.edu`. Ph.: 312-996-8777.

(For more details on Home Depot and PBS, see `http://suppliercenter.homedepot.com/wps/portal`). Similarly, Autozone, one of the biggest automobile parts resellers, adopts Pay-On-Scan contracts with its suppliers. Under such a contract, Autozone selects suppliers and sets the retail price for each product. Autozone pays the supplier only when a product has been scanned and sold, with payment terms of up to 90 days after the sale (Boorstin [2003], Fahey [2003]). These two business arrangements between retailer and supplier are essentially *consignment contracts with a fixed payment term*.

Several types of consignment contracts with various characteristics are being used, in varying degrees, across all product categories and in virtually all industries. A prime example of a consignment contract in Internet commerce is Amazon Marketplace (Wang et al. [2004]). Amazon.com not only sells products directly to consumers, but also offers an additional online site called “Amazon Marketplace”. Marketplace enables and invites other merchants, even those who could potentially compete with Amazon, to sell their products through the Amazon website. The merchants can decide how many units to sell and their selling price. The merchants incur no cost for listing their items, but Amazon.com receives a commission on products sold by deducting a certain percentage from the final sale price. Essentially, Amazon has adopted a *consignment contract with revenue share* with merchants in its Marketplace.

Moreover, consignment contracts have recently gained in popularity in healthcare venues, as they allow the supplier to retain ownership of the items in the healthcare retail facility at no charge until items are actually dispensed. Items with consignment potential include intraocular lenses, orthopedic implants and pulse generators. For example, the University of California San Francisco Medical Center implements consignment contracts with its suppliers of various products (of California at San Francisco Medical Center [2006]).

The APICS Dictionary (Blackstone and Cox [2005]) defines consignment as the process in which the supplier retains the full ownership of the inventory, places items at a retail location (or, virtually, anywhere) with no payment received until the goods are sold to consumers. The benefit of this process to the retailer is to incur no risk associated with the uncertainty of the demand other than the storage cost and the opportunity cost due to shelf space usage. The supplier bears all risk associated with demand uncertainty. If the merchandise does not sell, no money is exchanged. The supplier gains access to consumers and transfers the responsibility and the cost of storage to the retailer, which could potentially increase sales volume and profits.

One of the most critical features of a consignment contract is the payment mechanism specified in the contract. The specific mechanism for determining the supplier’s revenue impacts all parties in the contract. To our knowledge, two consignment contract terms are used in practiced, and have been discussed in the literature. On the one hand, in Wang et al. [2004], a Stackelberg game model is proposed in which the retailer, acting as the leader, offers the supplier a consignment contract which specifies the supplier’s revenue share as a percentage of the retailer’s revenue for each unit sold. On the other hand, in Ru and Wang [2010], the supplier is the leader and selects a fixed consignment price, specifying the amount of payment to the supplier for each unit sold at the retailer. The retailer acts as a follower and, based upon the consignment price selected by the supplier, decides the retail price and order quantity.

The large majority of research on consignment contracts has focused on a channel structure consisting of a single supplier and a single retailer (Wang et al. [2004], Li et al. [2009], Ru and Wang [2010]). Consignment contracts in the presence of retail competition have received little or no attention. A large number of researchers have considered horizontal competition in the supply chain and contracts (for example, Choi [1996], Dana Jr. and Spier [2001], Bernstein and Federgruen [2005], Yao et al. [2008b], Cachon and Kok [2010]), but none of them studied consignment contracts. Regarding horizontal competition in the framework of consignment contracts, only the effect of upstream competition among *suppliers* has been recently discussed (Wang [2006], Zhang [2008]), but not downstream competition among retailers. Our goal is to fill the gap in the literature by considering consignment in a setting where retailers compete horizontally, thereby extending the research in consignment contracts from a non-competing market to the more realistic setting in which downstream competition exists. This paper, thus, introduces horizontal competition at the retailer level in addition to vertical competition between the supplier and the retailers.

Our research focuses on a consignment contract under the two different payment schemes for the supplier previously mentioned: (i) a fixed consignment price per unit sold and (ii) a percentage of the revenue earned by the retailer as the supplier's revenue share. In order to understand the impact of these two types of consignment contracts, a price-only contract (wholesale price contract) is used as a benchmark to evaluate consignment contracts. Our research quantifies the benefits to all members of the supply chain under different contract settings and helps determine which contract terms are most beneficial to the entire system as well as to the different parties involved.

The purpose of this study is to investigate the effect of retail competition on the decision-making of supply chain members and the channel performance under consignment contracts. The results of the study are used to answer the following research questions:

1. How do consignment contracts compare with price-only contracts from the entire system's perspective? Do consignment contracts improve channel performance? Do consignment contracts benefit all supply chain members?
2. Which contract term (e.g., fixed consignment price or revenue sharing) should be used in a consignment contract to provide the greatest benefit to all supply chain members?
3. How does the presence of competition among retailers and the level of retailer differentiation affect decisions such as retail prices and consigned quantity?
4. Does more retail differentiation improve total channel profits and each supply chain member's profit? Are these patterns of improvement the same across different supplier payment mechanisms?

We show that the benefit of consignment contracts to the supply chain members depends upon the level of retailer differentiation. Specifically, the consignment price contract yields highest profits to both the supplier and the retailers when retailer differentiation is weak. Conversely, when retailer differentiation is strong, the price-only contract yields higher retailer profits than the two types of consignment contracts considered, and the consignment contract with revenue share yields higher supplier profit than the other contracts. Moreover, we find that the consignment price contract is more beneficial to the retailers than the consignment contract with revenue share, regardless of the level of retailer differentiation. The benefit of each type of consignment contracts to the supplier, however, depends on the level of retailer differentiation. Regarding the effect of retailer differentiation on the decisions and the profits of each of the supply chain members, we find that a decrease in the level of retailer differentiation leads to an increase in prices (or revenue share). As expected, the supplier benefits from a lower retailer differentiation for all types of contracts. The effect of retailer differentiation on the retailer profit, however, varies depending on the contract.

This paper is organized as follows. Section 2 provides a review of related literature. Section 3 formulates the model and provides equilibrium results. Section 4 provides a numerical study to investigate the equilibrium solutions for each contract. Section 5 concludes and suggests future research directions. The Appendix contains the detailed mathematical proofs, additional plots from numerical experiments, a summary table, a detailed study of a supply chain with 10 retailers, an extension to integrated consignment contracts (i.e., a combination of fixed consignment price and revenue-share), and a comparison between consignment and traditional revenue-sharing contracts.

2. Related literature

The problem considered in this paper draws ideas from four research areas: (1) supply chain coordination via contracts, (2) consignment contracts, (3) retailer-managed inventory (RMI) and (4) retail competition. We briefly review relevant literature in these areas.

It is well known in the literature on supply chain management that total supply chain profits in a decentralized channel are in general lower than those in an integrated or centralized channel–a characteristic known as double marginalization. In a decentralized chain, supply chain members maximize their own objective and their “selfish” actions result in poor overall channel performance (i.e., lower total channel profits than in a centralized channel). The loss in total supply chain profit in the decentralized chain is referred to as *supply chain inefficiency*. Several types of ***contracts*** have been introduced and implemented to coordinate the supply chain. A contract coordinates the supply chain if it provides incentives to all participants so that the total profits in the decentralized channel match those in the centralized channel (Cachon [2003]). A variety of contractual relationships for reducing supply chain inefficiency, their benefits and drawbacks have been discussed in the literature: e.g. the price-only contract (wholesale-price contract), buy-back contract, price-discount contract, revenue sharing contract and consignment contract.

The simplest and the most common contracts are *price-only contracts*, or wholesale price contracts. As described in Perakis and Roels [2007], the price-only contract specifies a constant price per unit purchased from a supplier by a retailer. Despite its popularity, Cachon [2003] shows that the price-only contract cannot coordinate the supply chain since the retailer does not order enough inventory to maximize the supply chain total profits. Buy-back contracts allow a retailer to return unsold merchandise up to a specified amount at an agreed-upon price. This contract gives the retailer an incentive to increase her order quantity, resulting in higher product availability and higher profits for both the supplier and the retailer. However, the buy-back contract may result in surplus inventory for the supplier and it may lead to inflated retail orders (Tsay [1999]). Bernstein and Federgruen [2005] study a price-discount contract, in both linear and nonlinear forms. Closely related to a buy-back contract, a linear price-discount contract specifies a wholesale price and a buy-back rate. These two terms are linear functions of the chosen retail price. Bernstein and Federgruen conclude that such a contract can coordinate the price-setting newsvendor problem in a supply chain with one supplier and multiple non-competing retailers. Furthermore, they demonstrate that a nonlinear version of the price-discount contract can coordinate the channel in the case of competing retailers.

Cachon and Lariviere [2005] introduce a ***revenue sharing contract*** with a single retailer and a single supplier. Under such a contract, the supplier charges the retailer a wholesale price for each unit purchased and a percentage of the revenue that the retailer generates. The authors find that revenue sharing is equivalent to buy-backs in the newsvendor case and is equivalent to price discounts in the price-setting newsvendor case. Yao et al. [2008a] find that the benefits of revenue sharing contracts for each supply chain member vary depending on the price sensitivity and demand variability. Linh and Hong [2009] generalize a revenue sharing contract with a single retailer and a single supplier to a two-period newsvendor problem. They find that the optimal revenue sharing ratio for the retailer is linearly increasing in wholesale prices. In every revenue sharing study cited above, the retailer makes decisions on the inventory level and the retail price. Pan et al. [2010] consider both the case when retailer(s) or supplier(s) make these decisions, and compare a revenue sharing contract with a wholesale price contract under deterministic demand in a supply chain with two different channel structures: (a) two manufacturers and one retailer; (b) one manufacturer and two retailers.

Consignment is different from other types of contracts in terms of inventory ownership and time of payment. Wang et al. [2004] propose a single product consignment contract with revenue sharing between a supplier and a retailer. The retailer first decides the fraction of the revenue to keep for each unit sold; the supplier then chooses the retail price and the quantity placed at the retailer's. The authors assess the impact of the retailer's share of the channel cost and the demand-price elasticity on channel profits. They conclude that the loss of profit in a decentralized supply chain decreases with the retailer's cost share and increases with the demand-price elasticity. Wang [2006] extends consignment contracts to a supply chain with multiple suppliers of complementary products and a single retailer. The suppliers decide on the price and product quantity, either simultaneously or sequentially. Contrary to Wang et al. [2004], Li et al. [2009] utilize a cooperative game approach (Nash bargaining model) to coordinate the decentralized consignment channel with a single supplier and a single retailer. They show that coordination between the two supply chain partners can be achieved. Ru and Wang [2010] further study a consignment contract in two different settings: a *retailer-managed* and a *vendor-managed* consignment inventory setting. They model these settings within the framework of a supplier Stackelberg game: the supplier first specifies a fixed consignment price for each product sold by the retailer. They find that both the supplier and the retailer benefit from a supplier-controlled inventory. The papers by Wang et al. [2004] and Ru and Wang [2010] are closely related to our work, in that they study the channel performance under consignment contracts. However, our work extends their research to a situation in which two retailers compete in the market.

It is common in the literature for a price-only contract to be used as a benchmark for comparing the channel performance (total profit earned) of several types of contracts. For example, Cachon [2004] studies and compares the allocation of inventory risk under several types of contracts (e.g., an advance-purchase contract) against a wholesale price contract. Özer and Wei [2006] focus on comparing capacity-reservation and advance-purchase contracts with a price-only contract. Pasternack [2002], Gerchak and Wang [2004], Yao et al. [2008a], Pan et al. [2010], Katok and Wu [2009] compare the channel performance of the revenue sharing contract with the performance of the price-only contract. Su and Zhang [2008] compare buyback, markdown money, and sales rebates contracts with a price-only contract. Chen [2011] compares the performance of a return-discount contract and a price-only contract. As a result, we also use the price-only contract in our study for evaluating the channel performance of consignment contracts.

Who controls the quantity decision is another critical aspect in supply chain coordination. The ***inventory decision*** (i.e., how much merchandise to order) may be made by the upstream supplier(s) or the downstream retailer(s). The former case is also known as a vendor-managed inventory (VMI) and the latter as retailer-managed inventory (RMI). Aviv and Federgruen [1998] and Fry et al. [2001] are prime examples of VMI models. Studies of a revenue sharing contract to coordinate the decentralized supply chain that adopt RMI models include Cachon and Lariviere [2005], Dana Jr. and Spier [2001], Veen and Venugopal [2005], Yao et al. [2008a], Linh and Hong [2009]. Wang et al. [2004] also adopt an RMI model in their study of a consignment contract with revenue sharing with a single supplier and a single retailer. Ru and Wang [2010] extend this paper by considering a VMI model as well. Our model focuses on an RMI model.

The literature on supply chain coordination with a single manufacturer selling to multiple ***competing retailers*** is very rich. Dana Jr. and Spier [2001] focus on a revenue sharing contract in the presence of perfectly competitive retailers. They conclude that revenue sharing can coordinate price-setting retailers, while each retailer earns zero profit in equilibrium. Bernstein and Federgruen [2005] investigate the equilibrium behavior of a decentralized channel with a monopolistic supplier and competing retailers under demand uncertainty. They employ a combination of wholesale price and buy-back contracts to coordinate the decentralized channel. Yao et al. [2008b] study a revenue sharing contract for coordinating a supply chain with one manufacturer and two competing retailers facing stochastic demand. They analyze the impact of demand variability, price-sensitivity, and level of competition on decisions such as retail prices, order quantity and profit sharing between manufacturer and retailers. The channel structure in one of the models considered in Pan et al. [2010] consists of one manufacturer interacting with two retailers. They focus their study on comparing the channel performance of the revenue-sharing contract (without consignment) with the channel performance of the price-only contract. None of these studies, however, considers consignment.

The effect of competition in a consignment setting has been recently discussed. Wang [2006] investigates the equilibrium price and stocking factor of multiple suppliers of complementary products and the revenue share decision of the retailer. He concludes that competition among suppliers leads to higher product prices and lower quantities. Zhang [2008] extends the work by Wang et al. [2004] by including competition between two manufacturers producing substitutable products. He finds that higher product substitutability benefits the retailer. Conversely, the suppliers only benefit from higher product substitutability when product substitutability is not too strong. While these papers investigate the effect of upstream supplier competition, we analyze in this paper the effect of downstream competition between two retailers.

3. Model and equilibrium results

3.1. Model assumptions

Consider a supply chain with one supplier and two retailers. We assume the supplier (S) produces one product and sells it through two competing differentiated retailers (R_1 and R_2). The supplier produces at a constant unit cost of \$$c_M$, and retailer i incurs a unit cost of \$$c_{R_i}$, $i = 1,2$ for handling and selling the product to consumers. Define $c = c_S + c_{R_1} + c_{R_2}$ as the total unit cost for the channel, and $\alpha_i = c_{R_i}/c$ as the share of the channel cost that is incurred at retailer i, $i = 1,2$. Note $\alpha_1 + \alpha_2 < 1$.

We consider a demand for the product at each retailer during a single selling season that is price-dependent and uncertain. We use a multiplicative model to capture the randomness in the demand. A multiplicative demand uncertainty is widely used in the literature (see for example Karlin and Carr [1962], Petruzzi and Dada [1999]). In the context of consignment, Wang et al. [2004], Wang [2006], Ru and Wang [2010] also adopt this model. We thus model the demand for the product at retailer i, denoted by $D_i(\mathbf{p})$ where $\mathbf{p} = (p_1, p_2)$, as:

$$D_i(\mathbf{p}) = y_i(\mathbf{p}) \cdot \epsilon, \quad i = 1,2, \tag{1}$$

where p_i, $i=1,2$ is the retail price charged by retailer i to consumers, $y_i(\mathbf{p})$ is the expected demand at retailer i and ϵ is a random scaling factor, representing randomness of the demand, with a mean value of 1, cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$ that have support $[A,B]\subset\Re^+$ with $B>A$. Let $h(x)=f(x)/[1-F(x)]$ denote the failure rate function.

We model the expected demand as an exponential function of both prices. In absence of competition, an exponential demand model (also called log-linear) $y(p)=ae^{-\beta p}$, where β is a price sensitivity parameter and p is the retail price, has been adopted in several studies in the supply chain management literature (Gallego and Van Ryzin [1994], Petruzzi and Dada [1999], Huang et al. [2006], Ru and Wang [2010], Besbes and Maglaras [2009]). This model is also common in the economics literature (Greenhut et al. [1988], Jeuland and Shugan [1988], Cowan [2008]). In the presence of competition, Talluri and Van Ryzin [2005], Simon [2007] propose a natural extension of this demand model in the form of $y_i(p_i,\mathbf{p_{-i}})=a_i\exp\left\{\sum_{j=1}^{N}\beta_{ij}p_j\right\}$, where β_{ij} is a price-elasticity coefficient and $\beta_{ii}>0,\ \beta_{ij}\le 0$ for $j\neq i$. The main motivation behind such a model is that it follows intuitive monotonicity properties (increasing in the competitor's price and decreasing in the price of the corresponding retailer) and that the logarithm of the demand is linear in prices. We also adopt this type of model in the case of two competing retailers by modeling the expected demand at retailer i as

$$y_i(\mathbf{p})=ae^{-\beta p_i+\gamma p_{-i}},\qquad a,\beta,\gamma>0;\ \beta>\gamma. \tag{2}$$

Note that the expected demand at retailer i is a decreasing function of the retailer's own price p_i, and an increasing function of its competitor's price p_{-i}, where $-i=2$ if $i=1$ and $-i=1$ if $i=2$.

In this formulation, a is the primary demand of each retailer (i.e., demand if both prices were zero), β is each retailer's own price sensitivity of demand, and γ is the price sensitivity of demand with respect to the competitor's price. The larger the value of β (resp. γ), the more a retailer's expected demand is affected by a change in her own (resp. the competitor's) price. The assumption $\beta>\gamma$ indicates that sales at a given retailer are relatively more sensitive to price changes at the same retailer than at the competitor's, which is a standard assumption in economics when sellers are differentiated. Parameter γ is related to the level of retailer differentiation. As γ increases (approaches β), the retailers are less differentiated (more substitutable), therefore the degree of retail competition is intensifying (see Adida and DeMiguel [2011]). In other words, when γ is zero, the two retailers are completely differentiated and therefore there is no competition.

Retail competition is incorporated into the model in a way in which retailers make simultaneous price and quantity decisions that maximize their own objectives and tactically compete for market demand. Each retailer's objective depends on the decisions of the competitor via the demand function because the demand for the product at each retailer depends not only on its own price, p_i, but also on its competitor price, p_{-i}. Note that the retailers are not constrained to select the same retail prices. We assume that unmet demand is lost to all retailers, i.e., unsatisfied consumers from one retailer do not switch to another retailer but simply leave the market. This type of retail competition model via demand dependency on the competitor's price is common to many studies in the operation management literature (McGuire and Staelin [1983], Jeuland and Shugan [1988], Choi [1991, 1996], Pan et al. [2010], Yao et al. [2008b], Zhang [2008]).

Similarly to Vives [1984, 1985], Petruzzi and Dada [1999], we impose a mild restriction on the demand distribution known as the increasing failure rate (IFR) condition.

Assumption 3.1. The demand distribution satisfies the IFR property: $h(x) = f(x)/[1 - F(x)]$ is increasing in x.

