AN INVESTIGATION IN REAL-TIME BUS HOLDING POLICY

Qin CHEN [a], Elodie ADIDA [b], Jie (Jane) LIN [a]

[a] *Department of Civil and Materials Engineering, University of Illinois at Chicago, 842 W. Taylor Street (M/C 246), Chicago, IL 60607. Email: qchen23@uic.edu; janelin@uic.edu*
[b] *Department of Mechanical and Industrial Engineering, University of Illinois at Chicago, 842 W. Taylor Street, Chicago, IL 60607. Email: elodie@uic.edu*

ABSTRACT

This paper investigates a control strategy of holding a group of buses at a single or multiple control points. By incorporating any possible passenger boarding activities during holding, a single control point problem (P) is developed and extended to multiple control points. The problem (P) is a non-convex optimization programming with linear constraints that minimizes the total passenger waiting time both on-board and at stops. A heuristic is then developed that is easy and fast to implement, which makes it suitable for real-time implementation. The model is evaluated with simulations of the real-time bus operation data from the Chicago Transit Authority. The simulation results show considering the boarding activities in the total waiting time mitigates the error propagation and stabilizes the holding performance, compared to the common models in the literature, which do not consider boarding while holding.

1. INTRODUCTION

Vehicle holding is a commonly used strategy among a variety of control strategies in transit operation in order to reduce bus bunching and regulate bus headways. Often it is formulated as an optimization problem to minimize the passenger waiting time after a holding policy is implemented. For example, Hickman (2001) formulated a model for holding one vehicle at a time at a given control point and treated the vehicle running time stochastically. Eberlein et al. (2001) used a rolling-horizon scheme to argue that a deterministic model can be used when the variance of the stochastic elements are sufficiently small. Sun and Hickman (2008) presented models with holding groups of buses at multiple stops and concluded that multiple holdings brought more benefit than just a single holding. By considering the vehicle capacity constraint, Delgado (2009) combined holding strategy and passenger boarding preventing strategy together to formulate a non-convex problem and a heuristic was developed to help solving the problem.

However, all of the studies above assumed that no boarding or alighting is allowed while holding. For example, Sun and Hickman (2008) enacted holding when bus arrives at the control point, the on-board waiting time for those boarding while holding is not considered. While that simplification has greatly reduced the complexity of the problem, it has often introduced inconsistency that pertains to how to treat the holding time in relation to dwell time, headway and subsequently the number of passengers boarding and alighting at the control point. Such inconsistency may become problematic especially when the demand is high at the stop and the holding time is relatively long. It is also worth noting that few studies used the real world data to validate their algorithms (e.g., Eberlein, 2001 and Fu and Yang, 2002). To the authors' best knowledge, there has been no real-world headway based bus holding study because of the limitation of data source and the input parameters that are not always available in the real world data.

In this study, we investigate a holding strategy for multiple buses at a given control point and formulate the problem similarly to the past studies with the exception that possible passenger boarding during holding is explicitly taken into account in our model. This significantly increases the complexity of the problem due to the induced interdependence of variables among vehicles and stops.

One of the important features of the proposed algorithm is its suitability for real-time application (i.e., fast computation). We have also successfully extended the model to multiple control points to reduce the variance of headways for the downstream stops – due to the page limitation the results of the multiple stops scenario will not be presented here. Lastly, the model is evaluated with a simulation case study by using the real-time bus operation data (i.e., automatic vehicle location (AVL) and automatic passenger Count (APC) data) from the Chicago Transit Authority (CTA).

2. PROBLEM FORMULATION

2.1 The Problem P

First, there are a number of assumptions made before the problem is formulated:

1) Running time between stops and service rate at stop are deterministic;
2) Passenger boarding during holding follows the same arrival rate as that during the regular dwell time. The arrivals are uniformly distributed;
3) Dwell time is an affine function of the number of boarding passengers only (i.e., assuming boarding is the dominant activity in a front-on rear-off boarding policy);
4) The number of alighting passengers from bus *i* at stop *j* is proportional to the number of passengers on board at the time of arrival;
5) No overtaking is allowed. This is a simplification commonly used in the literature (e.g., Sun and Hickman, 2008);
6) No vehicle capacity constraint is considered, which is based on the observations of the real-time bus AVL and APC data from the Chicago Transit Authority (CTA) to be described later.

