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Point Spreads versus Odds

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Why do bookies use the point-spread device for wagering on football games when odds betting on the winner is also feasible? One possibility is that point-spread betting is more fun than odds betting. The existence of point spreads, however, does not logically entail this "more fun" explanation. Given simple specifications of beliefs and betting behavior, and given that point spreads and odds are equally thrilling, it is demonstrated that the point-spread game can exist as the game produced by a profit-maximizing bookie. The disparate beliefs which create the demand for some form of betting market can also explain the particular betting game produced by the bookie.

I

The latest point spread on the Bears-Rams football game this weekend lists the Bears as nine-point underdogs. For a 10-cent fee a bookie will accept an even-odds one-dollar wager on the event $B + 9 > R$, where B and R are the Bears' and Rams' final point totals. Based on your knowledge of the teams' relative strengths, you assign a (personalistic) probability of .7 to the event $B + 9 > R$ (and a probability of zero to $B + 9 = R$). Hence, a one-dollar (plus the 10-cent bookie fee) wager on the Bears has an expected monetary value (EMV) of $.7(1 - .1) - .3(1 + .1)$, or a positive 30 cents. Given the

An earlier version of this paper was presented at the Econometric Society Meeting, New York City, December 1977. I want to acknowledge helpful comments from a referee at this *Journal*.

alternative of not betting, you as an *EMV* maximizer will wager that the Bears will beat the spread.

Point-spread schemes characterize virtually all of the wagering in football betting markets. So-called odds betting, however, is also possible and can provide for any particular individual an *EMV* identical to the point-spread game. If the odds are 6:1 on a Bear victory ($B > R$) and if you assign a probability of .2 to this event (and a probability of zero to $B = R$), then the *EMV* of a one-dollar (plus the 10-cent bookie fee) Bears bet is $.2(6 - .1) - .8(1 + .1)$, which is again 30 cents. *Ceteris paribus*, you are indifferent between the point-spread and odds wagers. Hence the question, Why do bookies produce the point-spread betting game?

The existence of point spreads is commonly attributed to bettor preferences. This explanation is implicit in news articles on point spreads, and it is also the most frequently offered explanation in the very informal settings where I have posed the point spreads versus odds question. Under this view bettor preferences are thought to depend on more than *EMV* or expected utility—utility being defined on income or wealth. Point spreads are said to be more fun or thrilling than odds so that bettors are willing to forgo expected return with odds in order to play the point-spread game. This appeal of point spreads could reflect a general preference for even-odds wagers, or it could be simply that point spreads are more complementary with viewing a football game. But the bias in favor of even-odds wagers does not extend to other lotteries, and many football bettors presumably wager on games they do not watch. It would appear that the majority (in the informal settings) think the intrinsic delight in beating the spread is obvious and requires no deeper analysis. The answer to the point spreads versus odds question, then, is simply that point spreads are more fun than the odds betting game.

Bettor preferences provide an answer to the point spreads versus odds question, but the explanation posits rather curious tastes which cannot be understood in terms of an *EMV* or expected utility decision criterion. It is possible, however, that preferences are in fact consistent with some more traditional decision criterion, and the stated views merely reflect the difficulty of imagining football betting without the point-spread device. Alternatively, the stated views might derive from the belief that bettor preferences are the only logical explanation consistent with the existence of point spreads. Without denying the possible empirical validity of the bettor-preferences explanation, the intent here is to generate alternative hypotheses for point spreads in a more traditional setting where bettors do not consider point spreads more fun. Disparate beliefs regarding the random variable $X = R - B$ are one reason for the existence of some

betting market, and they are examined here as also providing an explanation for the particular betting game produced by the bookie.

The market of potential bettors will be assumed fixed, and each bettor will be described by a probability distribution representing beliefs regarding X . In deciding on wagers the bettors are assumed to use an *EMV* criterion. The thrill of betting is therefore supposed constant over alternative betting games, and risk preference or aversion is ignored. (Bettors might be imagined to risk only a small fraction of their income or wealth.) Betting is facilitated by a profit-maximizing bookie who is paid a fee by participating bettors in exchange for either a point-spread, an odds, or a combined-betting game. To insulate himself from the outcome of X the bookie will also establish equilibrium parameters for the betting games. (The betting market is of some independent interest because it contains an agent, the bookie, who has an *incentive* to establish these equilibrium values.) The betting market is therefore envisioned as arising for purely speculative reasons. The market exists for the sole purpose of expediting the exchange of wagers whose *EMV*s are positive rather than because of variations in initial allocations of physical-location-dated commodities or the possibility of utility-increasing risk transfers. Wagering is rational in that each bettor is made better off according to his *EMV* criterion even though it is certain that the aggregate of bettors will lose money to the bookie.