In practice, it is possible for the supplier to offer different contract parameters to different retailers in the industry. However, Villas-Boas [2008] suggests that "if the manufactured products sold through different retailers are the same, then they should be set at the same wholesale price." Another reason to support the common use of this assumption is a policy of banning wholesale price discrimination (Meyer and Fischer [2004], Villas-Boas [2008], Hastings [2009]), in which the manufacturer or the supplier is constrained to set a uniform, nondiscriminatory wholesale price for a brand sold at any of the retailers. Moreover, the assumption that the single supplier offers two competing retailers identical contract terms (i.e., the same consignment price or revenue share) is common to many studies in the contract literature (Dana Jr. and Spier [2001], Yao et al. [2008a]. This assumption has also been made by a number of studies of two-part tariff in competitive supply chains (Tsay and Agrawal [2000], Xiao et al. [2005], Narayanan et al. [2005]). As a result, we also make the assumption that the supplier offers the same contract terms to the two competing retailers.

We model the decision making of this two-tier supply chain as a Supplier-Stackelberg game. Following the standard newsvendor model (Cachon [2003]), the following sequence of events takes place: (1) the supplier, acting as a leader, offers a contract specifying the terms of payment to him from the retailers upon sale of items to consumers; (2) each retailer, acting as a follower, chooses the quantity Q_i to order from the supplier and the retail price p_i; (3) before the start of selling season, the supplier produces $Q = Q_1 + Q_2$ units of the product and delivers Q_i units to retailer i, $i = 1, 2$; (4) demand realizes; (5) transfer payments are made between supplier and retailers according to the agreed contract.

In this study, the supplier and two retailers play, vertically, a Stackelberg game with the supplier as the leader and the two retailers as followers. Horizontally, the two retailers play a Nash game, i.e. they simultaneously decide their prices and stocking quantities. We solve this equilibrium problem to find the Stackelberg/Nash equilibrium. The next sections present equilibrium solutions for three types of contracts and derive their implications.

3.2. Price-only (PO) contracts

In this type of contract, the supplier charges each retailer a wholesale price w_p per unit ordered. The time of payment and the ownership of inventory are key differences between price-only and consignment contracts. In price-only contracts, the retailers have full ownership of the inventory ordered and thus bear all the risks for all unsold units. The supplier receives the payment for all the units ordered by retailers, regardless of whether the retailer sells them. However, in a consignment agreement, the supplier retains ownership of merchandise even though items are at retail locations. The supplier receives no payment until the items are sold by retailers. Therefore, the retailers incur no risk for any unsold units. We use the price-only (PO) contract as a benchmark for evaluating the performance of consignment contracts with two different payment schemes.

In this section, we adapt a PO contract in a single retailer situation Cachon and Lariviere [2005] to a setting with our demand model and with two competing retailers that act as followers. The sequence of events is as follows: (1) the supplier specifies the wholesale price w_p for each unit ordered; (2) each retailer i simultaneously selects the retail price p_i and order quantity Q_i; (3) demand is realized. We find the equilibrium solution by using backward induction. We first derive each retailer's best response price and inventory quantity to the supplier's wholesale price decision.

3.2.1. Retailer i's selling price and stocking factor best response At the second step of the decision sequence, for a given wholesale price w_p selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit:

$$\pi_{R_i}(p_i, Q_i|w_p) = p_i E\{\min(D_i, Q_i)\} - (c\alpha_i + w_p)Q_i. \tag{3}$$

Similarly to Petruzzi and Dada [1999], Wang et al. [2004], Ru and Wang [2010], we define $z_i = Q_i/y_i(\mathbf{p})$ the *stocking factor* of inventory. The stocking factor is defined as a surrogate for safety factor and is a measure of the deviation of the ordered quantity from the expected demand (see Petruzzi and Dada [1999] and the references therein). Using z_i as a decision variable instead of Q_i, we can rewrite retailer i's profit function (3) as

$$\begin{aligned}\pi_{R_i}(p_i, z_i|w_p) &= y_i(\mathbf{p})\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w_p)z_i\}\\ &= ae^{-\beta p_i + \gamma p_j}\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w_p)z_i\},\end{aligned}$$

where $\Lambda(z_i) = \int_A^{z_i}(z_i - x)f(x)\,dx$.

We provide two lemmas that will be useful in the remaining of the paper.

LEMMA 3.2. The quantity $z_i - \Lambda(z_i)$ is positive for any given stocking factor $z_i > 0$.

The following lemma is provided in Ru and Wang [2010].

LEMMA 3.3. Let $G(z_i) = \frac{1}{1-F(z_i)} - \frac{z_i}{z_i - \Lambda(z_i)}$. Under Assumption 3.1, $G(z_i)$ is increasing in z_i.

To find the best response, denoted by $(\bar{z}_i, \bar{p}_i)$, that maximizes $\pi_{R_i}(p_i, z_i|w_p)$ for a given w_p, we first derive the retailer's best response retail price $\tilde{p}_i(z_i|w_p)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)$. Note that $\bar{z}_i$ and $\bar{p}_i$ are functions of w_p but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

PROPOSITION 3.4. For any given stocking factor z_i, wholesale price $w_p > 0$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p_i}(z_i|w_p)$ is given by

$$\tilde{p_i}(z_i|w_p) = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)}. \tag{4}$$

Proposition 3.4 implies in particular that each retailer's best response price (for a given z_i and w_p) is independent of the competitor's price decision. A price strategy that is independent of the competitor's is a property that appears in previous literature. Moorthy [1988] and Choi [1991] found that the class of constant price elasticity (iso-elastic) demand functions, such as a multiplicative function $q_i = ap_i^{-\beta}p_j^{\gamma}$, $i,j = 1,2$; $i \neq j$, is known to result in price strategies that are independent of the competitor's strategy. Our demand model, $y_i(\tilde{\mathbf{p}}) = ae^{-\beta\tilde{p_i}+\gamma\tilde{p_j}}$ is, in fact, related to an iso-elastic demand function after a change of variable $\tilde{p_i} = log(p_i)$. Thus, this characteristic of the result in Proposition 3.4 is consistent with their findings.

According to Proposition 3.4, for a given stocking factor z_i and wholesale price w_p, retailer i's best response retail price $\tilde{p_i}(z_i|w_p)$ consists of two components: the first component $1/\beta$ is related to the sensitivity of consumers to price changes, and the second component $(c\alpha_i + w_p)z_i/(z_i - \Lambda(z_i))$ reflects the retailer's costs, that is, the wholesale price paid to the supplier and the holding cost, for each unit ordered. The first component increases in β because as consumers become more sensitive to price changes, the retailer lowers the price. The second component increases proportionately to the total cost per unit. Specifically, the effect of the retailer's costs on the retail price depends upon the ratio $z_i/(z_i - \Lambda(z_i)) = y_i(\mathbf{p})z_i/(y_i(\mathbf{p})(z_i - \Lambda(z_i)))$, representing the ratio of expected demand to the expected quantity sold. If this ratio is high, meaning that the retailer incurs a higher risk of over-ordering merchandise, then the retailer increases the retail price.

PROPOSITION 3.5. The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit $\pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)$ for a given w_p is uniquely determined as the solution of:

$$\frac{1}{(c\alpha_i + w_p)\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1 - F(\bar{z}_i)}. \tag{5}$$

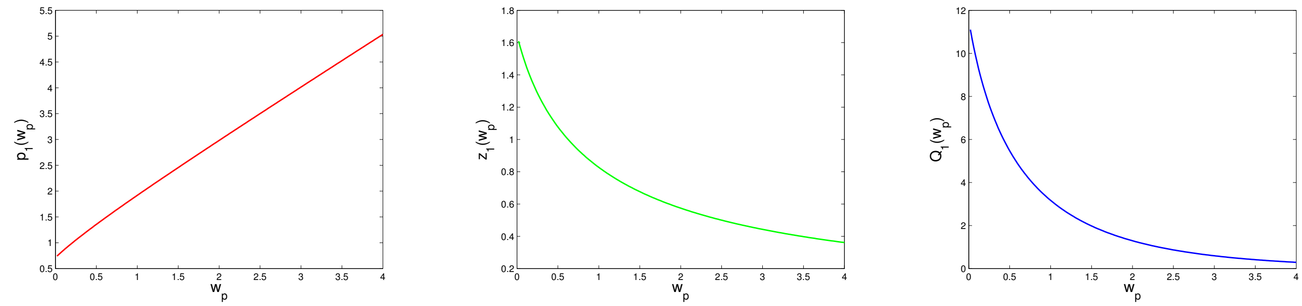
Similarly to the best response price, Proposition 3.5 suggests that the retailer's best response stocking factor $\bar{z}_i$ is independent of the competitor's stocking factor. Note that there is no closed form expression for $\bar{z}_i$. However, we are able to prove the following property.

COROLLARY 3.6. The best response stocking factor $\bar{z}_i$ is decreasing in w_p.

This result means that as the supplier charges the retailer more per item, the retailer orders less compared with the expected demand to lower her overstock risk exposure.

Using (4) and (5), we obtain that the best response retail price to a wholesale price w_p is

$$\bar{p}_i = \tilde{p_i}(\bar{z}_i|w_p) = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)}. \tag{6}$$



(a)Best response price $\bar{p_1}$ (b)Best response stocking factor $\bar{z_1}$ (c)Best response quantity $\bar{Q_1}$

Figure 1 **Retailer 1's best response price, stocking factor and quantity as a function of the wholesale price** w_p **when** $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ **in the PO contract**

Figure 1 illustrates the retailer's best response price, stocking factor and quantity as a function of the supplier's wholesale price. The retailer's best response price increases with w_p. This observation is intuitive because as the supplier's wholesale price increases, the retailer transfers this cost increase to consumers by increasing the retail price. The higher retail price causes the demand to go down, which leads to a lower quantity at each retailer. As a result, both the expected demand and quantity decrease with w_p. However, the order quantity decreases faster than the expected demand. Thus, the stocking factor decreases with the supplier's wholesale price (consistent with Corollary 3.6).

3.2.2. Supplier's wholesale price decision At the first step, anticipating the retailers' reaction, the supplier sets the wholesale price w_p to maximize her own expected profit:

$$\begin{aligned}\pi_S(w_p) &= w_p(\bar{Q_1} + \bar{Q_2}) - c(1 - \alpha_1 - \alpha_2)(\bar{Q_1} + \bar{Q_2}) \\ &= [w_p - c(1 - \alpha_1 - \alpha_2)]\{y_1(\bar{\mathbf{p}})\bar{z_1} + y_2(\bar{\mathbf{p}})\bar{z_2}\} \\ &= [w_p - c(1 - \alpha_1 - \alpha_2)]\{ae^{-\beta\bar{p_1} + \gamma\bar{p_2}}\bar{z_1} + ae^{-\beta\bar{p_2} + \gamma\bar{p_1}}\bar{z_2}\}.\end{aligned}$$

To find the equilibrium solution w_p^*, we seek to maximize $\pi_S(w_p)$ over w_p. Since $\bar{z_i}$ and $\bar{p_i}$ are only known as implicit functions of w_p given by (5) and (6), this problem has no analytical solution. Thus, we find w_p^* numerically; numerical results are discussed in Section 4.

3.3. Consignment price (CP) contracts

In the consignment setting, the supplier retains full ownership of the inventory that is placed at retailers'. Therefore, the supplier bears all the risk associated with demand uncertainty while the retailers incur only a holding cost for over-ordered merchandise. Two types of consignment contract exist in the literature. In this section, we consider *consignment price* (CP) contracts. We will discuss the second type, *consignment contract with revenue share* (CR) in the next section.

Our model is different from previous studies (Wang [2006], Zhang [2008]) in that we consider consignment contracts *with retail competition*. Furthermore, our focus differs from Wang et al. [2004] due to the fact that we focus on retail-managed inventory, meaning that retailers decide the inventory quantity. This agreement is commonly used in supply chains (Gümüs et al. [2008]). A prime example of such a setting is seen in the automotive market. For example, AutoZone, Inc., one of the biggest auto parts resellers, operates under a consignment contract (called pay-on-scan agreements) with suppliers.

Under CP contracts, decisions are made in two sequential steps. At the first step, the supplier decides the consignment price w corresponding to the amount of payment to be received from the retailers for each unit sold to consumers. At the second step, given this consignment price, each retailer simultaneously selects the retail price p_i and order quantity Q_i. We find the equilibrium solution by using backward induction. We first derive each retailer's best response price and inventory quantity to the supplier's consignment price decision.

3.3.1. Retailer i's selling price and stocking factor decision At the second step of the decision sequence, for a given consignment price w selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit:

$$\pi_{R_i}(p_i, Q_i|w) = [p_i - w]E\{min(D_i, Q_i)\} - c\alpha_i Q_i.$$

Notice that, in contrast with (3) for a PO contract, the price w paid to the supplier applies only to sold quantities and not to ordered quantities. Since $z_i = Q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned}\pi_{R_i}(p_i, z_i|w) &= y_i(\mathbf{p})\{(p_i - w)(z_i - \Lambda(z_i)) - c\alpha_i z_i\}\\ &= ae^{-\beta p_i + \gamma p_j}\{(p_i - w)(z_i - \Lambda(z_i)) - c\alpha_i z_i\}.\end{aligned}$$

To find the best response, denoted by $(\bar{p}_i, \bar{z}_i)$, that maximizes $\pi_{R_i}(p_i, z_i|w)$ for a given w, we first derive the retailer's best response retail price $\tilde{p}_i(z_i|w)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|w), z_i|w)$. Note that $\bar{z}_i$ and $\bar{p}_i$ are functions of w but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

Proposition 3.7. For any given stocking factor z_i, consignment price $w > 0$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|w)$ is given by

$$\tilde{p}_i(z_i|w) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{z_i - \Lambda(z_i)} + w. \tag{7}$$

For a given stocking factor z_i and consignment price w, the retailer's best response price $\tilde{p}_i(z_i|w)$ consists of two components. Similarly to the PO contract, the first component $1/\beta$ is related to the sensitivity of consumers to price changes. The second component $c\alpha_i z_i/(z_i - \Lambda(z_i)) + w$ reflects the retailer's total cost, including the holding cost for each unit ordered and the consignment price paid to the supplier for each unit sold. The effect of the retailer's holding cost on the retail price depends upon the ratio $z_i/(z_i - \Lambda(z_i))$, which is related to the risk of excess inventory. The effect of the consignment price, however, is independent of this ratio. This is because under the CP contract, each retailer only pays the consignment price to the supplier for each unit sold (not for each unit ordered). The retailer incurs no risk of loss associated with unsold merchandise, thus the retail price increases with an increase in the consignment price w, regardless of how the quantity ordered compares with the quantity sold.

Comparing the best response retail price in a CP contract (7) and a PO contract (4), we observe that for a fixed stocking factor z_i and a given supplier's price $w_p = w$, the PO best response retail price is higher than the CP best response retail price in a consignment price contract by $\frac{z_i}{z_i - \Lambda(z_i)}$. This finding reflects the fact that in a PO contract, the retailers incur more risk associated with over-ordered products than in a CP contract, and therefore charge consumers a higher retail price.

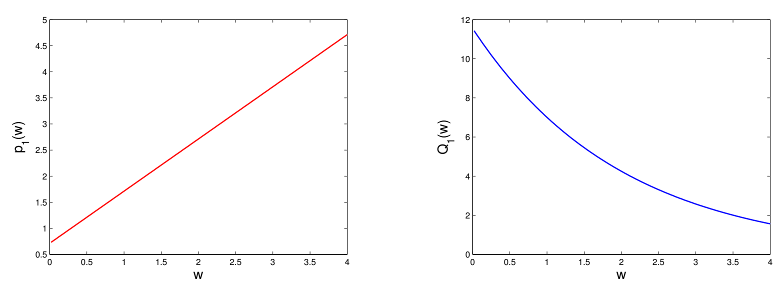
Proposition 3.8. The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit $\pi_{R_i}(\tilde{p}_i(z_i), z_i|w)$ for a given w is uniquely determined as the solution of:

$$\frac{1}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1 - F(\bar{z}_i)}. \tag{8}$$

Proposition 3.8 implies that $\bar{z}_i$ does not depend on the supplier's consignment price w, thus $\bar{z}_i$ is the retailer i's equilibrium stocking factor z_i^*. In particular, using (7), this implies that $\tilde{p}_i(z_i|w) - w$ is independent of w.

Using (7) and (8), we find that the best response retail price to a consignment price w is

$$\bar{p}_i = \tilde{p}_i(\bar{z}_i|w) = \frac{1}{\beta} + \frac{c\alpha_i\bar{z}_i}{z_i - \Lambda(\bar{z}_i)} + w. \tag{9}$$



(a)Best response price $\bar{p}_1$ (b)Best response quantity $\bar{Q}_1$

Figure 2 Retailer 1's best response price and quantity as a function of the consignment price w when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ in the CP contract

Figure 2 illustrates the retailer's best response price and quantity as a function of the consignment price. The best response retail price increases with w. Similarly to the PO contract, the retailers transfer any consignment price increase to consumers by increasing their retail prices, which causes the demand to decrease and thus the quantity to decrease.

3.3.2. Supplier's consignment price decision At the first step, anticipating the retailers' reaction to her decision, the supplier sets the consignment price w to maximize her own expected profit $\pi_S(w)$, given by

$$\begin{aligned}\pi_S(w) &= w[E\{min(D_1,\bar{Q}_1)\} + E\{min(D_2,\bar{Q}_2)\}] - c(1-\alpha_1-\alpha_2)[\bar{Q}_1+\bar{Q}_2]\\ &= w[y_1(\bar{\mathbf{p}})(\bar{z}_1-\Lambda(\bar{z}_1)) + y_2(\bar{\mathbf{p}})(\bar{z}_2-\Lambda(\bar{z}_2))] - c(1-\alpha_1-\alpha_2)[y_1(\bar{\mathbf{p}})\bar{z}_1 + y_2(\bar{\mathbf{p}})\bar{z}_2]\\ &= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[w(\bar{z}_1-\Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[w(\bar{z}_2-\Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2].\end{aligned}$$

To find the equilibrium solution, denoted by w^*, we maximize $\pi_S(w)$ over w.

PROPOSITION 3.9. The supplier's unique equilibrium consignment price w^* is given by

$$w^* = \frac{k_1[(\bar{z}_1-\Lambda(\bar{z}_1)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_1] + k_2[(\bar{z}_2-\Lambda(\bar{z}_2)) + (\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar{z}_2]}{(\beta-\gamma)[k_1(\bar{z}_1-\Lambda(\bar{z}_1)) + k_2(\bar{z}_2-\Lambda(\bar{z}_2))]}$$

where $k_i = e^{-c\left(\frac{\beta\alpha_i\bar{z}_i}{\bar{z}_i-\Lambda(\bar{z}_i)} - \frac{\gamma\alpha_{-i}\bar{z}_{-i}}{\bar{z}_{-i}-\Lambda(\bar{z}_{-i})}\right)}$, $i=1,2$.

We interpret the result above in a special case.