We first formulate the problem (denoted as P) of holding a group of buses (called controlled buses) at a predefined control point (denoted as stop *k*). We want to determine the holding time for each bus in the bus set (denoted as Im for any controlled bus $i \in \mathrm{Im}$) at stop *k* to minimize the passenger waiting time both on-board and at the downstream stops. The downstream impacted stop set on the study route is denoted as In for any impact stop $j \in \mathrm{In}$. The rest of the notation is listed below:

r_j ---deterministic passenger arrival rate at stop j;

q_j ---alighting fraction to the number of on-board passengers at stop j;

$A_{i,j}$ ---number of alighting passengers off bus i at stop j;

$B_{i,j}$ ---number of boarding passengers onto bus i at time of departure from stop j;

$b_{i,j}$ ---number of passengers boarding bus i at the end of the regular dwell time at stop j; observe that $b \le B$ at the control point, and $b = B$ at other stops;

$L_{i,j}$ ---passenger load on bus i at time of departure from stop j;

R_j ---deterministic running time between stops $j-1$ and j;

α, β ---parameters in determining dwell time as a function of boarding;

$S_{i,j}$ ---regular dwell time for bus i at stop j (i.e., no holding time);

$t_{i,k}$ ---holding time for bus i at control point k;

$a_{i,j}$ ---arrival time of bus i at stop j;

$d_{i,j}$ ---departure time of bus i at stop j: $d_{i,j} = a_{i,j} + S_{i,j}$ if $j \ne k$ and $d_{i,k} = a_{i,k} + S_{i,k} + t_{i,k}$.

The problem (P) of multi-bus holding at a control point k can be formulated as follows:

$$\min Z = \sum_{In}\sum_{\mathrm{Im}} (a_{i,j} - d_{i-1,j})^2 \times r_j / 2 + \sum_{\mathrm{Im}} t_{i,k} \times (L_{i,k} - r_k \times t_{i,k}) + \sum_{\mathrm{Im}} t_{i,k}{}^2 \times r_k / 2 \qquad (1)$$

St.
$$S_{i,j} = \alpha + \beta \times b_{i,j}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (2)$$
$$A_{i,j} = q_j \times L_{i,j-1}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (3)$$
$$B_{i,j} = r_j \times (d_{i,j} - d_{i-1,j}), \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (4)$$
$$b_{i,j} = B_{i,j}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n, j \neq k \quad (5)$$
$$b_{i,k} = r_k \times (d_{i,k} - t_{i,k} - d_{i-1,k}), \forall i \in \mathrm{Im} \quad (6)$$
$$d_{i,j} = a_{i,j} + S_{i,j}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n, j \neq k \quad (7)$$
$$d_{i,k} = a_{i,k} + S_{i,k} + t_{i,k}, \forall i \in \mathrm{Im} \quad (8)$$
$$a_{i,j} = d_{i,j-1} + R_j, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (9)$$
$$d_{i,j} \leq a_{i+1,j}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (10)$$
$$L_{i,j} = L_{i,j-1} + B_{i,j} - A_{i,j}, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (11)$$
$$t_{i,k} \geq 0, \forall i \in \mathrm{Im} \quad (12)$$
$$a_{i,j} \geq 0, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (13)$$
$$d_{i,j} \geq 0, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (14)$$
$$L_{i,j} \geq 0, \forall i \in \mathrm{Im}, j \in \mathrm{I}n \quad (15)$$

In the objective function, both at-stop and on-board passenger waiting times due to holding are considered. The first term in equation (1) is the at-stop passenger waiting time since the departure of the preceding bus (*i-1*) till the arrival of the current bus (*i*) at all impacted stops; the second term is the on-board waiting time excluding the additional passengers that boarded during holding; and the last term is the on-board waiting time for those additional boarding passengers during holding.