3.3.3. Special case: Consignment price contracts for symmetric retailers We now consider the case where the two retailers have symmetric cost structure, i.e., $\alpha_1=\alpha_2\equiv\alpha$, where $0<\alpha<0.5$. It follows from Proposition 3.8 that retailer i's equilibrium stocking factor z_i^* satisfies $z_1^*=z_2^*\equiv z^*$ where

$$\frac{1}{c\alpha\beta} + \frac{z_i^*}{z^*-\Lambda(z^*)} = \frac{1}{1-F(z^*)}. \tag{10}$$

It follows from Proposition 3.9 that, since $k_1=k_2=e^{-\frac{c(\beta-\gamma)\alpha z^*}{z^*-\Lambda(z^*)}}$, the supplier's unique equilibrium consignment price w^* is given by

$$w^* = \frac{1}{\beta-\gamma} + \frac{c(1-2\alpha)z^*}{z^*-\Lambda(z^*)}. \tag{11}$$

Let $p_i^* = \tilde{p}_i(z_i^*|w^*)$; It follows from Proposition 3.8 that $p_1^*=p_2^*=p^*$ where

$$p^* = \frac{1}{\beta} + \frac{1}{\beta-\gamma} + \frac{c(1-\alpha)z^*}{z^*-\Lambda(z^*)}. \tag{12}$$

Notice that at equilibrium, the retailers' margin

$$p^*-w^* = \frac{1}{\beta} + \frac{c\alpha z^*}{z^*-\Lambda(z^*)}$$

does not depend on the competitor's price sensitivity γ. Therefore, the level of retail competition affects the retail price only through the consignment price selected by the supplier.

We first focus on the impact of the price sensitivity on the supplier's equilibrium consignment price and the retailers' equilibrium retail price.

PROPOSITION 3.10.

- The equilibrium stocking factor z^* decreases in β.
- The equilibrium supplier's consignment price w^* decreases in β.
- The equilibrium retail price p^* decreases in β.

Proposition 3.10 indicates that the equilibrium stocking factor z^* decreases with the consumers' sensitivity to the retail price. Since the expected demand decreases when consumers become more sensitive to the retail price, retailers reduce their order quantity to reduce the risk of excess inventory. Specifically, the quantity ordered by each retailer Q^* decreases in β faster than the (expected) demand $y^*(\mathbf{p}^*)$. Furthermore, the consignment price w^* and the retail price p^* are decreasing functions of β: as consumers are more sensitive to the retail price, the supplier charges each retailer a lower consignment price so that retailers can lower their retail prices.

We now focus on the impact of retailer differentiation. Since the equilibrium stocking factor z_i^* is independent of the price cross-sensitivity, we study how the supplier's equilibrium consignment price w^*, the retailer's equilibrium selling price p_i^*, the retailer's equilibrium profit, the supplier's equilibrium profit and the total profit of the channel vary with the level of retailer differentiation.

Proposition 3.11.

- The supplier's consignment price at equilibrium w^* increases in γ.
- The retail price at equilibrium p^* increases in γ.
- The ratio p^*/w^* decreases in γ.
- The retailer's order quantity at equilibrium Q^* increases in γ.
- The retailer's profit at equilibrium $\pi_R^{d^*}$ increases in γ.
- The supplier's profit at equilibrium $\pi_S^{d^*}$ increases in γ.

Proposition 3.11 indicates that the supplier's consignment price increases in the price cross-sensitivity. This suggests that the supplier takes advantage of the increased competitiveness between less differentiated retailers (large γ) by charging a higher consignment price. The retailers transfer this price increase to consumers by increasing their retail price. This result is consistent with several existing studies (Jeuland and Shugan [1988], Choi [1991]). Moreover, the ratio of the retail price to the consignment price p^*/w^* decreases when retailer differentiation decreases, implying that the retail price does not increase as fast as the consignment price when retailers are less differentiated. Furthermore, Proposition 3.11 indicates that the quantity ordered by each retailer increases in γ. The effect of retail differentiation on the order quantity is subject to two opposing effects: a direct effect, and an indirect effect through the retail price. On the one hand, the direct effect of an increase of γ is to *increase* the (expected) demand which could drive the ordered quantity to go up. On the other hand, as γ increases, the retail price increases which tends to make the (expected) demand *decrease* and thus would drive the quantity to go down. Because the direct effect is stronger, overall the order quantity increases when γ increases. Since both the supplier's consignment price, the retailers' selling price and the order quantity increase in γ, the profits for the supplier and the retailers increase as the level of retailer differentiation decreases.

We can quantify the effect of retail competition on the retailers' share of the decentralized channel profits (Lariviere and Porteus [2001])

$$\frac{\pi_{R_1}^{d^*}+\pi_{R_2}^{d^*}}{\pi_S^{d^*}}=1-\frac{\gamma}{\beta}.$$

Note that the retailers' share of the channel profits is linearly decreasing in γ but increasing in β. That is, the retailers jointly earn proportionally less in channel profits when the competition is more intense, as expected. However, the retailers collect a larger share of channel profits when the demand is more sensitive to a change in their own prices. Surprisingly, the retailers' share of channel profits does not depend on the retailers' cost αc.

3.4. Consignment contracts with revenue share (CR)

In this section, we consider another type of consignment contract, known as *consignment contract with revenue share* (CR). Under CR contracts, decisions are made in two sequential steps. At the first step, the supplier decides the revenue share r of the retailers' revenue that she will receive for each unit sold to consumers. At the second step, given this revenue share, each retailer simultaneously selects the retail price p_i and order quantity Q_i. We find the equilibrium solution by using backward-induction. We first derive each retailer's best response price and inventory quantity to the supplier's revenue share decision.

3.4.1. Retailer i's selling price and stocking factor decision At the second step, for a given revenue share r selected by the supplier, retailer i selects the retail price p_i and order quantity Q_i to maximize her own expected profit which is given by

$$\pi_{R_i}(p_i,Q_i|r)=(1-r)p_iE\{min(D_i,Q_i)\}-c\alpha_iQ_i.$$

Since $z_i=Q_i/y_i(\mathbf{p})$, the profit can be rewritten as

$$\begin{aligned}\pi_{R_i}(p_i,z_i|r)&=y_i(\mathbf{p})\{(1-r)p_i(z_i-\Lambda(z_i))-c\alpha_iz_i\}\\&=ae^{-\beta p_i+\gamma p_{-i}}\{(1-r)p_i(z_i-\Lambda(z_i))-c\alpha_iz_i\}.\end{aligned}\tag{13}$$

To find the best response, denoted by $(\bar{p}_i,\bar{z}_i)$, that maximizes $\pi_{R_i}(p_i,z_i|r)$ for a given r, we first derive the retailer's best response retail price $\tilde{p}_i(z_i|r)$ for a given stocking factor z_i; we then find the best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|r),z_i|r)$. Note that $\bar{z}_i$ and $\bar{p}_i$ are functions of r but we omit to explicitly show the dependency to keep the notation simpler. The results are summarized in the following propositions.

Proposition 3.12. For any given stocking factor z_i, revenue sharing proportion $0<r<1$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|r)$ is given by

$$\tilde{p}_i(z_i|r)=\frac{1}{\beta}+\frac{c\alpha_iz_i}{(1-r)(z_i-\Lambda(z_i))}.\tag{14}$$

For a given stocking factor z_i and revenue share r, the retailer's best response price $\tilde{p}_i(z_i|r)$ consists of two components. Similarly to the PO and CP contracts, the first component $1/\beta$ is related to the consumers' sensitivity to price changes. The second component $\frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}$ reflects the retailer's cost for each unit ordered. The effect of the retailer's cost on the retail price depends upon the ratios $\frac{z_i}{z_i-\Lambda(z_i)} = \frac{y_i(\mathbf{p})z_i}{y_i(\mathbf{p})(z_i-\Lambda(z_i))}$ and $\frac{1}{1-r}$. The first ratio represents the risk of excess inventory. The second ratio can be considered a "markup" factor due to the revenue share owed to the supplier.

PROPOSITION 3.13. The retailer i's best response stocking factor $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)$ for a given revenue share r is uniquely determined as the solution of:

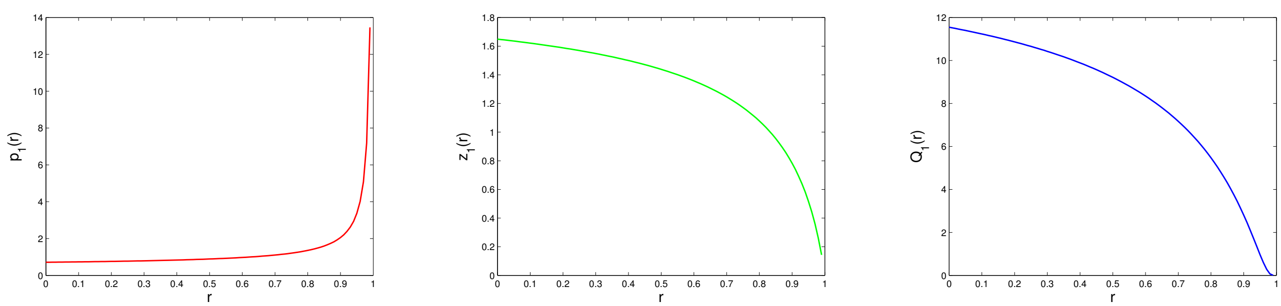
$$\frac{1-r}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1-F(\bar{z}_i)}. \tag{15}$$

Proposition 3.13 indicates that the retailer's best response stocking factor does depend on the supplier's decision in a CR contract, as opposed to a CP contract (Proposition 3.8). One explanation for this difference is that at the best response in a CP contract the retailer's margin is independent of the supplier's decision, as noted in Section 3.3.3, while in a CR contract it depends on r.

Using (14) and (15), we find that the best response retail price to a revenue share r is

$$\bar{p}_i = \tilde{p}_i(\bar{z}_i|r) = \frac{1}{\beta} + \frac{c\alpha_i\bar{z}_i}{(1-r)(\bar{z}_i - \Lambda(\bar{z}_i))}. \tag{16}$$

COROLLARY 3.14. The best response stocking factor $\bar{z}_i(r)$ is decreasing in r.



(a)Best response price $\bar{p}_1$ (b)Best response stocking factor $\bar{z}_1$ (c)Best response quantity $\bar{Q}_1$

Figure 3 Retailer 1's best response price, stocking factor and stocking quantity as a function of the revenue share r where $\beta = 2.0,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ in the CR contract

Figure 3 illustrates that the retailer's best response retail price increases with the supplier's revenue share r: when the supplier keeps a higher share of the retailers' revenue, the retailers transfer this increasing revenue loss to consumers. The higher retail price causes the demand to go down, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's revenue share r, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's revenue share (consistent with Corollary 3.14).

3.4.2. Supplier's revenue sharing fraction decision At the first step, anticipating the retailers' reaction to her decision, the supplier sets the revenue sharing fraction r to maximize her own expected profit $\pi_S(r)$, given by

$$\begin{aligned}\pi_S(r) &= r\bar{p}_1 E\{\min(D_1, \bar{Q}_1)\} - c(1-\alpha_1-\alpha_2)\bar{Q}_1 + r\bar{p}_2 E\{\min(D_2, \bar{Q}_2)\} - c(1-\alpha_1-\alpha_2)\bar{Q}_2 \\ &= y_1(\bar{\mathbf{p}}))[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] + y_2(\bar{\mathbf{p}}))[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2] \\ &= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[r\bar{p}_1(\bar{z}_1 - \Lambda(\bar{z}_1)) - c(1-\alpha_1-\alpha_2)\bar{z}_1] \\ &\quad + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[r\bar{p}_2(\bar{z}_2 - \Lambda(\bar{z}_2)) - c(1-\alpha_1-\alpha_2)\bar{z}_2]. \end{aligned} \tag{17}$$

To find the equilibrium solution, denoted by r^*, we would have to maximize $\pi_S(r)$ over r. Obtaining an analytical solution for this maximization problem is intractable, therefore, we use numerical methods as shown in Section 4.

4. Numerical analysis

In this section, we obtain numerically the equilibrium quantities that could not be obtained in closed-form in previous sections, and we interpret the findings. Plots are included in Appendix B.

Our numerical study is geared at understanding the impact of price sensentivity parameters β and γ on the performance of consignment contracts in comparison to the price-only contract. It is common in the literature to focus on the impact of such parameters (Yao et al. [2008a,b], Lau and Lau [2002]). We first investigate the effect of the price sensitivity parameter β on the equilibrium decisions and profits. In order to properly evaluate this effect, we need to isolate it from the effect of other parameters (such as the cross-price sensitivity γ and each retailer's share of the channel cost α_i) by keeping all these parameters constant. Likewise, when we next consider the effect of retailer differentiation on the equilibrium decisions and profits through parameter γ, we keep β and α_i constant. In both cases, we choose values of α_1 and α_2 that are equal to avoid introducing a cost difference that could bias the effect of the parameter of interest.

The random perturbation on the demand, ϵ, is assumed to follow the uniform distribution on $[A, B]$. Following previous numerical studies (Choi [1991], Li et al. [2009], Zhang et al. [2010]), we set $a = 10$, $c = 1$, and $\alpha_1 = \alpha_2 = 0.125$. Moreover, we choose $A = 0$ and $B = 2$ in order to ensure that the perturbation on the demand has a mean value of 1.

4.1. The effect of the price sensitivity factor

We study the impact of the price sensitivity parameter β on the supplier's equilibrium price (PO and CP contracts), equilibrium revenue share (CR contract), equilibrium retail price, equilibrium quantity and equilibrium profits. The values of the parameters are chosen to ensure that $\beta > \gamma$. The value of γ is fixed at $\gamma = 2$.

Figure 4(a) demonstrate that the supplier's wholesale price (PO contract), consignment price (CP contract), and revenue share (CR contract) decrease in β. Indeed, the supplier must decrease the price/ revenue share charged to the retailers when consumers are more sensitive to price changes. As a result, the retail price decreases in β under all types of contracts, as seen in Figure 4(b), because the retailers transfer their decreased loss of revenue to consumers. The fact that the consignment price and the retail price decrease in β under the CP contract is consistent with Proposition 3.10.

The effect of the price sensitivity parameter β on the order quantity Q_i^* depends on the type of contract. An increase of β drives the (expected) demand to decrease, everything being kept constant (direct effect). On the other hand, as β increases, retail prices decrease, which could cause the (expected) demand to increase (indirect effect). The cumulative effect of β is thus a combination of two opposite effects. Figure 5(a) illustrates that in the CP contract, the direct effect is stronger, leading to a decrease of the order quantity in β. However, in the PO and CR contracts the order quantity is not a monotonic function of β: it first increases then decreases. One explanation is that the direct effect of β (which causes the demand to decrease) dominates the the indirect effect of β (which drives the demand to increase) when price sensitivity β is low. However, the indirect effect prevails when price sensitivity β is high.

Figure 5(b) shows the effect of the price sensitivity parameter on the stocking factor. The stocking factor in the PO and CR contracts is not monotonic with β, due to the non-monotonicity of the order quantity with β. The stocking factor in the CP contract, however, decreases in β, which is consistent with Proposition 3.10.

Figure 6 depicts the effect of β on the retailers' profits. The retailers' profits decrease in β under the CP contract because the retail price and the order quantity decrease in β. The retailers' profits under the PO and CR contracts, however, do not display a monotonic pattern because the order quantity is not monotonic. No contract yields higher profits to the retailers than the other two contracts for any level of price sensitivity. Specifically, when the price sensitivity is low, the CP contract yields a higher profit to the retailers than both the PO and the CR contracts; when the price sensitivity is high, the PO yields higher retailers' profits than both consignment contracts.

The supplier's profit decreases in β in all contracts since the supplier's price/ revenue share decreases in β, as depicted in Figure 7. Surprisingly, the share of the channel profits for the retailers $\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$ increases in β for all contracts (Figure 8). This indicates that the supplier's profit decreases in β at a higher rate than the retailers' profits.

The effect of the price sensitivity parameter on the decisions and profits for each of the supply chain members is summarized in Table 1.

4.2. The effect of retailer differentiation

The major difference between our work and previous studies in consignment contracts Wang et al. [2004], Wang [2006], Zhang [2008], Ru and Wang [2010] is that we incorporate retailer differentiation into our model. The price cross-sensitivity γ represents the price sensitivity with respect

Decisions and profits	PO contract	CP contract	CR contract	Remark
w_p^* or w^* or r^*	decreasing	decreasing	decreasing	$w^* > w_p^*$
p_i^*	decreasing	decreasing	decreasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*}$
Q_i^*	not monotonic	decreasing	not monotonic	$Q_i^{CP^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for small β $Q_i^{CR^*} > Q_i^{CP^*} > Q_i^{PO^*}$ for large β
z_i^*	not monotonic	decreasing	not monotonic	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	decreasing	not monotonic	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for small β $\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$ for large β
π_S^*	decreasing	decreasing	decreasing	$\pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for small β $\pi_S^{CR^*} > \pi_S^{CP^*} > \pi_S^{PO^*}$ for large β
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	increasing	increasing	increasing	$PO > CP > CR$

Table 1 The effect of the price sensitivity parameter β on the equilibrium decisions and profits

to the competitor's price, and captures retailer differentiation. That is, if the retailers are less differentiated, then γ is larger (closer to β).

The values of the parameters are chosen to ensure that $\beta > \gamma$. The value of β is fixed at $\beta = 4$.

We now examine the effect of retailer differentiation on equilibrium prices, quantities and profits. Figure 9 suggests that the supplier's wholesale price (PO contract), the consignment price (CP contract) and the revenue share (CR contract) increase in γ. This indicates that the supplier takes advantage of lower retailer differentiation (a higher value of γ) by increasing her price or revenue share. The retail price p_i^*, in all three contracts, also increases in γ. This reflects the fact that retailers transfer to consumers their increased supplier costs, and is consistent with the intuition that consumer surplus is lower when retailers are less competitive. Since the consignment price in a CP contract is always higher than the wholesale price in a PO contract, the retail price is also higher in a CP contract than in a PO contract, for any level of retailer differentiation.

Figure 10 depicts the effect of retailer differentiation on the order quantity Q_i^*. This effect depends on the type of contract. An increase of γ drives the (expected) demand to increase, everything being kept constant (direct effect). On the other hand, as γ increases, retail prices increase, which could cause the (expected) demand to decrease (indirect effect). The cumulative effect of γ is thus a combination of two opposite effects. Figure 10 illustrates that in the CP contract, the direct effect is stronger, leading to an increase of the order quantity in γ (which is consistent with Proposition 3.11). However, in the PO and CR contracts the order quantity is not a monotonic function of γ: it first increases then decreases. One explanation is that the direct effect of γ (which drives the demand to increase) dominates the the indirect effect of γ (which causes the demand to decrease) when the level of retailer differentiation is strong (small values of γ). On the other hand, when retailer differentiation is less intense (larger values of γ), the indirect effect prevails.

Figure 11 demonstrates that the effect of retailer differentiation on the stocking factor depends on the contract. The stocking factor in the PO and CR contracts decreases in γ while in the CP contract it is independent of γ. The explanation is that under the PO and the CR contracts, the best response stocking factor depends on the supplier's wholesale price w_p and revenue share r, respectively. However, the best response stocking factor in a CP contract does not depend on the consignment price w. Therefore retailer differentiation affects the equilibrium stocking factor in the PO and the CR contracts through the equilibrium wholesale price and revenue share, respectively, but has no effect on the equilibrium stocking factor in the CP contract. The fact that the stocking factor in the PO and CR contracts decreases in γ means that the order quantity does not increase as fast as the (expected) demand when the level of retailer differentiation decreases.