Constraints (2) and (3) define dwell time and passenger alighting respectively. (4) to (6) define boarding separately for the control point (i.e., stop *k*) and the non-control points (i.e., downstream impacted stops in In) . (7) and (8) are departure times at the non-control points and the control point respectively. With the deterministic running time assumption, (9) defines arrival time at the impacted stops and (10) says the arrival time of bus *i+1* at stop *j* can never precede the departure time of bus *i* at stop *j*, i.e. no overtaking is allowed. (11) means the loading when departs from stop *j* equals to the loading when departs from stop *j-1* plus the boarding and alighting activity at stop *j* . Lastly, (12)-(15) say that all the variables are non-negative.

The problem is a non-linear program with a non-convex quadratic objective function under linear constraints. The inclusion in the formulation of boarding activities while holding makes the quadratic term matrix not positive defined. Furthermore, the on-board cost (the last two terms) is a non-linear addition to the non-convexity of the problem. There is no closed form solution to the problem. To solve it, we have developed a heuristic as follows.

2.2 The Heuristic P0

The idea is to transform the original problem (P) to a convex one and solve it iteratively and heuristically. To achieve that, an iterative holding time variable $T_{i,k}(p)$ is introduced as the current holding time (of bus *i* at stop *k*) in iteration *p*. Specifically, here are the steps in the proposed heuristic:

Step 0: Initialization: iteration number $p = 0$, and initial holding time $T_{i,k}(0) = 0$;

Step 1: Iteration: at iteration p , combine the latter two items in the objective function, and rewrite the objective function as
$$\min Z = \sum_{In} \sum_{\mathrm{Im}} (a_{i,j} - d_{i-1,j})^2 \times r_j / 2 + \sum_{\mathrm{Im}} T_{i,k}(p) \times (L_{i,k} - r_k \times t_{i,k} / 2) \quad (16)$$

Notice that by changing the decision variable $t_{i,k}$ to an iterative term $T_{i,k}(p)$, the revised objective function as shown in equation (16) becomes convex and easy to solve. Solve for holding time $t_{i,k}$; and Step 2: If $|t_{i,k} - T_{i,k}(p)| > \varepsilon$ (a predefined tolerance level), then $p = p + 1$, and $T_{i,k}(p+1) = t_{i,k}$, go to Step 1; or else stop.

An important feature of the proposed algorithm is its fast computing time, which makes it suitable for online real-time implementation of bus holding strategies, even when the problem scale is comparatively large, i.e., with large number of controlled buses and impacted stops. For example, when ε=1.0e-5, the algorithm converges within twenty iterations in less than 0.5 seconds with the problem size of ten controlled buses and ten impacted stops. Moreover, we noticed that by carefully selecting the starting point for the search the algorithm converges even faster with less number of iterations.

It is worth mentioning that we have also developed an algorithm (MP) for multiple-stop bus holding. Due to the space limitation, neither the algorithm nor the MP case study results are presented in this paper.

3. CASE-STUDY OF REAL-TIME BUS HOLDING IMPLEMENTATION

To demonstrate the implementation of the proposed bus holding policy, especially in the real time environment, a simulation case study is designed to use the online bus location and count data and determine bus holding in real time.

3.1 Real-time Bus data from Chicago Transit Authority (CTA)

Chicago Transit Authority (CTA) serves the city of Chicago and the surrounding suburbs; all buses are equipped with the AVL system and majority has the APC system. In this study we use the same data set as used in Lin and Ruan (2009). The study route is a CTA route on a major urban street on the west side of downtown Chicago, which intersects with many other bus routes and connects to the subway system, leading to a relatively high demand of passenger boarding and alighting. It takes about 80 minutes to run on the 14 mile route and in total there are about 110 stops and 13 time points, which are planned geographic locations (often physical stops) in a bus route with the scheduled arrival times for on-time service monitoring and scheduling purposes by CTA. To reduce the amount of data processing without losing the reality, control points for holding are chosen among time points.

One month of original weekday AVL and APC (about 15% of all bus runs) records in September 2006, which contain bus operational events (e.g., serviced a stop, dwell time, passenger counts, arrival time, departure time) at the time-point level, is used to compute and calibrate the model parameters: arrival rate r_j ,alighting fraction q_j, parameters in dwell time-boarding function α and β .The passenger load exceeding thirty-five during the peak hours only accounts for 3% of all the observed bus trips during the same period. Hence, the assumption of no vehicle capacity constraint is considered satisfied in the study. The average headway is found to be 7 to 11 minutes. The average arrival rate r is calculated to be 1.7 persons per minute and the alighting fraction q to be 0.06. A linear regression model based on the dwell time and number of boarding's estimates the values of α =0.05 and β =0.08 in Eq. (2). With the computed parameters, the simulation is based on one-day southbound bus trip records during the morning peak hours on Wednesday, Sep 13, 2006 from 7:00AM to 10: 00AM. The running time was typically 10 to 12 minutes between time points and there were 12 buses running in the route with a 6-10 min headway during the peak hours.

3.2 Performance Measures

Following other studies in the literature (eg. Fu and Yang 2002, Hickman 2001), this study uses the following metrics to evaluate the effectiveness of the holding strategy: 1) *Average saved time* (passenger-minutes per bus per stop) compared with no holding scenario. It is formulated as the changes of the objective value minus the extra on-board waiting time, averaged by the controlled buses and impacted stops; 2) *Holding time* for bus *i* (minutes); 3) *Average holding time* (minutes per bus); 4) *Variance of headway* at the downstream stop *j*; and 5) *Total computing time* (CPU seconds) coded in Matlab on a 2.2GHz Intel Core 2 Duo CPU computer. .

3.3 Simulation Results

3.3.1 The Effectiveness of the Proposed Model

Applying the model (P) to the bus operation data, we set the control point at stop 2 with downstream impacted stops {3,4,5} and used the controlled bus groups {2,3,4,5,6} bounded by buses 1 and 7. With seven iterations and a computing time of 0.213 seconds, the simulation gives the converged solution of holding time for buses {2, 3, 4, 5, 6} to be {5.24, 3.13, 4.51, 4.39, 3.83} minutes respectively at the control point (stop 2). The average holding time per bus is 4.22 minutes and the average saved time per bus per stop is 201.17 passenger-minutes, 15.32% reduction from the no holding waiting time. The before and after holding vehicle trajectories are presented in Figure 1, showing more regular headways after implementing holding than that without holding. Note that in the vertical axis the control point (stop 2) is marked as 0 and subsequently the downstream impacted stops 3, 4, and 5 are marked as 1, 2, 3 in the figures.

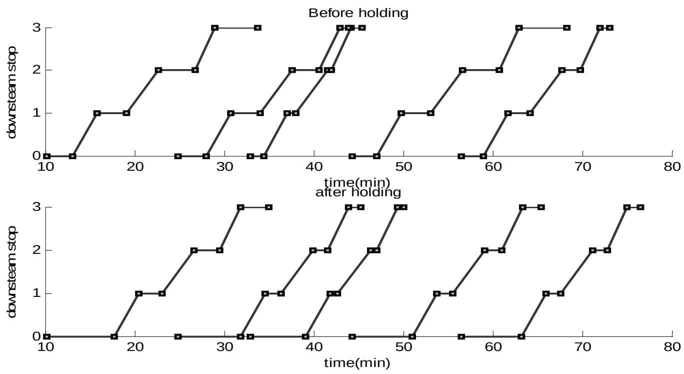

Figure 1. Vehicle trajectories with/without holding strategy

3.3.2 Advantages of Consider Boarding Activity While Holding

An important feature that distinguishes our model to other models in the literature is the inclusion of boarding activity during holding. An obvious advantage of our model is its close replication of the reality. In addition, it is of great interest to investigate the performance of our model as compared to those that typically do not consider boarding activities while holding (referred to as "traditional model" hereafter) in terms of the performance measures defined above. To do so, we have also constructed a traditional model (T) with the similar structure to our proposed model with the exception that model T does not include passenger activities while holding by changing the objective function (1) to

$$\min Z = \sum_{In} \sum_{\mathrm{Im}} (a_{i,j} - d_{i-1,j})^2 \times r_j / 2 + \sum_{\mathrm{Im}} t_{i,k} \times L_{i,k} \tag{22}$$

Therefore, under the constraints (2)-(15), the new model is a convex quadratic problem, which could be readily solved using standard optimization software packages. To be consistent, we again used Matlab to code the model (T).

In this comparison, we created six scenarios by varying the arrival rate from 0.7 to 3.2 persons per minute, i.e., from half to twice of the observed arrival rates. The number of impacted stops and controlled buses were fixed at four and three, respectively.