Since the retail price and the order quantity in the CP contract increase when the level of retailer differentiation decreases, the retailers' profits, consequently, increases in γ, as illustrated in Figure 12, which is consistent with Proposition 3.11. The retailers' profits under the PO and CR contracts, however, are non-monotonic functions of γ because the order quantity is not monotonic. No contract yields higher profits to the retailers than the other two contracts for any level of retailer differentiation. Specifically, when the retailer differentiation is high (small γ), the PO contract yields a higher profit to the retailers than both consignment contracts; when the retailer differentiation is low, the CP yields higher retailers' profits than the PO and CR contracts.

Figure 13 indicates that the supplier's profit under all types of contracts increases in γ, meaning that the supplier profits more when the retailers are less differentiated, which is consistent with Proposition 3.11 (in a CP contract with symmetric retailers). No contract yields higher profits to the supplier than the other two contracts for any level of retailer differentiation. If retailer differentiation is strong, the CR contract is the most beneficial; otherwise the CP contract yields highest supplier profit.

The retailers' share of the total channel profits, $\frac{\pi^*_{R_1}+\pi^*_{R_2}}{\pi^*_S}$, as illustrated in Figure 14, decreases in γ, implying that as retailer differentiation decreases, the retailers' share of the total channel profits decreases under all contracts. In particular, when the retailers are completely differentiated and there is thus no competition among them ($\gamma = 0$), the retailers jointly earn close to 100% of the total channel profits in the PO and CP contracts. As the level of retailer differentiation decreases, the retailers compete more and their profits do not increase as fast as the supplier's profit. Furthermore, under any level of retailer differentiation, the retailers earn a higher share of the total profits under the PO than under either the CP or CR contracts.

The retailers' share of the total channel profits decreases as the level of retailer differentiation decreases, implying that the supplier collects a larger share of the total profits than the retailers as γ increases. Thus, the supplier's profit has a stronger impact on the total channel profits than the retailers'. As a result, the total channel profits exhibit the same trend as the supplier's profit. That is, the total channel profits under all contracts increase in γ (shown in Fig. 15). When the level of retailer differentiation is high, the CR contract yields higher total channel profits than both the PO and the CP contracts. As the level of retailer differentiation decreases, the CP contract prevails over the other contracts.

The effect of retailer differentiation on the decisions and the profits for each of the supply chain members (with a fixed $\beta = 4$) is summarized in Table 2.

Decisions and profits	PO contract	CP contract	CR contract	Remark
w_p^* or w^* or r^*	increasing	increasing	increasing	$w^* > w_p^*$
p_i^*	increasing	increasing	increasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*}$
Q_i^*	not monotonic	increasing	not monotonic	$Q_i^{CR^*} > Q_i^{PO^*} > Q_i^{CP^*}$ for small γ $Q_i^{CP^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for large γ
z_i^*	decreasing	independent	decreasing	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	increasing	not monotonic	$\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$ for small γ $\pi_{R_i}^{CP^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for large γ
π_S^*	increasing	increasing	increasing	$\pi_S^{CR^*} > \pi_S^{PO^*} > \pi_S^{CP^*}$ for small γ $\pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for large γ
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	decreasing	decreasing	decreasing	$PO > CP > CR$

Table 2 The effect of parameter γ on the equilibrium decisions and profits

4.3. The effect of the price sensitivity and retailer differentiation in a supply chain with 10 retailers

Thus far, we have considered three different contracts (the PO contract, the CP contract, and the CR contract) under a supply chain with one supplier and two competing retailers. In the previous section, we solved for the equilibrium solutions numerically and we drew managerial insights on the effect of the price sensitivity factor and the effect of retail differentiation on the equilibrium decisions and profits of the supply chain members. In practice, however, there are often more than two retailers in competition. One question of interest is whether our conclusions remain valid in the case of multiple (more than two) retailers. In this section, we consider an extension of our results to the case of 10 competing retailers, and we investigate numerically whether our findings still hold. The details of this study can be found in Appendix E and the results are summarized below.

We obtain that the conclusions of the effect of the price sensitivity on the equilibrium decisions and profits of the supply chain members in the case of two retailers (as summarized in Table 1) remain valid for the case of 10 retailers. For example, we still see that the retail price is highest in the CP contract, for any level of price sensitivity. Moreover, the effect of the price sensitivity parameter on the order quantity varies, depending on the type of contract. That is, the order quantity in the CP contract decreases in β but the order quantity, in the PO and CR contracts, is not monotonic.

The results on the effect of retailer differentiation on the equilibrium decisions and profits for a channel of one supplier and 10 retailers lead us to conclusions similar to those obtained in the case of two retailers (as summarized in Table 2). For instance, we still observe that the consignment price in the CP contract is always higher than the wholesale price in the PO contract, regardless of level of retailer differentiation. Furthermore, the retailers' share of the total channel profits is highest in the PO contract, for any level of retailer differentiation.

Moreover, we observe that, as intuitively expected, increasing number of retailers (i.e., an increasing level of retail competition) increases the supplier's profit for all contracts as the supplier exploits the increased competition among retailers.

5. Conclusion

Consignment contracts have received increased attention in the recent Supply Chain Management literature. While upstream competition has been discussed, downstream retailer competition has not. Our study contributes to research in consignment contracts and retail competition by providing insights on how the presence of retail competition and retailer differentiation affect the decisions and performance of the supply chain. In this paper, we build a game-theoretic model in order to analyze the channel decisions and performance in three different contracts: price-only, consignment price, and consignment with revenue share contracts. We summarize our findings below.

(1) There is no particular type of contract that dominates the others from all players' perspective. The benefit of each consignment contract critically depends upon the level of retailer differentiation. The CP contract is preferable to all supply chain members when retailer differentiation is weak. When retailer differentiation is strong, the PO contract yields higher profits to the retailers than the two types of consignment contracts. The supplier, however, earns the highest profit in the CR contract when retailer differentiation is strong.

(2) In order to understand how the different payment terms in consignment contracts affect the decisions of the channel members and supply chain performance, we consider two different payment schemes: fixed (CP contract) and proportional (CR contract). The comparison of these two types of contracts is summarized in Table 3 (Appendix C). Our numerical study shows that the CP contract yields higher profits to the retailers than the CR contract, regardless of the level of retailer differentiation. The benefit of each type of consignment contract to the supplier, however,

depends upon the level of retailer differentiation. When retailer differentiation is strong, the CR contract yields a higher profit to the supplier; when it is weak, the CP contract is more beneficial.

(3) The effect of retailer differentiation on the decisions of the supplier and the retailers are as follows: with less retailer differentiation, the supplier increases the price (wholesale or consignment) or revenue share charged to the retailers. An increase of the supplier's price or revenue share leads the retailers to increase the retail price. The order quantity in the CP contract increases when the level of retailer differentiation decreases, but it is non monotonic in the PO and CR contracts.

(4) The supplier earns a higher profit when the level of retailer differentiation decreases, for all three contracts. The retailers also earn a higher profit when the level of retailer differentiation decreases in the CP contract; however, the retailers' profits in the PO and CR contracts is not monotonic with retailer differentiation. Furthermore, our numerical study suggests that the benefits of the a lower retailer differentiation are not equally distributed across all supply chain members: the retailers collect a smaller share of the total profits as the differentiation decreases.

Clearly, the results and insights obtained in our paper are based on a specific demand function. As suggested in Wang et al. [2004], Wang [2006], obtaining closed form solutions for other demand functional forms, namely the linear and additive demand model, is intractable even for the simplest setting with one retailer and one supplier. However, Wang et al. [2004] conduct numerical experiments and show that the properties and insights generated from their iso-price-elastic and multiplicative demand model still hold strongly for other demand models such as the linear with multiplicative demand model. For future work, it would be interesting to examine whether their conclusion still remains valid for the case of two retailers.

Our model focuses on a situation in which the retailers choose the consigned inventory level. This setting is known as Retailer Managed Inventory (RMI). In future research, one could consider a setting where the supplier makes that decision (Vendor Managed Inventory or VMI) (Wang et al. [2004], Wang [2006], Ru and Wang [2010]). We also assume that the retailers face symmetric demands. It may be of interest to study the effect of demand asymmetry among retailers on decisions and profits. Finally, this model could be extended to a situation in which the supplier faces multiple competing retailers or the retailer faces multiple competing suppliers, and possibly to an even more general setting with multiple agents at both levels of the supply chain.

References

Adida, E., V. DeMiguel. 2011. Supply chain competition with multiple manufacturers and retailers. *Operations Research* **59**(1) 156–172.

Aviv, Y., A. Federgruen. 1998. The operational benefits of information sharing and vendor managed inventory (VMI) programs. John M. Olin School of Business working paper, Washington University, St. Louis, MO.

Bernstein, F., A. Federgruen. 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Management Science* **51**(1) 18–29.

Besbes, O., C. Maglaras. 2009. Revenue optimization for a make-to-order queue in an uncertain market environment. *Operations Research* **57**(6) 1438–1450.

Blackstone, J. H. Jr., J. F. Cox, eds. 2005. *APICS Dictionary*. Eleventh ed. APICS, Virginia.

Boorstin, J. 2003. An auto-parts store your mother could love. *Fortune* **148**(10) 163.

Cachon, G. P. 2003. Supply chain coordination with contracts. S. C. Graves, A. G. Kok, eds., *Handbook in Operations Research and Management Science: Supply Chain Management*. Elsevier, Amsterdam, 229–339.

Cachon, G. P. 2004. The allocation of inventory risk in a supply chain: push, pull, and advance-purchase discount contracts. *Management Science* **50**(2) 222–238.

Cachon, G. P., A. G. Kok. 2010. Competing manufacturers in a retail supply chain: On contractual form and coordination. *Management Science* **56**(3) 571–589.

Cachon, G. P., M. A. Lariviere. 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science* **51**(1) 30–44.

Chen, J. 2011. Return with wholesale-price-discount contract in a newsvendor problem. *International Journal of Production Economics* **130**(1) 104–111.

Choi, S. C. 1991. Price competition in a channel structure with a common retailer. *Marketing Science* **10**(4) 271–296.

Choi, S. C. 1996. Price competition in a duopoly common retailer channel. *Journal of Retailing* **72**(2) 117–134.

Cowan, S. 2008. When does third-degree price discrimination reduce social welfare, and when does it raise it? Economics series working paper 410, Washington University of Oxford, Department of Economics.

Dana Jr., J. D., K. E. Spier. 2001. Revenue sharing and vertical control in the video rental market. *Journal of Industrial Economics* **49**(3) 223–245.

Fahey, J. 2003. Stock now, pay later. *Forbes* **172**(9) 60.

Fry, J. M., R. Kapuscinski, T. L. Olsen. 2001. Coordinating production and delivery under a (z, z)-type vendor-managed inventory contract. *Manufacturing & Service Operations Management* **3**(2) 151–173.

Gallego, G., G. Van Ryzin. 1994. Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management Science* **40**(8) 999–1020.

Gerchak, Y., Y. Wang. 2004. Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and Operations Management* **13**(1) 23–33.

Greenhut, M. L., G. Norman, C. Hung. 1988. The economics of imperfect competition: a spatial approach. *Journal of Economic Behavior & Organization* **9**(3) 321–323.

Gümüs, M., E. M. Jewkes, J. H. Bookbinder. 2008. Impact of consignment inventory and vendor-managed inventory for a two-party supply chain. *International Journal of Production Economics* **113**(2) 502–517.

Hastings, J. 2009. Wholesale price discrimination and regulation: Implications for retail gasoline prices. Yale University Working Paper.

Huang, C., G. Yu, S. Wang, X. Wang. 2006. Disruption management for supply chain coordination with exponential demand function. *Acta Mathematica Scientia* **26**(4) 655–669.

Jeuland, A., S. Shugan. 1988. Competitive pricing behavior in distribution systems. T. M. Devinney, ed., *Issues in Pricing: Theory and Research*. Lexington Books, 219–238.

Karlin, S., C. R. Carr. 1962. Prices and optimal inventory policies. K. J. Arrow, S. Karlin, H. Scarf, eds., *Studies in Applied Probability and Management Science*, chap. 10. Stanford University Press, 159–172.

Katok, E., D. Y. Wu. 2009. Contracting in supply chains: a laboratory investigation. *Management Science* **55**(12) 1953–1968.

Lariviere, M. A., E. L. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing and Service Operations Management* **3**(4) 293–305.

Lau, H. S., A. H. L. Lau. 2002. The effect of reducing demand uncertainty in a manufacturer-retailer channel for single-period products. *Computers and Operations Research* **29**(11) 1583–1620.

Li, S., Z. Zhu, L. Huang. 2009. Supply chain coordination and decision making under consignment contract with revenue sharing. *International Journal of Production Economics* **120**(1) 88–99.

Linh, C. T., Y. Hong. 2009. Channel coordination through a revenue sharing contract in a two-period newsboy problem. *European Journal of Operational Research* **198**(3) 822–829.

McGuire, T. W., R. Staelin. 1983. Industry equilibrium analysis of downstream vertical integration. *Marketing Science* **2**(2) 161–191.

Meyer, D., J. Fischer. 2004. Strategic decentralization in channels. Federal Trade Commission Bureau of Economics Working Paper #271.

Moorthy, K. S. 1988. Strategic decentralization in channels. *Marketing Science* **7**(4) 335–355.

Narayanan, V. G., A. Raman, J. Singh. 2005. Agent costs in a supply chain with demand uncertainty and price competition. *Management Science* **51**(1) 120–132.

of California at San Francisco Medical Center, University. 2006. Materiel service or resources and education guide. `http://www.ucsfmedicalcenter.org/matserv/misc/or_guides/or_resource_and_information_guide.doc`. [Online; accessed April 12, 2011].

Özer, O., W. Wei. 2006. Strategic commitments for an optimal capacity decision under asymmetric forecast information. *Management Science* **52**(8) 1238–1257.

Pan, K., K. K. Lai, S. C. H. Leung, D. Xiao. 2010. Revenue-sharing versus wholesale price mechanisms under different channel power structures. *European Journal of Operational Research* **203**(2) 532–538.

Pasternack, B. A. 2002. Using revenue sharing to achieve channel coordination for a newsboy type inventory model. J. Geunes, P. M. Pardalos, H. E. Romeijn, eds., *Supply Chain Management: Models, Applications, and Research Directions*. Kluwer Academic Publishers, 117–136.

Perakis, G., G. Roels. 2007. The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Science* **53**(8) 1249–1268.

Petruzzi, N., M. Dada. 1999. Pricing and the newsvendor problem: A review with extensions. *Operations Research* **47**(2) 184–194.

Ru, J., Y. Wang. 2010. Consignment contracting: Who should control inventory in the supply chain? *European Journal of Operational Research* **201**(3) 760–769.

Simon, C. 2007. Dynamic pricing with demand learning under competition. Ph.D. thesis, Massachusetts Institute of Technology.

Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science* **54**(10) 1759–1773.

Talluri, K. T., G. J. Van Ryzin. 2005. *The theory and practice of revenue management*. Springer, Boston.

Tsay, A. A. 1999. The quantity flexibility contract and supplier-customer incentives. *Management Science* **45**(10) 1339–1358.

Tsay, A. A., N. Agrawal. 2000. Channel dynamics under price and service competition. *Manufacturing and Service Operations Management* **2**(4) 372–391.

Veen, J. V. D., V. Venugopal. 2005. Using revenue sharing to create win-win in the video rental supply chain. *Journal of the Operational Research Society* **56**(7) 757–762.

Villas-Boas, S. B. 2008. An empirical investigation of the welfare effects of banning wholesale price discrimination. Department of Agricultural and Resource Economics, UCB, UC Berkeley.

Vives, X. 1984. Duopoly information equilibrium: Cournot and bertrand. *Journal of Economic Theory* **34**(1) 71–94.

Vives, X. 1985. On the efficiency of bertrand and cournot equilibria with product differentiation. *Journal of Economic Theory* **36**(1) 166–175.

Wang, Y. 2006. Joint pricing-production decisions in supply chains of complementary products with uncertain demand. *Operations Research* **54**(6) 1110–1127.

Wang, Y., L. Jiang, Z. Shen. 2004. Channel performance under consignment contract with revenue sharing. *Management Science* **50**(1) 34–47.

Xiao, T., G. Yu, Z. Sheng, Y. Xia. 2005. Coordination of a supply chain with one-manufacturer and two retailers under demand promotion and disruption management decisions. *Annals of Operations Research* **135**(1) 87–109.

Yao, Z., S. C. H. Leung, K. K. Lai. 2008a. The effectiveness of revenue-sharing contract to coordinate the price-setting newsvendor products' supply chain. *Supply Chain Management: An International Journal* **134**(4) 263–271.

Yao, Z., S. C. H. Leung, K. K. Lai. 2008b. Manufacturer's revenue sharing contract and retail competition. *European Journal of Operational Research* **186**(2) 637–651.

Zhang, D. 2008. Essays on supply contracts and dynamic pricing. Ph.D. thesis, University of Iowa.

Zhang, D., R. de Matta, T. J. Lowe. 2010. Channel coordination in a consignment contract. *European Journal of Operational Research* **207**(2) 897–905.

Material for on-line appendix

Appendix A: Proofs of the results

Proof of Lemma 3.2. We consider three possible cases.

1. For $z_i < A$, $z_i - \Lambda(z_i) = z_i - \int_A^{z_i}(z_i - x)f(x)\,dx = z_i - 0 = z_i > 0$.

2. For $A \le z_i \le B$, $z_i - \Lambda(z_i) = z_i - \int_A^{z_i}(z_i - x)f(x)\,dx = z_i - z_iF(z_i) + \int_A^{z_i} xf(x)\,dx$. Thus, $z_i - \Lambda(z_i) > 0$ as $F(z_i) \le 1$ and $\int_A^{z_i} xf(x)\,dx \ge 0$.

3. For $z_i > B$, $z_i - \Lambda(z_i) = z_i - \int_A^{B}(z_i - x)f(x)\,dx = z_i - z_i + \int_A^B xf(x)\,dx = \int_A^B xf(x)\,dx$. Thus, $z_i - \Lambda(z_i) > 0$ since $\int_A^B xf(x)\,dx = E(\epsilon) = 1$.

Proof of Proposition 3.4. For any given stocking factor z_i, we take the partial derivative of $\pi_{R_i}(p_i, z_i|w)$ with respect to p_i as

$$\frac{\partial \pi_{R_i}(p_i, z_i|w_p)}{\partial p_i} = ae^{-\beta p_i + \gamma p_j}\left[z_i - \Lambda(z_i) - \beta\{p_i(z_i - \Lambda(z_i)) - (c\alpha_i + w_p)z_i\}\right].$$

Since $ae^{-\beta p_i + \gamma p_j} > 0$, $\frac{\partial \pi_{R_i}(p_i,z_i|w)}{\partial p_i} = 0$ when $p_i = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)} \equiv \tilde{p_i}(z_i|w_p)$. Moreover, $\frac{\partial \pi_{R_i}}{\partial p_i} > 0$ for all $p_i < \tilde{p_i}(z_i|w_p)$ and $\frac{\partial \pi_{R_i}}{\partial p_i} < 0$ for all $p_i > \tilde{p_i}(z_i|w_p)$, so $\tilde{p_i}(z_i|w_p)$ is the unique maximizer of $\pi_{R_i}(p_i, z_i|w)$ for fixed z_i, w_p and p_{-i}.

Proof of Proposition 3.5. We want to derive $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{dz_i} = \frac{\partial \pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{\partial p_i} \cdot \frac{d\tilde{p_i}(z_i|w_p)}{dz_i} + \frac{\partial \pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{\partial z_i}.$$

The first term is zero since $\frac{\partial \pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{\partial p_i} = 0$, due to optimality of $\tilde{p_i}(z_i|w_p)$.