Because the waiting times in the objective function are formulated differently, it makes no sense to compare the two models by extra waiting time and saved time. Hence, only average holding times and headway variances at the subsequent three downstream stops are compared between the two models. Table 1 shows the results. The average holding times at stops are quite similar in the two models, suggesting both models converge to the similar holding strategies, which is a desirable feature in either model. However, the headway variances at the downstream stops are much smaller in our model than that in the traditional model. Additionally, with the increase of arrival rate, the headway variance increases much faster in the traditional model than in the proposed model. These findings seem to suggest that by taking boarding activity into account in total waiting time determination our model provides the holding strategies with more stable holding performance due to less error propagation that the traditional model do.

Table 1 Holding time and headway variation for proposed model and traditional model

Arrival rate (persons/min)	**Proposed model (P0)**				**Traditional model (T)**			
	Hold time (min)	**Variance of headway**			**Hold time (min)**	**Variance of headway**		
0.7	4.1202	0.1343	0.1645	0.1973	4.0645	0.6164	0.6214	0.6340
1.2	3.5661	0.1736	0.2370	0.3200	3.5662	0.7967	0.7953	0.8106
1.7	2.7185	0.3101	0.6178	1.0139	2.7853	1.2039	1.2201	1.2807
2.2	1.9306	0.4925	0.5617	0.6863	1.5418	1.8362	1.9758	1.9899
2.7	3.4145	0.8421	1.8021	2.2177	3.8009	2.3342	2.3515	2.2944
3.2	2.3587	1.4959	1.3238	1.0807	2.1711	4.7148	3.6174	3.9604

3.3.3 Sensitivity Analysis for Single Control Point Problem

The first objective of sensitivity analysis was to investigate the bus holding performance with respect to the number of impacted stops. In the simulations, we used the similar setting to that in section 3.2.2 (the number of controlled buses was set to be three) and set the arrival rate at 1.7 persons per minute. By varying the number of impacted stops we created up to seven scenarios, the results of which are presented in Table 2. As the number of impacted stops increases, the average holding time decreases as expected because more at-stop waiting time is considered. Consequently, the average saved time increases, which suggests increased benefit of more impacted stops considered. On the other hand, when the number of impacted stops exceeds four, the improvements of holding time and saved time become marginal. As expected, the headway variance increases as the stop moves further away from the control point. This finding suggests that in practice no more than five impacted stops would be sufficient to be taken into consideration.

Table 2. Simulation results by number of impacted stops

# impacted stops	**CPU time (Sec)**	**Average holding time**	**Average Saved time**	**Headway variance**						
1	0.069	6.78	304.00	1.42						
2	0.073	6.35	372.15	0.89	0.73					
3	0.163	6.02	324.04	0.56	0.39	0.92				
4	0.251	4.27	488.78	0.33	0.96	1.26	1.93			
5	0.339	4.27	481.98	0.33	0.96	1.26	1.93	2.43		
6	0.341	4.27	481.98	0.33	0.96	1.26	1.93	2.43	3.75	
7	0.340	4.27	481.98	0.33	0.96	1.26	1.93	2.43	3.75	5.42

Secondly, we investigated how demand and headway affected the holding performance. In this part we used randomly generated dispatching time instead of the CTA bus dispatching data to generate different levels of headway: short headways of 2 to 5 minutes; medium headways of 8-12 minutes, and large headways of 15-20 minutes. Similar to the setting in section 3.2.2, the number of impacted stops and controlled buses were fixed at four and three. And passenger demand (arrival rate) is varied from 0.7 persons per minute to 3.2 persons per minute at each level. Average saved time and average holding time by level of headway and arrival rate are shown in Figures 2 and 3 respectively. It is seen that the benefit of saved time increases as demand goes up. Meanwhile the average holding time also increases, which is a result of heavier passenger activities at stops. By varying the headways, the holding time increases as headway increases. The reason may be due to the wider operational headways apparently requiring long holding time if holding is necessary.