Thus, we have

$$\begin{aligned}\frac{d\pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{dz_i} &= \frac{\partial \pi_{R_i}(\tilde{p_i}(z_i|w_p), z_i|w_p)}{\partial z_i} \\ &= ae^{-\beta\tilde{p_i}(z_i|w_p)) + \gamma p_{-i}(z_{-i})}\left\{\left[\frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{z_i - \Lambda(z_i)}\right][1 - F(z_i)] - (c\alpha_i + w_p)\right\} \\ &= \frac{ae^{-\beta\tilde{p_i}(z_i|w_p) + \gamma p_{-i}(z_{-i})}}{\beta(z_i - \Lambda(z_i))} \cdot g(z_i),\end{aligned}$$

where $g(z_i) = [z_i - \Lambda(z_i) + \beta(c\alpha_i + w_p)z_i][1 - F(z_i)] - \beta(c\alpha_i + w_p)(z_i - \Lambda(z_i))$. Since the ratio $\frac{ae^{-\beta\tilde{p_i}(z_i|w_p) + \gamma p_{-i}(z_i)}}{\beta(z_i - \Lambda(z_i))}$ in the above expression is always positive since we know from Lemma 1. that $z_i - \Lambda(z_i)$ is positive, first-order condition requires that the optimal $\bar{z}_i$ satisfy $g(\bar{z}_i) = 0$, which gives (8). Such a $\bar{z}_i$ always exists in the support interval (A, B) of $F(\cdot)$, because $g(z_i)$ is continuous, and $g(A) = A > 0$ and $g(B) = -\beta c\alpha_i < 0$, since the mean value of ϵ is equal to 1. To verify the uniqueness of $\bar{z}_i$, we have

$$g'(z_i) = [1 - F(z_i)]\Big\{[1 - F(z_i)] - h(z_i)\big[\beta(c\alpha_i + w_p)z_i + z_i - \Lambda(z_i)\big]\Big\}$$

$$\begin{aligned} g''(z_i) &= -h(z_i)g'(z_i) \\ &\quad +[1-F(z_i)]\Big\{-f(z_i)-h'(z_i)\big[\beta(c\alpha_i+w_p)z_i+z_i-\Lambda(z_i)\big]-h(z_i)\big[\beta(c\alpha_i+w_p)+1-F(z_i)\big]\Big\}, \end{aligned}$$

where $h(z_i)=f(z_i)/[1-F(z_i)]$ is the failure rate of the demand distribution. ¿From Assumption 3.1, $h'(z_i)>0$, then $g''(z_i)<0$ whenever $g'(z_i)=0$, implying that $g(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|w_p), z_i|w_p)$.

Proof of Corollary 3.6. (5) can be rearranged as

$$\frac{1}{1-F(\bar{z}_i)}-\frac{\bar{z}_i}{\bar{z}_i-\Lambda(\bar{z}_i)}=\frac{1}{(c\alpha_i+w_p)\beta}.$$

The right-hand side is a decreasing function of w_p, thus $G(\bar{z}_i)$ decreases in w_p, where G was defined in Lemma 2. The result then follows from Lemma 3.3.

Proof of Proposition 3.7. The proof is similar to the proof of Proposition 1 and is therefore omitted.

Proof of Proposition 3.8. We want to derive $\bar{z}_i$ that maximizes $\pi_{R_i}(\tilde{p}_i(z_i|w), z_i|w)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{dz_i}=\frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{\partial p_i}\cdot\frac{d\tilde{p}_i(z_i|w)}{dz_i}+\frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{\partial z_i}.$$

The first term is zero since $\frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{\partial p_i}=0$, due to optimality of $\tilde{p}_i(z_i|w)$. Thus, we have

$$\begin{aligned} \frac{d\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{dz_i} &= \frac{\partial\pi_{R_i}(\tilde{p}_i(z_i|w),z_i|w)}{\partial z_i} \\ &= ae^{-\beta\tilde{p}_i(z_i|w)+\gamma p_{-i}(z_{-i})}\left\{\left[\frac{1}{\beta}+\frac{c\alpha_i z_i}{z_i-\Lambda(z_i)}\right][1-F(z_i)]-c\alpha_i\right\} \\ &= \frac{ae^{-\beta\tilde{p}_i(z_i|w)+\gamma p_{-i}(z_i)}}{\beta(z_i-\Lambda(z_i))}\cdot L(z_i), \end{aligned}$$

where $L(z_i)=[z_i-\Lambda(z_i)+\beta c\alpha_i z_i][1-F(z_i)]-\beta c\alpha_i(z_i-\Lambda(z_i))$. Since the ratio $\frac{ae^{-\beta\tilde{p}_i(z_i|w)+\gamma p_{-i}(z_i)}}{\beta(z_i-\Lambda(z_i))}$ in the above expression is always positive, first-order condition requires that the optimal $\bar{z}_i*$ satisfy $L(\bar{z}_i)=0$, which gives (8). Such a $\bar{z}_i$ always exists in the support interval (A,B) of $F(\cdot)$, because $L(z_i)$ is continuous, and $L(A)=A>0$ and $g(B)=-\beta c\alpha_i\mu<0$, where μ is the mean value of ϵ. To verify the uniqueness of $\bar{z}_i$, we have

$$\begin{aligned} L'(z_i) &= [1-F(z_i)]\Big\{[1-F(z_i)]-h(z_i)\big[\beta c\alpha_i z_i+z_i-\Lambda(z_i)\big]\Big\} \\ L''(z_i) &= -h(z_i)g'(z_i)+[1-F(z_i)]\Big\{-f(z_i)-h'(z_i)\big[\beta c\alpha_i z_i+z_i-\Lambda(z_i)\big]-h(z_i)\big[\beta c\alpha_i+1-F(z_i)\big]\Big\}, \end{aligned}$$

where $h(z_i)=f(z_i)/[1-F(z_i)]$ is defined as the failure rate of the demand distribution. ¿From Assumption 3.1, $h'(z_i)>0$, then $L''(z_i)<0$ whenever $L'(z_i)=0$, implying that $L(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|w), z_i|w)$.

Proof of Proposition 3.9. At the second step, $\bar{z}_i$ chosen by the retailer i does not depend on the consignment price w set by the supplier at the first step. Since $\bar{p}_i=\tilde{p}_i(\bar{z}_i|w)=\frac{1}{\beta}+\frac{c\alpha_i\bar{z}_i}{\bar{z}_i-\Lambda(\bar{z}_i)}+w$, the first derivative of $\pi_S(w)$ with respect to w can be written as

$$\begin{aligned} \frac{d\pi_S(w)}{dw} &= ae^{-\beta\left[\frac{1}{\beta}+\frac{c\alpha_1\bar{z}_1}{\bar{z}_1-\Lambda(\bar{z}_1)}+w\right]+\gamma\left[\frac{1}{\beta}+\frac{c\alpha_2\bar{z}_2}{\bar{z}_2-\Lambda(\bar{z}_2)}+w\right]}\Big[(\bar{z}_1-\Lambda(\bar{z}_1))-(\beta-\gamma)\big[w(\bar{z}_1-\Lambda(\bar{z}_1))-c(1-\alpha_1-\alpha_2)\bar{z}_1\big]\Big] \\ &\quad +ae^{\beta\left[\frac{1}{\beta}+\frac{c\alpha_2\bar{z}_2}{\bar{z}_2-\Lambda(\bar{z}_2)}+w\right]+\gamma\left[\frac{1}{\beta}+\frac{c\alpha_1\bar{z}_1}{\bar{z}_1-\Lambda(\bar{z}_1)}+w\right]}\Big[(\bar{z}_2-\Lambda(\bar{z}_2))-(\beta-\gamma)\big[w(\bar{z}_2-\Lambda(\bar{z}_2))-c(1-\alpha_1-\alpha_2)\bar{z}_2\big]\Big] \\ &= ae^{-\beta[1/\beta+w]+\gamma[1/\beta+w]}\Big\{e^{-\left(\frac{\beta\alpha_1\bar{z}_1}{(\bar{z}_1-\Lambda(\bar{z}_1))}-\frac{\gamma\alpha_2\bar{z}_2}{(\bar{z}_2-\Lambda(\bar{z}_2))}\right)c}\Big[(\bar{z}_1-\Lambda(\bar{z}_1)) \\ &\qquad -(\beta-\gamma)\big[w(\bar{z}_1-\Lambda(\bar{z}_1))-c(1-\alpha_1-\alpha_2)\bar{z}_1\big]\Big] \\ &\qquad +e^{-\left(\frac{\beta\alpha_2\bar{z}_2}{(\bar{z}_2-\Lambda(\bar{z}_2))}-\frac{\gamma\alpha_1\bar{z}_1}{(\bar{z}_1-\Lambda(\bar{z}_1))}\right)c}\Big[(\bar{z}_2-\Lambda(\bar{z}_2)) \\ &\qquad -(\beta-\gamma)\big[w(\bar{z}_2-\Lambda(\bar{z}_2))-c(1-\alpha_1-\alpha_2)\bar{z}_2\big]\Big]\Big\}. \end{aligned}$$

Since $ae^{-\beta[1/\beta+w]+\gamma[1/\beta+w]}>0$, $\frac{d\pi_S(w)}{dw}=0$ implies that

$$w^*=\frac{k_1[(\bar z_1-\Lambda(\bar z_1))+(\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar z_1]+k_2[(\bar z_2-\Lambda(\bar z_2))+(\beta-\gamma)c(1-\alpha_1-\alpha_2)\bar z_2]}{(\beta-\gamma)[k_1(\bar z_1-\Lambda(\bar z_1))+k_2(\bar z_2-\Lambda(\bar z_2))]},$$

where $k_i=e^{-\left(\frac{\beta\alpha_i\bar z_i}{(\bar z_i-\Lambda(\bar z_i))}-\frac{\gamma\alpha_{-i}\bar z_{-i}}{(\bar z_{-i}-\Lambda(\bar z_{-i}))}\right)c}$, $i=1,2$ and $\bar z_i$ as in (8). Moreover, $\frac{d\pi_S}{dw}>0$ for all $w<w^*$ and $\frac{d\pi_S}{dw}<0$ for all $w>w^*$, so w^* is the unique maximizer of π_S.

Proof of Proposition 3.10.

- From (8), $\frac{1}{c\alpha\beta}$ is a decreasing function in β. Thus $G(z^*)$ decreases in β. The result then follows from Lemma 3.3.
- To show that $w^*$is decreasing in β, we show that $\frac{\partial w^*}{\partial\beta}\le 0$

$\frac{\partial w^*}{\partial\beta}=-\frac{1}{(\beta-\gamma)^2}+c(1-\alpha)\left[\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2}\right]\frac{\partial z^*}{\partial\beta}\le 0$ since $\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2}=\frac{\int_A^z xf(x)\,dx}{(z-\Lambda(z))^2}\ge 0$ and $\frac{\partial z^*}{\partial\beta}\le 0$.

- To show that $p^*$is decreasing in β, we show that is equivalent to $\frac{\partial p^*}{\partial\beta}\le 0$

$\frac{\partial p^*}{\partial\beta}=-\frac{1}{\beta^2}+c\alpha\left[\frac{zF(z)-\Lambda(z)}{(z-\Lambda(z))^2}\right]\frac{\partial z^*}{\partial\beta}+\frac{\partial w^*}{\partial\beta}\le 0$ since $\frac{\partial z^*}{\partial\beta}\le 0$ and $\frac{\partial w^*}{\partial\beta}\le 0$.

Proof of Proposition 3.11.

- To show that $w^*$is increasing in γ, we show that $\frac{\partial w^*}{\partial\gamma}\ge 0$ (see (11)).
- To show that $p^*$is increasing in γ, we show that $\frac{\partial p^*}{\partial\gamma}\ge 0$ (see (12)).
- To show that $p^*/w^*$is decreasing in γ, we show that $\frac{\partial[p^*/w^*]}{\partial\gamma}\le 0$.

$\frac{\partial[p^*/w^*]}{\partial\gamma}=-\frac{1}{(\beta-\gamma)^2}\left[\frac{1}{\beta}+\frac{c\alpha z^*}{z^*-\Lambda(z^*)}\right]\le 0.$

- To show that $Q^*$is increasing in γ, we show that $\frac{\partial Q^*}{\partial\gamma}\ge 0$.

$\frac{\partial Q^*}{\partial\gamma}=ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}z^*\left\{-(\beta-\gamma)\frac{\partial p^*}{\partial\gamma}+[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]\right\}.$
$=ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}z^*\{\frac{1}{\beta}+\frac{cz^*}{z^*-\Lambda(z^*)}\}.$

- To show that $\pi_R^{d^*}*$is increasing in γ, we show that $\frac{\partial\pi_{R_i}^{d^*}}{\partial\gamma}\ge 0$.

$\frac{\partial\pi_{R_i}^{d^*}}{\partial\gamma}=ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}\frac{1}{\beta}\left(\frac{1}{\beta}+\frac{cz^*}{z^*-\Lambda(z^*)}\right)[z^*-\Lambda(z^*)]\ge 0$, where $i=1,2$.

- To show that $\pi_S^{d^*}$is increasing in γ, we show that $\frac{\partial\pi_S^{d^*}}{\partial\gamma}\ge 0$.

$\frac{\partial\pi_S^{d^*}}{\partial\gamma}=2ae^{-(\beta-\gamma)[\frac{1}{\beta}+\frac{1}{\beta-\gamma}+\frac{cz^*}{z^*-\Lambda(z^*)}]}\left\{\frac{1}{(\beta-\gamma)^2}+\frac{1}{\beta-\gamma}\left(\frac{1}{\beta}+\frac{cz^*}{z^*-\Lambda(z^*)}\right)\right\}[z^*-\Lambda(z^*)]\ge 0.$

Proof of Proposition 3.12. We take the partial derivative of (13) with respect to p_i and get

$$\frac{\partial\pi_{R_i}(p_i,z_i|r)}{\partial p_i}=ae^{-\beta p_i+\gamma p_j}\left[(1-r)(z_i-\Lambda(z_i))-\beta\{(1-r)p_i(z_i-\Lambda(z_i))-c\alpha_i z_i\}\right].$$

Since $ae^{-\beta p_i+\gamma p_j}>0$, $\frac{\partial\pi_{R_i}(p_i,z_i/r)}{\partial p_i}=0$ implies that $\tilde p_i(z_i|r)=\frac{1}{\beta}+\frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}$, which gives us (14). Moreover, $\frac{\partial\pi_{R_i}}{\partial p_i}>0$ for all $p_i<\tilde p_i(z_i|r)$ and $\frac{\partial\pi_{R_i}}{\partial p_i}<0$ for all $p_i>\tilde p_i(z_i|r)$, so $\tilde p_i(z_i|r)$ is the unique maximizer of $\pi_{R_i}(p_i,z_i|r)$ for fixed z_i,r and p_{-i}.

Proof of Proposition 3.13. We want to derive $\bar z_i$ that maximizes $\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)$. By the chain rule, we have

$$\frac{d\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{dz_i}=\frac{\partial\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{\partial p_i}\cdot\frac{d\tilde p_i(z_i|r)}{dz_i}+\frac{\partial\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{\partial z_i}.$$

The first term is zero, $\frac{\partial\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{\partial p_i}=0$, due to optimality of $\tilde p_i(z_i|r)$. Thus, we have

$$\begin{aligned}\frac{d\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{dz_i}&=\frac{\partial\pi_{R_i}(\tilde p_i(z_i|r),z_i|r)}{\partial z_i}\\&=ae^{-\beta\tilde p_i(z_i|r)+\gamma p_{-i}}\left\{(1-r)\left[\frac{1}{\beta}+\frac{c\alpha_i z_i}{(1-r)(z_i-\Lambda(z_i))}\right][1-F(z_i)]-c\alpha_i\right\}\\&=\frac{ae^{-\beta\tilde p_i(z_i|r)+\gamma p_{-i}}}{\beta(z_i-\Lambda(z_i))}\cdot H(z_i),\end{aligned}$$

where $H(z_i) = [(1-r)(z_i - \Lambda(z_i)) + \beta c \alpha_i z_i]\,[1 - F(z_i)] - \beta c \alpha_i (z_i - \Lambda(z_i))$.Since the ratio $\frac{ae^{-\beta \tilde{p}_i(z_i)+\gamma p_{-i}}}{\beta(1-r)(z_i-\Lambda(z_i))}$ in the above expression is always positive, first-order condition requires that the optimal $\bar{z}_i$ satisfy $H(z_i) = 0$, which gives us (15). Such a $\bar{z}_i$ always exists in the support interval (A, B) of $F(\cdot)$, because $H(z_i)$ is continuous for any given r where $0 < r < 1$, and $H(A) = A(1 - r + \alpha_i c \beta r) > 0$ and $H(B) = -\beta c \alpha_i (1-r) < 0$, since the mean value of ϵ is 1. To verify the uniqueness of $\bar{z}_i$, we have

$$
\begin{aligned}
H'(z_i) &= [1-F(z_i)]\Big\{(1-r)(1-F(z_i)) - h(z_i)\big[\beta c \alpha_i z_i + (1-r)(z_i - \Lambda(z_i))\big]\Big\} \\
H''(z_i) &= -h(z_i)H'(z_i) + [1-F(z_i)]\Big\{-f(z_i)(1-r) - h'(z_i)\big[\beta c \alpha_i z_i + (1-r)(z_i - \Lambda(z_i))\big] \\
&\quad - h(z_i)\big[\beta c \alpha_i + (1-r)(1-F(z_i))\big]\Big\},
\end{aligned}
$$

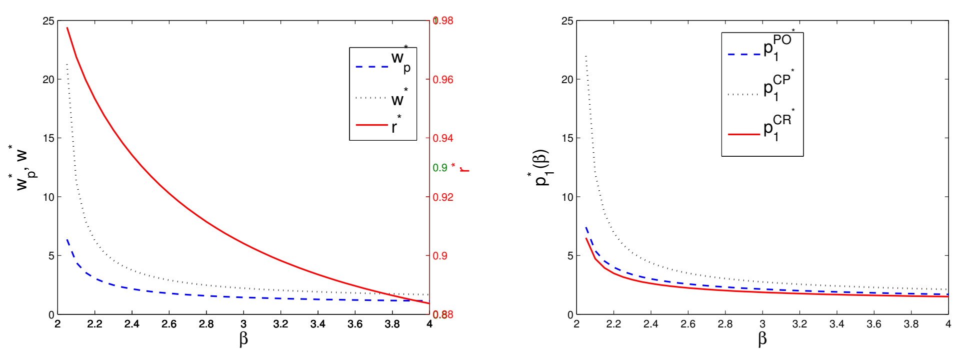
where $h(z_i) = f(z_i)/[1 - F(z_i)]$ is defined as the failure rate of the demand distribution. ¿From Assumption 3.1, $h'(z_i) > 0$, then $H''(z_i) < 0$ whenever $H'(z_i) = 0$, implying that $H(z_i)$ is a unimodal function. We have proved that $\bar{z}_i$ is a unique maximizer of $\pi_{R_i}(\tilde{p}_i(z_i|r), z_i|r)$.

Proof of Corollary 3.14. (15) can be rearranged as

$$\frac{1}{1-F(\bar{z}_i)} - \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1-r}{c\alpha_i\beta}.$$

$\frac{1-r}{c\alpha_i\beta}$ is a monotonic decreasing function in r. In conjunction with Lemma 3.3, there is a one-to-one correspondence between $\bar{z}_i$ and r, given that any other parameters remain unchanged. Therefore, we can further say that decreases $\bar{z}_i$ in r. The proof is complete.

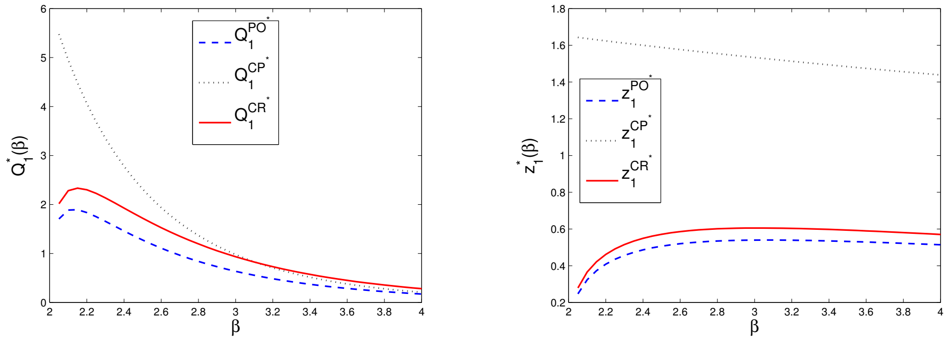
Appendix B: Plots form numerical study



(a)Supplier's wholesale price w_p, consignment price w and revenue share r

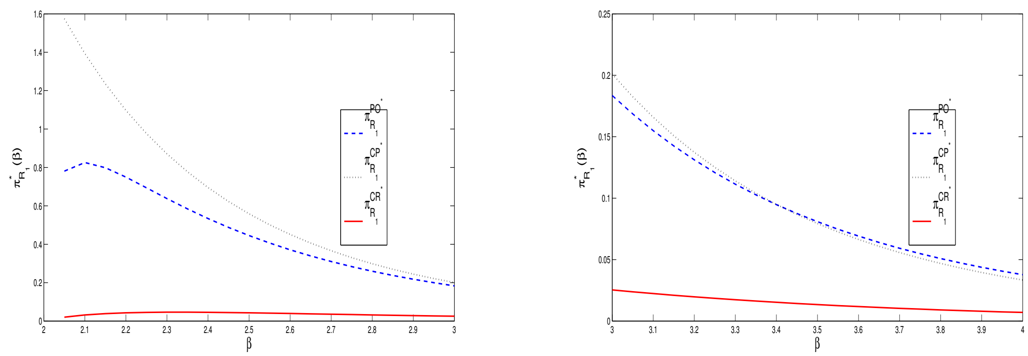
(b)Retailer i's selling price p_i^*

Figure 4 **Supplier's price (or revenue share) and retailer's selling price as a function of β under three contracts, where $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$**



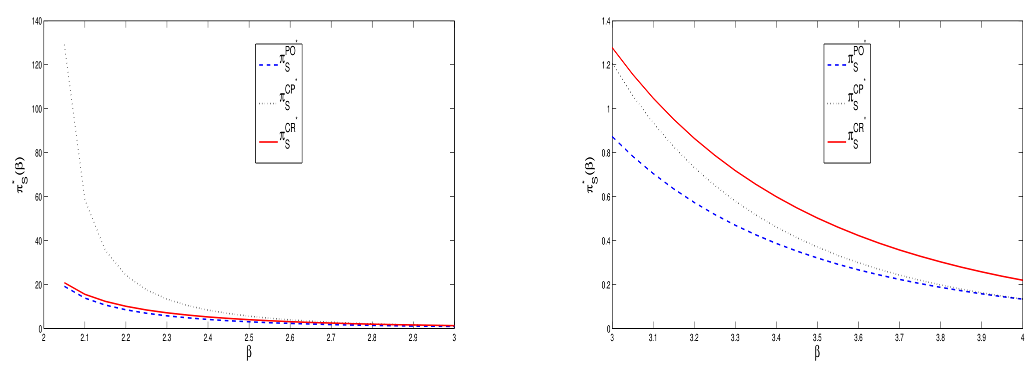
(a)Retailer i's order quantity Q_i^* (b)Retailer i's stocking factor z_i^*

Figure 5 **Retailer i's stocking quantity and stocking factor as a function of β under three contracts, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Retailer i's profit $\pi_{R_i}^*$ for low price sensitivity (b)Retailer i's profit $\pi_{R_i}^*$ for high price sensitivity

Figure 6 **Retailer i's profit as a function of β under three contracts, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Supplier's profit π_S^* for low price sensitivity (b)Supplier's profit π_S^* for high price sensitivity

Figure 7 **Supplier's profit as a function of β under three contracts, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

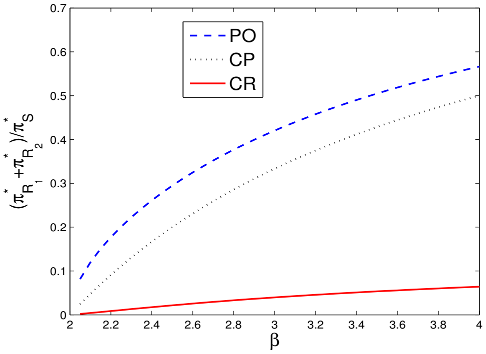
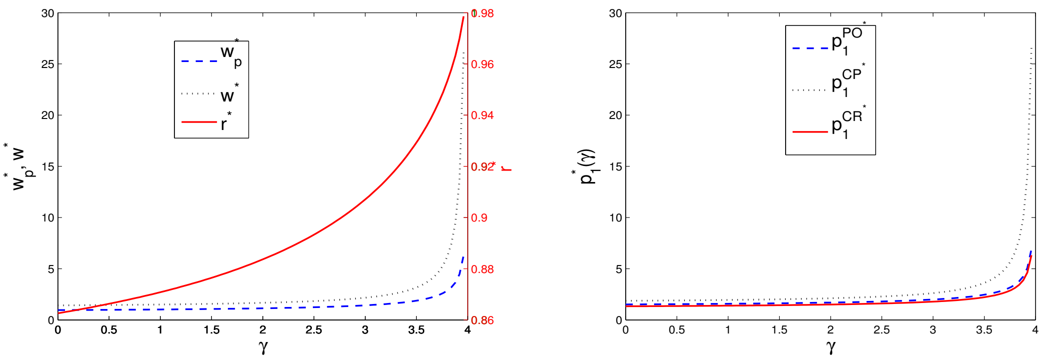


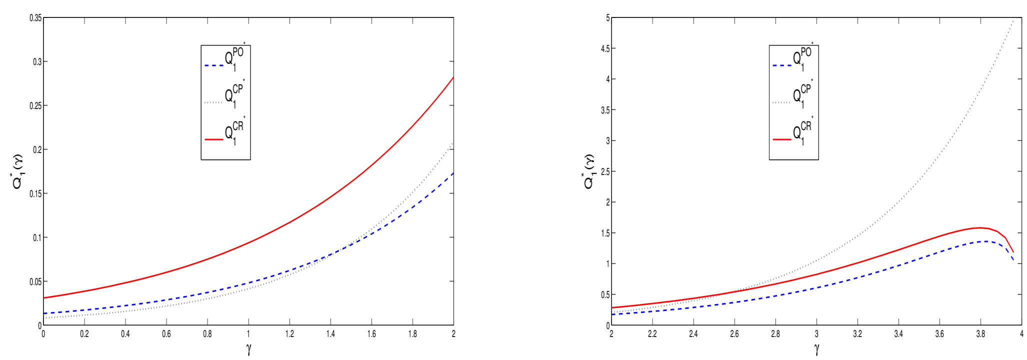
Figure 8 **Retailer's share of the total channel profits as a function of** β **under three contracts** $\frac{\pi^*_{R_1}+\pi^*_{R_2}}{\pi^*_S}$**, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Supplier's wholesale price w_p, consignment price w and revenue share r

(b)Retailer i's selling price p_i^*

Figure 9 **Supplier's price (or revenue share) and retailer's selling price as functions of** γ **under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Retailer i's order quantity Q_i^* for high retailer differentiation

(b)Retailer i's order quantity Q_i^* for low retailer differentiation

Figure 10 **Retailer** i**'s order quantity as a function of** γ **under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

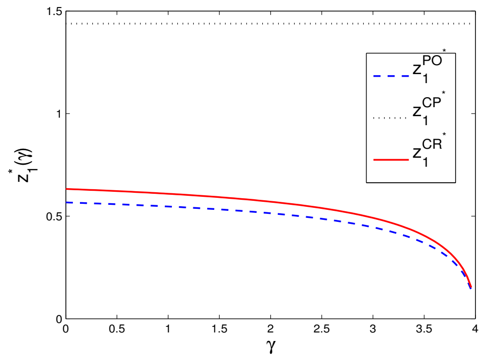
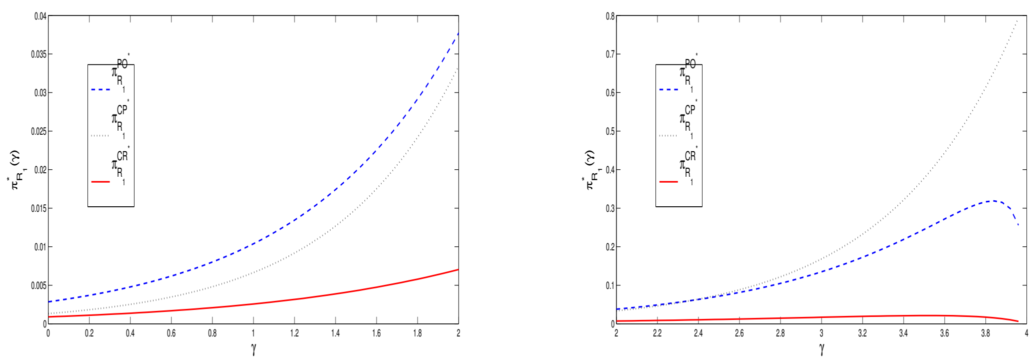


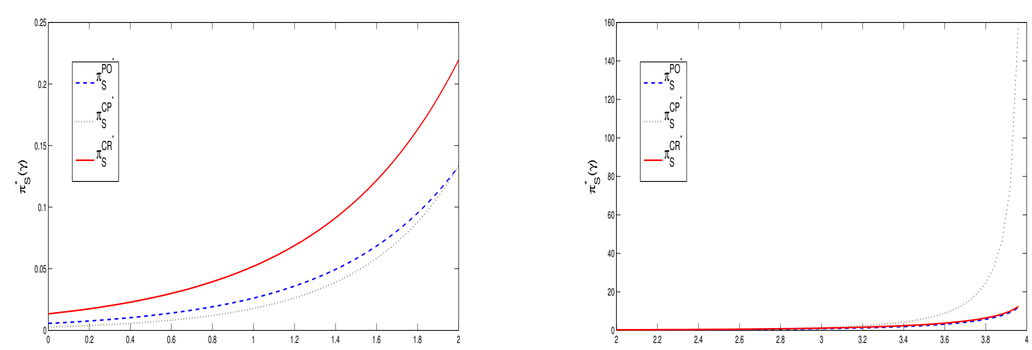
Figure 11 **Retailer i's stocking factor as a function of γ under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Retailer i's profit $\pi^*_{R_i}$ for high retailer differentiation

(b)Retailer i's profit $\pi^*_{R_i}$ for low retailer differentiation

Figure 12 **Retailer i's profit as a function of γ under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Supplier's profit π^*_S for high retailer differentiation

(b)Supplier's profit π^*_S for low retailer differentiation

Figure 13 **Supplier's profit as a function of γ under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

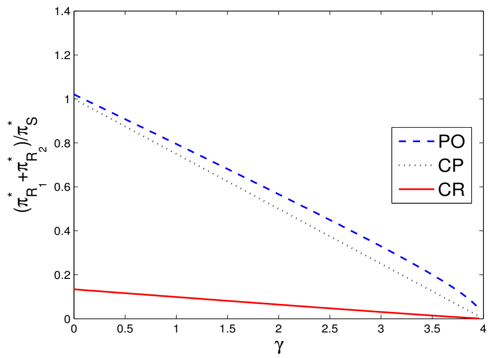
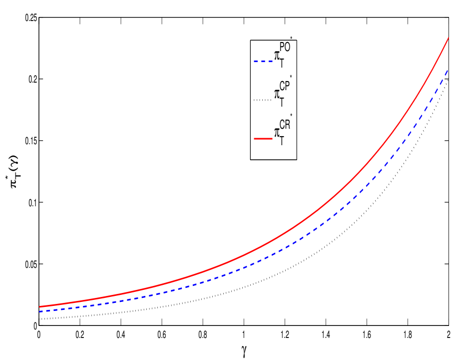
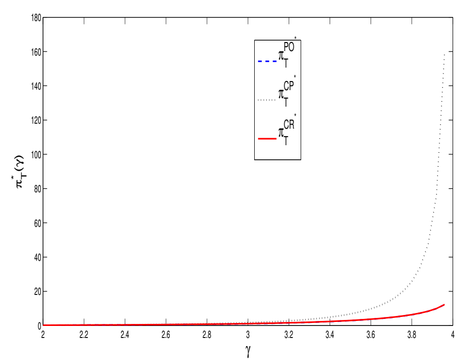


Figure 14 **The retailers' share of the total channel profits as a function of γ under three contracts, where** $\beta=4,\ a=10,\ \alpha_1=\alpha_2=0.125,\ c=1$



(a)Total channel profits $\pi_T^*=\pi_{R_1}^*+\pi_{R_2}^*+\pi_S^*$ under low retail competition



(b)Total channel profits $\pi_T^*=\pi_{R_1}^*+\pi_{R_2}^*+\pi_S^*$ under high retail competition

Figure 15 **Total channel profits as a function of γ under three contracts, where** $\beta=4,\ a=10,\ \alpha_1=\alpha_2=0.125,\ c=1$

Appendix C: Summary Table

Decisions and profits	Retailer differentiation
p_i^*	$p_i^{CP^*} > p_i^{CR^*}$
Q_i^*	$Q_i^{CR^*} > Q_i^{CP^*}$ for more retailer differentiation (small γ) $Q_i^{CR^*} < Q_i^{CP^*}$ for less retailer differentiation (large γ)
z_i^*	$z_i^{CP^*} > z_i^{CR^*}$
$\pi_{R_i}^*$	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*}$
π_S^*	$\pi_S^{CR^*} > \pi_S^{CP^*}$ for more retailer differentiation (small γ) $\pi_S^{CR^*} < \pi_S^{CP^*}$ for less retailer differentiation (large γ)
$\frac{\pi_{R_1}^*+\pi_{R_2}^*}{\pi_S^*}$	$CP > CR$

Table 3 The comparison of two types of consignment contracts

Appendix D: Special case: Consignment contracts with revenue share in the case of deterministic demand

In order to shed some light on the supplier's equilibrium revenue share r^*, we consider a special, more tractable case. In this section, we assume that the demand function is deterministic, i.e., $\epsilon = 1$ and $D_i(p) = y_i(p)$. Since there is no stochasticity, the quantity ordered by the retailers matches the demand (i.e. the stocking factor equals 1) and the only decision left to retailers is the retail price.

It follows from Proposition 3.12 that the best response retail price of retailer i, $\bar{p}_i$, to the supplier's revenue share r is given by

$$\bar{p}_i = \frac{1}{\beta} + \frac{\alpha_i c}{1-r}. \tag{18}$$

It follows from (17) that the supplier's profit can be expressed as

$$\pi_S(r) = ae^{-\beta p_1 + \gamma p_2}[rp_1 - (1-\alpha_1-\alpha_2)c] + ae^{-\beta p_2 + \gamma p_1}[rp_2 - (1-\alpha_1-\alpha_2)c]. \tag{19}$$

Using (18) into (19), we find

$$\begin{aligned}\pi_S(r) = {} & ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right] \\ & + ae^{\beta[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right].\end{aligned}$$

Proposition D.1. If $\frac{\gamma}{\beta} < \frac{\alpha_1}{\alpha_2} < \frac{\beta}{\gamma}$, the supplier's equilibrium revenue share r^* is the solution of

$$\sum_{i=1}^{2} e^{\frac{-(\beta\alpha_i - \gamma\alpha_{-i})c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_i c}{(1-r)}+\frac{\alpha_i cr}{(1-r)^2}-\left[\frac{(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_i c}{(1-r)}\right)-(1-\alpha_i-\alpha_{-i})c\right]\right\}=0.$$

In the proposition above, the condition on α_1/α_2 guarantees that the supplier's profit function is bounded from above. If $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, then the supplier's profit function is unbounded.

Proof of Proposition D.1. Consider the supplier's profit function

$$\pi_S(r) = ae^{-(\beta-\gamma)1/\beta}\left\{\sum_{i=1}^{2} e^{\frac{-(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)}}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_i-\alpha_{-i})c\right]\right\}.$$

If $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, meaning that $-\beta c\left(\alpha_i - \frac{\gamma}{\beta}\alpha_{-i}\right) > 0$, then

$$\begin{aligned}\lim_{r\to 1}\pi_S(r) &= ae^{-(\beta-\gamma)1/\beta}\left\{\sum_{i=1}^{2}\lim_{r\to 1} e^{\frac{-(\beta\alpha_i-\gamma\alpha_{-i})c}{(1-r)}}\left[\lim_{r\to 1} r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-\lim_{r\to 1}(1-\alpha_i-\alpha_{-i})c\right]\right\}\\ &= +\infty.\end{aligned}$$

Thus, if $\frac{\alpha_1}{\alpha_2} \notin \left(\frac{\gamma}{\beta}, \frac{\beta}{\gamma}\right)$, then the supplier's profit function is unbounded.

Assume that $\frac{\gamma}{\beta} < \frac{\alpha_1}{\alpha_2} < \frac{\beta}{\gamma}$, the first order condition requires the optimal revenue share r^* satisfy the following equation:

$$\begin{aligned}\frac{d\pi_S}{dr} &= ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]}\Big\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\\ &\qquad+\left[\frac{-\beta\alpha_1 c}{(1-r)^2}+\frac{\gamma\alpha_2 c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ &\quad+ae^{-\beta[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}]+\gamma[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}]}\Big\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\\ &\qquad+\left[\frac{-\beta\alpha_2 c}{(1-r)^2}+\frac{\gamma\alpha_1 c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ &= ae^{-(\beta-\gamma)\frac{1}{\beta}}\Big\{e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\Big\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ &\quad+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\Big\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\Big\}\\ &= 0.\end{aligned}$$

There exists $r^* \in [0,1]$ such that $\frac{d\pi_S}{dr}|_{r=r^*} = 0$ because $\frac{d\pi_S}{dr}$ is continuous in r, $\frac{d\pi_S}{dr}|_{r=0} > 0$ and $\frac{d\pi_S}{dr}|_{r=1} < 0$. To verify that $\pi_S(r)$ is strictly increasing for $0 \le r < r^*$ and strictly decreasing for $r^* < r \le 1$, we need to show that $\pi_S(r)$ is a unimodal function.