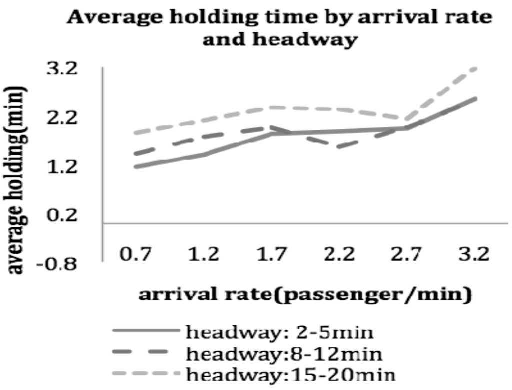

Figure 2. Holding time by headway level

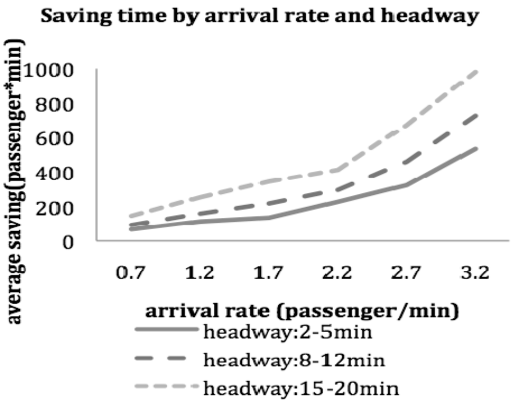

Figure 3. Saved time by headway level

4. CONCLUSION

This study has presented a holding strategy of groups of buses at multiple control points. First we formulated the problem of holding a group of buses at a predefined control point. Considering possible passenger boarding during holding, the model was formulated as a non-convex optimization program with linear constrains and a heuristic was developed to help solve the problem. Furthermore, the algorithm was expanded to multiple control points. The model was evaluated with a simulation case study by using the real-time bus operation data from CTA. There are several findings in this study:

(1) By simulating the proposed model based on the CTA real-time operation data, it demonstrated that the holding strategy indeed reduce the waiting time and improve the performance. The fast computing time of the proposed algorithm is another attraction in terms of online implementation of bus holding polices;
(2) Comparing our model with the traditional model which does not consider boarding activity while holding, it is found that our model decreases the variance of downstream headways and outperforms the traditional model in that regard, especially when the demand is high; and,
(3) Sensitivity analysis shows that in practice implementing holding strategies in four consecutive downstream stops seem to be sufficient for achieving stable performance (in terms of average holding time, average saved time, and head variance). The benefit of holding increases as demand goes up.

There are several limitations that are worth noting so that the findings should be interpreted within the context. Firstly, the deterministic running time assumption is a very strong one. In reality, running time is stochastic. In future work that assumption may be relaxed. Secondly, where to set up the control point(s) is critical and must be investigated further in future work. The demand distribution along the route may greatly affect the holding strategies at the chosen control point(s). Moreover, major transfer stops must be taken into consideration of the control point selection. For example, if a major transfer point is at the downstream of the control point, large amount of delays may occur due to holding. Lastly, holding strategy is intended to reduce bus bunching by inserting slack into bus schedule, when a bus is falling behind the schedule, holding does not help with the schedule adherence. Therefore holding strategies may be combined with other control strategies, e.g., stop skipping and adjusting bus cruising speed, in order to archive a better performance.

REFERENCES

Delgado, F., Muñoz, J.C. and Giesen, R. (2009) Real-Time Control of Buses in a Transit Corridor Based on Vehicle Holding and Boarding Limits .*Transportation Research Record.* Vol. 2090, pp. 59-67.

Eberlein, X.J., Wilson, N.H.M. and Bernstein, D. (2001) The Holding Problem with Real-time Information Available. *Transportation Science* 35(1): 1-18.

Fu, L. and Yang, X.(2002) Design and Implementation of Bus-holding Control Strategies with Real-Time Information. *Transportation Research Record.* Vol. 1791, pp. 6-10.

Hickman, M. (2001) An Analytical Stochastic Model for the Transit Vehicle Holding Problem. *Transportation Science* 35(3): 215-237.

Lin, J. and Ruan, M. (2009) Probability-based bus headway regularity measure. *IET Intelligent Transport Systems*, Vol. 3, Iss. 4, pp. 400-408

Sun, A. and Hickman, M. (2008) The Holding Problem at Multiple Holding Stations. *Computer-aided Systems in Public Transport*, SPRINGER VERLAG, pp. 339-362.