Consider

$$\begin{aligned}\frac{d^2\pi_S}{dr^2} &= e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\Big\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\\ &\qquad-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\left[\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right]\\ &\qquad+\frac{(\beta\alpha_1-\gamma\alpha_2)^2c^2}{(1-r)^4}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ &\quad+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\Big\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\\ &\qquad-\frac{2(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\left[\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right]\\ &\qquad+\frac{(\beta\alpha_2-\gamma\alpha_1)^2c^2}{(1-r)^4}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ \frac{d^2\pi_S}{dr^2} &= e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\Big\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\Big\}\\ &\quad-e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\Big\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\Big\}\end{aligned}$$

$$-e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$-e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\}$$
$$-e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}.$$

At $\bar{r}^*$ such that $\frac{d\pi_S}{dr}=0$, the third and sixth terms become zero.

$$\frac{d^2\pi_S}{dr^2}|_{r=\bar{r}^*} = e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$-e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right\}$$
$$+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$-e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\}.$$

At $\bar{r}^*$,

$$e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}-\left[\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right]\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}=0.$$

Thus,

$$e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left\{\frac{2\alpha_1 c}{(1-r)^2}+\frac{2\alpha_1 cr}{(1-r)^3}-\frac{2(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left\{\frac{2\alpha_2 c}{(1-r)^2}+\frac{2\alpha_2 cr}{(1-r)^3}-\frac{2(\beta\alpha_2-\gamma\alpha_1 c}{(1-r)^3}\left[r\left(\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}\right)-(1-\alpha_1-\alpha_2)c\right]\right\}$$
$$=-\frac{2}{\beta(1-r)}\left(e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\right).$$

Therefore,

$$\frac{d^2\pi_S}{dr^2}|_{\bar{r}^*} = -\frac{2}{\beta(1-r)}\left(e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}+e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\right)$$
$$-e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}\left(\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_1 c}{(1-r)}+\frac{\alpha_1 cr}{(1-r)^2}\right\}$$
$$-e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}\left(\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}\right)\left\{\frac{1}{\beta}+\frac{\alpha_2 c}{(1-r)}+\frac{\alpha_2 cr}{(1-r)^2}\right\}.$$

Since $e^{\frac{-(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)}}$, $e^{\frac{-(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)}}$, $\frac{(\beta\alpha_1-\gamma\alpha_2)c}{(1-r)^2}$ and $\frac{(\beta\alpha_2-\gamma\alpha_1)c}{(1-r)^2}$ are positive, $\frac{d^2\pi_S}{dr^2}|_{\bar{r}^*}$ is negative.

$\frac{d^2\pi_S}{dr^2}<0$ whenever $\frac{d\pi_S}{dr}=0$. Therefore, $\pi_S(r)$ itself is a unimodal function. The proof is complete.

Appendix E: Extension: the case of ten competing retailers

In this section, we extend the analysis to a supply chain with 10 competing retailers. In order to keep the problem tractable, we assume that the cross price-elasticity of demand γ is symmetric for all of the 10 retailers. The generalized demand function can then be written as:

$$D_i(\mathbf{p}) = y_i(\mathbf{p}) \cdot \epsilon, \quad i = 1, \ldots, 10,$$

where

$$y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma \sum_{j=1,\ j\neq i}^{10} p_j}, \qquad a, \beta, \gamma > 0;\ \beta > 9\gamma.$$

Note that in this model the expected demand at retailer i, $y_i(\mathbf{p})$, is a decreasing function of the retailer's own price (p_i), and an increasing function of any of its competitor's price (p_j, where $1 \leq j \leq 10,\ j \neq i$).

We also assume that the 10 retailers have a symmetric cost structure, i.e., $\alpha_1 = \ldots = \alpha_{10} \equiv \alpha$. As a result, each retailer i incurs a cost $\alpha_i c = \alpha c$, to retain the products from the supplier, and the supplier's production cost is $(1 - 10\alpha)c$, where $0 < \alpha < 0.1$.

We compute the best response decisions under the the PO contract and find that the retailer i's unique best response stocking factor $\bar{z}_i$ for a given w_p is given by equation (5) and the retailer i's best response price for any given wholesale price w_p is given by equation (6). This is consistent with the fact that, in the case of two retailers, the retailer's best response to the supplier's decision was independent of the other retailer's price.

Similarly, under the CP contract, we find that retailer i's unique best response stocking factor $\bar{z}_i$ for a given w is given by equation (8) and the best response price for any given consignment price w_p is given by equation (9). Under the CR contract, the retailer i's unique best response stocking factor $\bar{z}_i$ for a given r is given by equation (15) and the retailer i's best response price for any given consignment price r is given by equation (16). Consequently, the retailer's best response price and quantity functions to a given supplier's decision exhibit the same trend as in the case of two retailers.

Since it is intractable to obtain the equilibrium solution in closed-form, including the supplier's price, the retail price, and the quantity for each contract, we solve for the equilibrium quantities numerically.

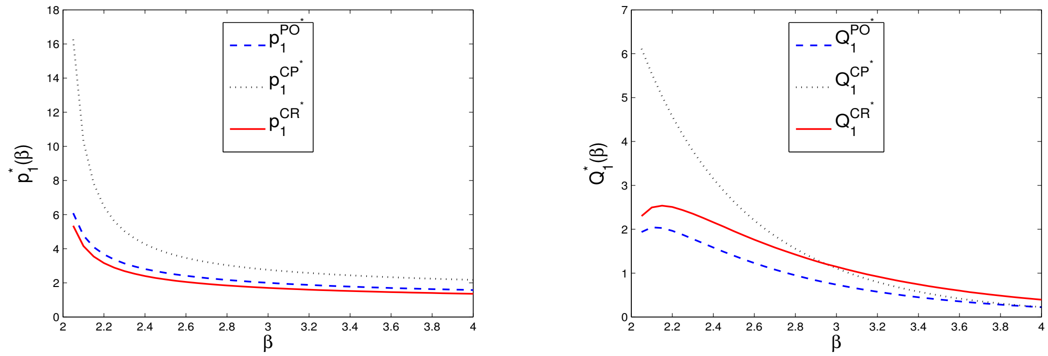
Numerical analysis

Following our previous numerical study, we set $a = 10,\ c = 1$, and $\alpha_1 = \ldots = \alpha_{10} = 0.025$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0, 2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = \frac{2}{9}$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

The effect of the price sensitivity factor

Overall, the conclusions on the effect of the price sensitivity parameter β on the equilibrium decisions and profits in the case of two retailers (as summarized in Table 1) remain valid for the case of 10 retailers. For example, we still see that the retail price is highest in the CP contract, for any level of price sensitivity (as depicted in Figure 16(a)). Moreover, the effect of β on the order quantity varies, depending on the type of contract. Figure 16(b) illustrates that the order quantity decreases in β, under the CP contract. However, in the PO and CR contracts, the order quantity is not a monotonic function of β.

The effect of retailer differentiation



(a)Retailer i's price p_i^* (b)Retailer i's order quantity Q_i^*

Figure 16 **Retailer i's equilibrium price and order quantity as a function of β under three contracts, where** $\gamma = \frac{2}{9},\ a = 10,\ \alpha_1 = \alpha_2 = \ldots = \alpha_{10} = 0.025,\ c = 1$

The numerical results on the effect of retailer differentiation on the equilibrium decisions and profits, for a channel of one supplier and 10 retailers under all types of contract, lead us to conclusions similar to those obtained in the case of two retailers (as summarized in Table 2). For instance, we still observe that the consignment price in the CP contract is always higher than the wholesale price in the PO contact, regardless of level of retailer differentiation. Furthermore, the retailers' share of the total channel profits, $\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$, decreases in γ, as illustrated in Figure 17. The retailers' share of the total channel profits is highest in the PO contract, for any level of retailer differentiation.

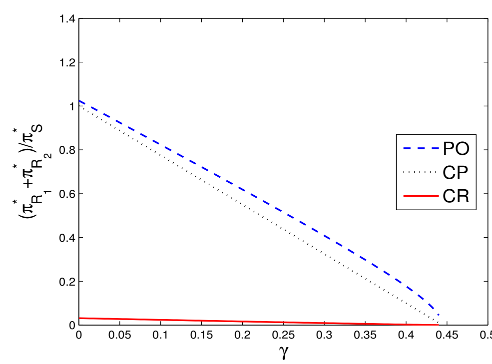


Figure 17 **The retailers' share of the total channel profits as a function of γ under three contracts, where** $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = \ldots = \alpha_1 0 = 0.025,\ c = 1$

Appendix F: Combination of the consignment price and the consignment with revenue share contracts

In this section, we study a more general consignment contract, i.e., a combination of the consignment price and the revenue share contracts, which we refer to as consignment price with revenue share (CPR) contract.

In this contract, decisions are made in two steps. In the first step, the supplier decides the consignment price w_r and the revenue share $\breve{r}$ to be received from the retailers for each unit sold to consumers. In the second step, given this consignment price and revenue share, each retailer simultaneously chooses the retail price p_i and order quantity Q_i. We first derive each retailer's best response price and inventory quantity to the supplier's consignment price and revenue share decisions.

Retailer i's selling price and stocking factor decision

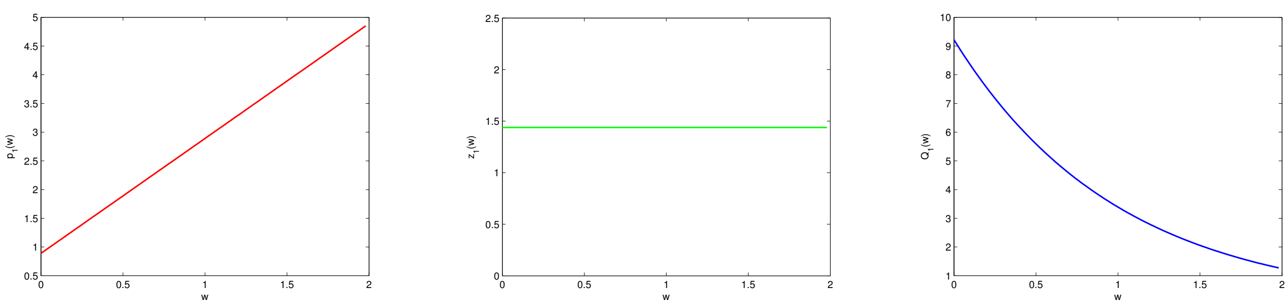
For a given stocking factor z_i, consignment price $w_r > 0$, revenue share $\breve{r}$ and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|w_r, \breve{r})$ is given by

$$\tilde{p}_i(z_i|w_r, \breve{r}) = \frac{1}{\beta} + \frac{c\alpha_i z_i}{(1-\breve{r})(z_i - \Lambda(z_i))} + \frac{w_r}{1-\breve{r}}.$$

Retailer i's best response stocking factor $\bar{z}_i$ that maximizes the profit for a given w_r and a given $\breve{r}$ is uniquely determined as the solution of:

$$\frac{1-\breve{r}}{c\alpha_i\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1-F(\bar{z}_i)}.$$

The CPR best response stocking factor does not depend on the supplier's consignment price w_r, but does depend on the supplier's revenue share $\breve{r}$. This is consistent with the best response stocking factor in both the CP and the CR contracts. By definition, the retailer's best response quantity is $y_i(\mathbf{p})\bar{z}_i$, where $y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma p_{-i}}$.

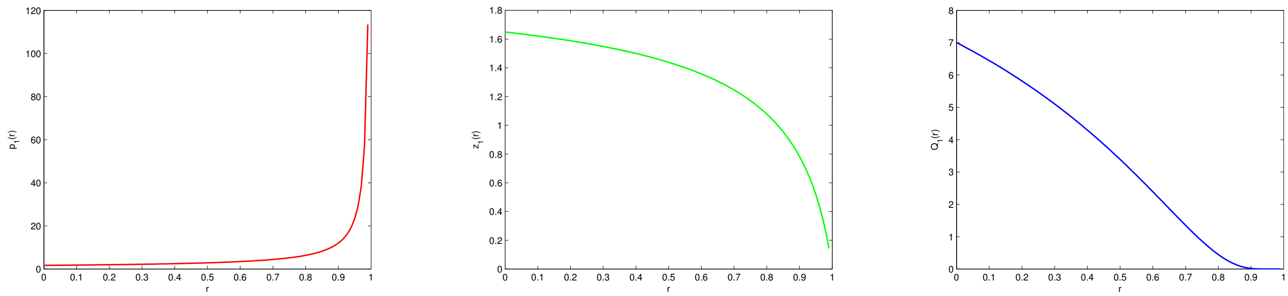


(a)Best response price $\bar{p}_1$ (b)Best response stocking factor $\bar{z}_1$ (c)Best response quantity $\bar{Q}_1$

Figure 18 **Retailer 1's best response price, stocking factor and quantity as a function of the consignment price w_r when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and $\breve{r}$ is fixed at 0.5 in the CPR contract**

Figures 18 and 19 depict that the retailer's best response retail price increases with the supplier's consignment price w_r and revenue share $\breve{r}$: when the supplier keeps a higher consignment price and/or a higher share of the retailer's revenue, the retailers transfer the increasing cost to consumers. The higher price causes the demand to decrease, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's consignment price and revenue share, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's consignment price and revenue share.

Supplier's consignment price and revenue share decision



(a)Best response price $\bar{p}_1$ (b)Best response stocking factor $\bar{z}_1$ (c)Best response quantity $\bar{Q}_1$

Figure 19 Retailer 1's best response price, stocking factor and quantity as a function of the revenue share $\breve{r}$ when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and w is fixed at 1 in the CPR contract

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the consignment price w_r and revenue share $\breve{r}$ to maximize her own expected profit. For any given revenue share $\breve{r}$, the supplier's unique equilibrium consignment price $\tilde{w_r}(\breve{r})$ is given by

$$\tilde{w_r}(\breve{r}) = \frac{k_1 v_1 + k_2 v_2}{(\frac{\beta-\gamma}{1-\breve{r}})(\frac{1}{1-\breve{r}})[k_1(\bar{z}_1 - \Lambda(\bar{z}_1)) + k_2(\bar{z}_2 - \Lambda(\bar{z}_2))]},$$

where $k_i = e^{-\frac{c}{1-\breve{r}}\left(\frac{\beta\alpha_i\bar{z}_i}{\bar{z}_i-\Lambda(\bar{z}_i)} - \frac{\gamma\alpha_{-i}\bar{z}_{-i}}{\bar{z}_{-i}-\Lambda(\bar{z}_{-i})}\right)}$, and

$$v_i = \frac{1}{1-\breve{r}}(\bar{z}_i - \Lambda(\bar{z}_i)) - \frac{\beta-\gamma}{1-\breve{r}}\left[\breve{r}(\frac{1}{\beta} + \frac{c\alpha_i\bar{z}_i}{\bar{z}_i-\Lambda(\bar{z}_i)})(\bar{z}_i - \Lambda(\bar{z}_i) - c(1-\alpha_i-\alpha_{-i})\bar{z}_i\right],\ i = 1, 2.$$

To find the equilibrium solution, denoted by $\breve{r}^*$ and $w_r^* = \tilde{w_r}(\breve{r}^*)$, we have to maximize $\pi_S(\tilde{w_r}(\breve{r}), \breve{r})$ over $\breve{r}$. Obtaining an analytical solution for this maximization is intractable, therefore, we use numerical methods.

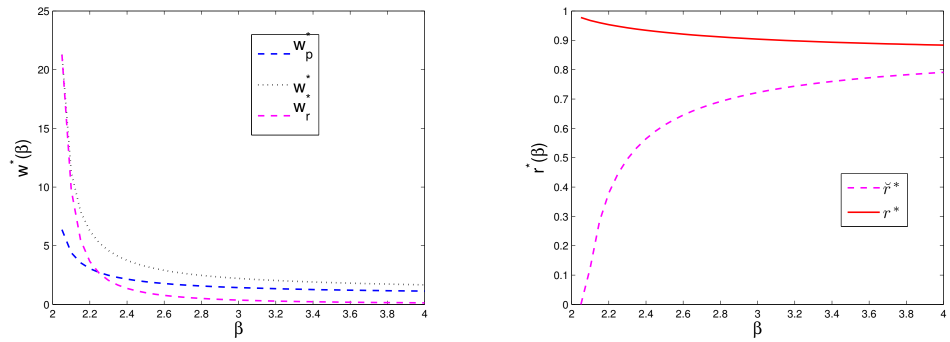
Numerical analysis

Following our previous numerical study, we set $a = 10,\ c = 1$, and $\alpha_1 = \alpha_2 = 0.125$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0, 2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = 2$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

The effect of the price sensitivity factor

Figures 20(a) and (b) depict the effect of the price sensitivity parameter on the supplier's consignment price w_r and revenue share $\breve{r}$, respectively. The supplier's consignment price w_r decreases in β, which is consistent with our finding in the CP contract. Interestingly, the supplier's revenue share $\breve{r}$ increases in β (while the revenue share r in the CR contract decreases in β). One explanation is that as β increases the consignment price w_r decreases, the supplier needs to compensate this loss by increasing the revenue share $\breve{r}$.

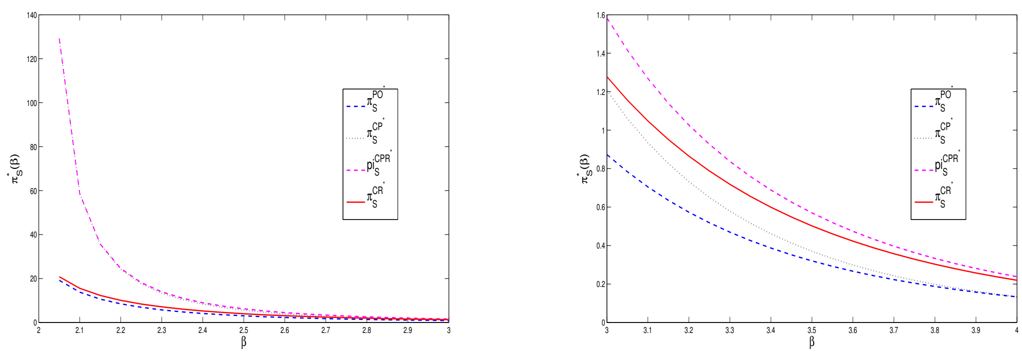
We also find that the effect of the price sensitivity parameter β on the retail price, the order quantity, the stocking factor, the retailers' profits, the supplier profit, and the share of the channel profits for the retailers are consistent with the findings in the CP contract (i.e., they are decreasing functions of β). Figure 21 shows the effect of the price sensitivity parameter on the supplier's profit. Interestingly, the supplier earns the highest profit in the CPR contract, for any level of price sensitivity. The effect of the price sensitivity parameter on the decisions and profits for each of the supply chain members is summarized in Table 4.



(a)Supplier's prices w_p^* (CP), w^* (CR) and w_r^* (CPR)

(b)Supplier's revenue share $\breve{r}^*$ (CPR) and r^* (CR)

Figure 20 **Supplier's price or revenue share at equilibrium as a function of β under four contracts, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Supplier's profit π_S^* for low price sensitivity

(b)Supplier's profit π_S^* for high price sensitivity

Figure 21 **Supplier's profit at equilibrium as a function of β under four contracts, where** $\gamma = 2,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

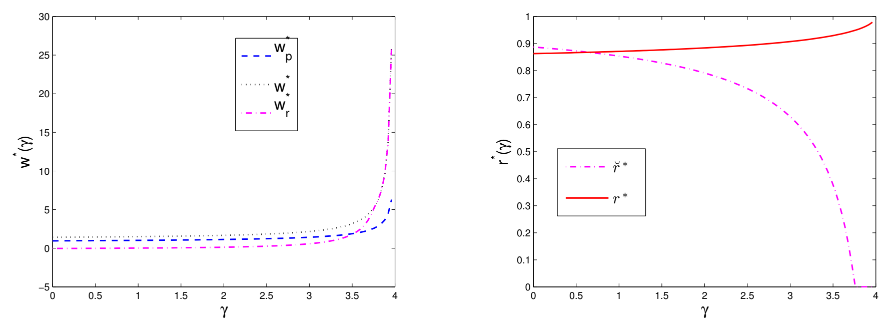
Decisions and profits	PO contract	CP contract	CR contract	CPR contract	Remark
w_p^* or w^* or w_r^*	decreasing	decreasing	-	decreasing	$w^* > w_r^* > w_p^*$ for small β $w^* > w_p^* > w_r^*$ for large β
r^* or $\breve{r}^*$	-	-	decreasing	increasing	$r^* > \breve{r}^*$
p_i^*	decreasing	decreasing	decreasing	decreasing	$p_i^{CP^*} > p_i^{CPR^*} > p_i^{PO^*} > p_i^{CR^*}$ for small β $p_i^{CP^*} > p_i^{PO^*} > p_i^{CPR^*} > p_i^{CR^*}$ for large β
Q_i^*	not monotonic	decreasing	not monotonic	decreasing	$Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for small β $Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{CP^*} > Q_i^{PO^*}$ for large β
z_i^*	not monotonic	decreasing	not monotonic	decreasing	$z_i^{CP^*} > z_i^{CPR^*} > z_i^{CR^*} > z_i^{PO^*}$
$\pi_{R_i}^*$	not monotonic	decreasing	not monotonic	decreasing	$\pi_{R_i}^{CP^*} \geq \pi_{R_i}^{CPR^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for small β $\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_{R_i}^{CR^*}$ for large β
π_S^*	decreasing	decreasing	decreasing	decreasing	$\pi_S^{CPR^*} > \pi_S^{CP^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for small β $\pi_S^{CPR^*} > \pi_S^{CR^*} > \pi_S^{CP^*} > \pi_S^{PO^*}$ for large β
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	increasing	increasing	increasing	increasing	$PO > CP > CPR > CR$

Table 4 **The effect of the price sensitivity parameter β on the equilibrium decisions and profits**

The effect of retailer differentiation

We find that the supplier's revenue share $\breve{r}$ is a non-increasing function of γ while the consignment price w_r is increasing in γ (as depicted in Figure 22). This reflects the fact that the supplier cannot simultaneously increase both the consignment price and the revenue share because this would lead to a very high retail price.

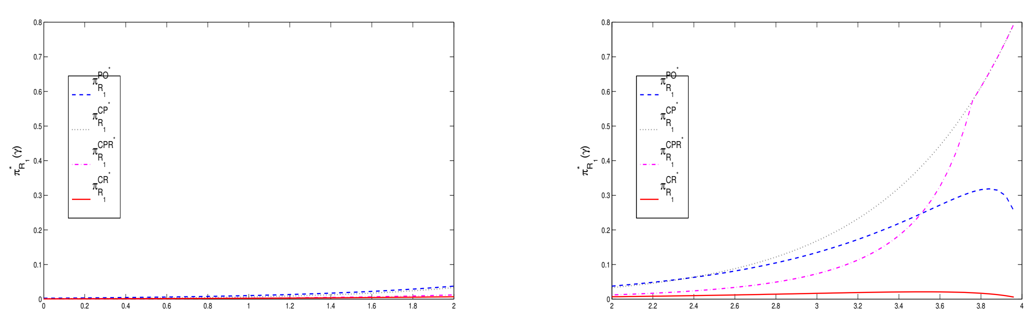
The effect of retailer differentiation on the retail price, the order quantity, the retailers' profits, and the supplier profit are consistent with the findings in the CP contract (i.e., they are increasing in γ). Figure 23 depicts that the retailer suffers the highest profit loss in the CPR contract, when retailers are more differentiated. This loss in profit decreases as retailers are less differentiated (a higher value of γ). On the other hand, figure 24 shows that the supplier earns a highest profit from the CPR contract, when the retailers are more differentiated. This benefit decreases as retailers become less differentiated. The effect of retailer differentiation on the decisions and profits for each of the supply chain members is summarized in Table 5.



(a)Supplier's prices w_p^* (CP), w^* (CR) and w_r^* (CPR)

(b)Supplier's revenue share $\breve{r}^*$ (CPR) and r^* (CR)

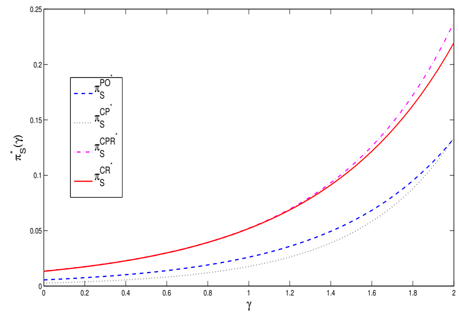
Figure 22 Supplier's price or revenue share at equilibrium as a function of γ under four contracts, where $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



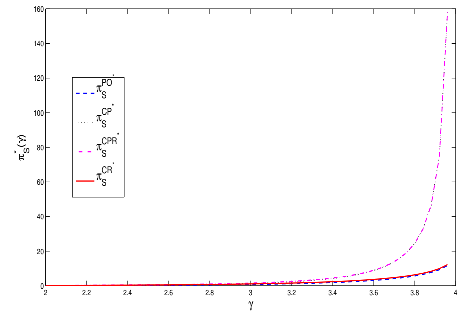
(a)Retailer i's profit $\pi_{R_i}^*$ for high retailer differentiation

(b)Retailer i's profit $\pi_{R_i}^*$ for low retailer differentiation

Figure 23 Retailer i's profit at equilibrium as a function of γ under four contracts, where $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$



(a)Supplier's profit π_S^* for high retailer differentiation



(b)Supplier's profit π_S^* for low retailer differentiation

Figure 24 Supplier's profit at equilibrium as a function of γ under four contracts, where $\beta = 4,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125,\ c = 1$

Conclusion

Our numerical study shows that the CPR contract yields the highest profit to the supplier, as compared with the other types of consignment contracts considered, namely CP and CR contracts, for any level of price sensitivity. Our numerical study also illustrates that the benefit of the CPR contract to the supplier and the retailers depends upon the level of retailer differentiation. When retailer differentiation is strong, the CP contract yields highest profits to the retailers; when it is weak, the CPR contract yields highest profits. Conversely, the supplier earns a higher profit in the CPR contract than the other types of consignment contracts when retailer differentiation is strong. The CP contract is more beneficial to the supplier as retailers become less differentiated. The comparison of these three types of consignment contracts is summarized in Table 6.

Decisions and profits	PO contract	CP contract	CR contract	CPR contract	Remark
w_p^* or w^* or w_r^*	increasing	increasing	-	increasing	$w^* > w_p^* > w_r^*$ for small γ $w^* > w_r^* > w_p^*$ for large γ
r^* or $\breve{r}^*$	-	-	increasing	non-increasing	$\breve{r}^* > r^*$ for small γ $r^* > \breve{r}^*$ for large γ
p_i^*	increasing	increasing	increasing	increasing	$p_i^{CP^*} > p_i^{PO^*} > p_i^{CR^*} > p_i^{CPR^*}$ for small γ $p_i^{CP^*} > p_i^{CPR^*} > p_i^{PO^*} > p_i^{CR^*}$ for large γ
Q_i^*	not monotonic	increasing	not monotonic	increasing	$Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*} > Q_i^{CP^*}$ for small γ $Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{PO^*}$ for large γ
z_i^*	decreasing	independent	decreasing	non-decreasing	$z_i^{CP^*} > z_i^{CR^*} > z_i^{PO^*} > z_i^{CPR^*}$ $z_i^{CP^*} \geq z_i^{CPR^*} > z_i^{CR^*} > z_i^{PO^*}$ for large γ
$\pi_{R_i}^*$	not monotonic	increasing	not monotonic	increasing	$\pi_{R_i}^{PO^*} > \pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*} > \pi_{R_i}^{CPR^*}$ for small γ $\pi_{R_i}^{CP^*} \geq \pi_{R_i}^{CPR^*} > \pi_{R_i}^{PO^*} > \pi_{R_i}^{CR^*}$ for large γ
π_S^*	increasing	increasing	increasing	increasing	$\pi_S^{CPR^*} > \pi_{R_i}^{CR^*} > \pi_S^{PO^*} > \pi_S^{CP^*}$ for small γ $\pi_S^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_S^{CR^*} > \pi_S^{PO^*}$ for large γ
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	decreasing	decreasing	decreasing	non-increasing	$PO > CP \geq CR > CPR$ for small γ $PO > CP > CPR > CR$ for large γ

Table 5 The effect of parameter γ on the equilibrium decisions and profits

Decisions and profits	Retailer differentiation
p_i^*	$p_i^{CP^*} > p_i^{CR^*} > p_i^{CPR^*}$ for more retailer differentiation (small γ) $p_i^{CP^*} > p_i^{CPR^*} > p_i^{CR^*}$ for less retailer differentiation (large γ)
Q_i^*	$Q_i^{CPR^*} > Q_i^{CR^*} > Q_i^{CP^*}$ for more retailer differentiation (small γ) $Q_i^{CP^*} > Q_i^{CPR^*} > Q_i^{CR^*}$ for less retailer differentiation (large γ)
z_i^*	$z_i^{CP^*} > z_i^{CR^*} > z_i^{CPR^*}$ for more retailer differentiation (small γ) $z_i^{CP^*} \geq z_i^{CPR^*} > z_i^{CR^*}$ for less retailer differentiation (large γ)
$\pi_{R_i}^*$	$\pi_{R_i}^{CP^*} > \pi_{R_i}^{CR^*} > \pi_{R_i}^{CPR^*}$ for more retailer differentiation (small γ) $\pi_{R_i}^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_{R_i}^{CR^*}$ for more retailer differentiation (small γ)
π_S^*	$\pi_S^{CPR^*} > \pi_{R_i}^{CR^*} > \pi_S^{CP^*}$ for more retailer differentiation (small γ) $\pi_S^{CP^*} > \pi_{R_i}^{CPR^*} > \pi_S^{CPR^*}$ for less retailer differentiation (large γ)
$\frac{\pi_{R_1}^* + \pi_{R_2}^*}{\pi_S^*}$	$CP \geq CR > CPR$ for more retailer differentiation (small γ) $CP > CPR > CR$ for less retailer differentiation (large γ)

Table 6 The comparison of three types of consignment contracts

Appendix G: Comparison between consignment contracts and revenue-sharing contracts

In the paper, we use the price-only contract as a benchmark for evaluating consignment contracts. In this section, we compare consignment contracts with revenue-sharing contracts.

The key difference between revenue-sharing and consignment contracts is the time of payment and the ownership of inventory. In revenue-sharing contracts, the retailers pay the supplier a wholesale price for each unit ordered in addition to a percentage of the revenue the retailers generate (Cachon and Lariviere [2005], Yao et al. [2008b], Linh and Hong [2009], Pan et al. [2010]). Thus, the retailers have full ownership of the inventory and bear all the risk of unsold units remaining . The supplier receive the payment for all the units ordered by retailers, regardless of whether or not the retailer sells them, in addition to a percentage of the retailers' revenue. In a consignment contract, the supplier retains ownership of merchandise even though items are at retail locations. The supplier receives no payment until the items are sold by retailers. Therefore, the risk of underselling is now born by the supplier.

In the revenue sharing (RS) contract, decisions are made in two steps. In the first step, the supplier decides the wholesale price w_p for each unit ordered by retailers and the revenue share r to be received from the retailers for each unit sold to consumers. In the second step, given this wholesale price and revenue share, each retailer simultaneously chooses the retail price p_i and order quantity Q_i. We first derive each retailer's best response and inventory quantity to the supplier's wholesale price and revenue share decisions.

Retailer i's selling price and stocking factor decision

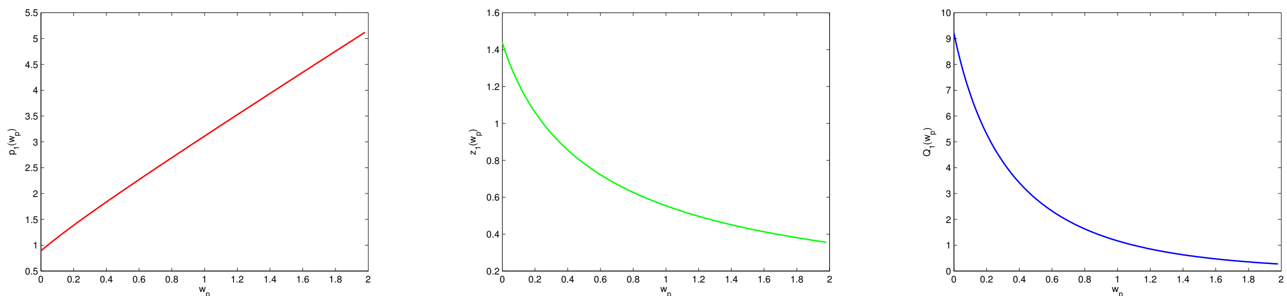
For a given stocking factor z_i, consignment price $w_p > 0$, revenue share r and price p_{-i} of retailer $-i$, retailer i's unique best response price $\tilde{p}_i(z_i|w_p, r)$ is

$$\tilde{p}_i(z_i|w_p, r) = \frac{1}{\beta} + \frac{(c\alpha_i + w_p)z_i}{(1-r)(z_i - \Lambda(z_i))}.$$

The retailer i's best response stocking factor $\bar{z}_i$ that maximizes the retailer i's profit for a given w_p and a given r is uniquely determined as the solution of:

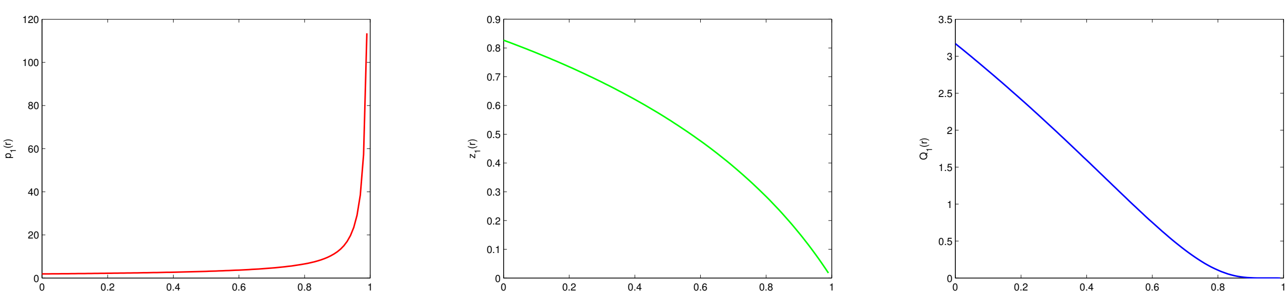
$$\frac{1-r}{(c\alpha_i + w_p)\beta} + \frac{\bar{z}_i}{\bar{z}_i - \Lambda(\bar{z}_i)} = \frac{1}{1 - F(\bar{z}_i)}.$$

In particular, the RS best response stocking factor depends on both the supplier's wholesale price w_p and the supplier's revenue share r. This result is consistent with best response obtained in both the PO and the CR contracts. The retailer's best response quantity is $y_i(\mathbf{p})\bar{z}_i$, where $y_i(\mathbf{p}) = ae^{-\beta p_i + \gamma p_{-i}}$.



(a)Best response price $\bar{p_1}$ (b)Best response stocking factor $\bar{z}_1$ (c)Best response quantity $\bar{Q}_1$

Figure 25 **Retailer 1's best response price, stocking factor and quantity as a function of the wholesale price w_p when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and r is fixed at 0.5 in the RS contract**



(a)Best response price $\bar{p_1}$ (b)Best response stocking factor $\bar{z}_1$ (c)Best response quantity $\bar{Q}_1$

Figure 26 **Retailer 1's best response price, stocking factor and quantity as a function of the revenue share r when $\beta = 2,\ \gamma = 1.5,\ a = 10,\ \alpha_1 = \alpha_2 = 0.125$ and w_p is fixed at 1 in the RS contract**

Figures 25 and 26 illustrate that the retailer's best response retail price increases with both the supplier's wholesale price w_p and revenue share r: when the supplier keeps a higher wholesale price and/or a higher share of the retailer's revenue, the retailers transfer the increasing cost to consumers. The higher price causes the demand to decrease, which leads to a lower quantity at each retailer. While both the expected demand and order quantity decrease with the supplier's wholesale price and revenue share, the order quantity decreases faster than the expected demand. Therefore, the stocking factor decreases with the supplier's revenue.

Supplier's revenue sharing fraction decision

At the first step, anticipating the retailers' reaction to her decision, the supplier sets the wholesale price w_p and revenue share r to maximize her own expected profit $\pi_S(w,r)$, given by

$$\begin{aligned}\pi_S(w,r) &= r\bar{p_1}E\{\min(D_1,\bar{Q_1})\} + r\bar{p_2}E\{\min(D_2,\bar{Q_2})\} + (w_p - c(1-\alpha_1-\alpha_2))(\bar{Q_1}+\bar{Q_2})\\ &= y_1(\bar{\mathbf{p}}))[r\bar{p_1}(\bar{z_1}-\Lambda(\bar{z_1})) + (w_p - c(1-\alpha_1-\alpha_2))\bar{z_1}] + y_2(\bar{\mathbf{p}}))[r\bar{p_2}(\bar{z_2}-\Lambda(\bar{z_2})) + (w_p - c(1-\alpha_1-\alpha_2))\bar{z_2}]\\ &= ae^{-\beta\bar{p}_1+\gamma\bar{p}_2}[r\bar{p}_1(\bar{z}_1-\Lambda(\bar{z}_1)) + (w_p - c(1-\alpha_1-\alpha_2))\bar{z}_1]\\ &\quad + ae^{\beta\bar{p}_2+\gamma\bar{p}_1}[r\bar{p}_2(\bar{z}_2-\Lambda(\bar{z}_2)) + (w_p - c(1-\alpha_1-\alpha_2))\bar{z}_2].\end{aligned}$$

To find the equilibrium solutions, denoted by r^* and $w_p^* = \tilde{w_p}(r^*)$, we have to maximize $\pi_S(\tilde{w_p}(r), r)$ over r. Obtaining an analytical solution for this maximization problem is intractable, therefore, we use numerical methods.

Numerical analysis

Following our previous numerical study, we set $a = 10$, $c = 1$, and $\alpha_1 = \alpha_2 = 0.125$ and the random perturbation on the demand ϵ is assumed to follow a uniform distribution on $[0, 2]$. For the analysis of the effect of the price sensitivity parameter, we fix $\gamma = 2$; for the analysis of the effect of retailer differentiation, we fix $\beta = 4$.

Our numerical study shows that the equilibrium supplier's wholesale price w_p^* is very close to zero for any given β and γ. This means that at equilibrium, the supplier's income (or profit) would almost entirely come from the fraction of the retailers' revenue. The retailers pay almost nothing for each unit purchased from the supplier but remit a share of their revenue to the supplier (for each unit sold to consumers). Indeed, the supplier is better off not charging the retailers for each unit ordered to incentivize them to order more and price adequately. Essentially, the revenue-sharing contract is equivalent to our consignment with revenue share (CR) contract. The results and conclusions of the effect of the price sensitivity parameter and the effect of retailer differentiation on the equilibrium decisions and profits of the supply chain members for the CR contract remain valid for the RS contract.