The bookie's profit-maximizing objective translates in the games described below into the objective of maximizing the betting pool—the total dollars wagered on the football teams. Given the assumptions that no wagering scheme is intrinsically more fun than any other and that all individuals wager rationally according to an *EMV* criterion, and given the example above which illustrates that for any individual there is some odds game in which betting behavior is identical to the point-spread game, it might seem that the bookie would be indifferent as to which game to produce. In the model, point spreads would then exist merely as the outcome of the bookie's random choice from a set of equivalent betting games. If a deterministic rationale for point spreads is sought, the model would then have to be abandoned in favor of the bettor-preference explanation. We will see, however, that depending on beliefs, certain betting games generate greater profits for the bookie so that an appeal to the supposed delights of point spreads is not logically necessary in order to understand the bookie's production decision.

The rationale for assuming that individual beliefs are consistent with a probability distribution comes from the personalistic or subjective theory of probability, and Winkler (1971) has found some empirical support for the theory in the football context. In the economics

literature, I know of only two references dealing directly with football betting.¹ Pankoff (1968) used football betting to analyze issues relating to the efficient-market hypothesis. Smith (1971) examined betting markets because of the equilibrating role played by the bookie and because football wagering could provide an unusual framework for examining decision making under uncertainty. He used the expected-utility hypothesis to analyze how betting behavior under various betting schemes might be influenced by wealth, beliefs, and the pleasures of gambling. While Smith did consider behavior under point spreads and odds games and his model is similar to the one presented here, he did not examine the conditions under which one betting game is produced over another.

No attempt is made here to associate differing opinions with different or incomplete information among potential bettors. In fact, potential bettors might be envisioned as having virtually identical information on the football teams' current situation and past history. Still, bettors are not presumed to agree on the distribution of X . While my casual observations suggest that this is a reasonable working hypothesis, it does conflict with the spirit of rational expectations where differing opinions are associated with and analyzed in terms of some more fundamental differences among individuals. Besides casual empiricism, the fact that the probability assigned to a future event like the winner of a football game cannot be as easily inferred as the probability assignments in, say, roulette might also serve to justify the existence and persistence of differing opinions. Despite being able to continuously update their theories by observing the outcomes of football games, potential bettors do not reach agreement. Theories are sufficiently vague or depend on so many exogenous factors that, while each bettor possesses a well-defined distribution for X , difference of opinion exists and persists. Difference of opinion is what makes for a horse race and is what is examined here as an explanation for the point-spreads versus odds question.

The plan of the rest of the paper is as follows. Section II describes the bookie's objectives and production possibilities, that is, the games which can be offered to bettors. A simple parameterization of market beliefs and a betting criterion are presented in Section III. The assumptions are adopted in order to illustrate how point spreads might beat odds and also to determine in a simple model the conditions for the optimality of a betting game. Section IV begins the exercise by working through a special case. Propositions in Section V express equilibrium values for the betting games and bookie profits as a function of the parameters describing market beliefs. Section VI

¹ I am indebted to Stephen Salant for making me aware of these references.

then presents conditions under which point spreads beat odds. The assumptions on beliefs and betting criteria are relaxed in Section VII and another example is presented. The final section contains some concluding remarks.

II

The bookie brings together individuals with disparate beliefs regarding X by producing a betting game called spread-odds (s, z) . Depending on the bookie's choice of the spread parameter s and the odds parameter z , the spread-odds (s, z) game is either the point-spread game, the odds game, or a game combining features of both point spreads and odds. The spread-odds (s, z) device, apparently first used by Smith (1971), is convenient for stating results and for obtaining conditions under which point spreads beat odds and any other combined-betting game.

The bookie collects a fee or commission c for each bet purchased in the spread-odds (s, z) game. The commission is assumed fixed so that s and z are the bookie's only choice variables.² Furthermore, the bookie's administrative and other costs per bet are assumed to not depend on the particular spread-odds (s, z) game which is produced.

Bettors are offered a Rams bet or a Bears bet in the spread-odds (s, z) game (bettors will also have the option of not betting). The bets are a function of (s, z) where $-\infty < s < \infty$ and $0 < z < 1$. Each bet is described by its net gain, its net loss, and the event under which the gain is realized. The spread-odds (s, z) bets are: Rams bet, $[2z - c : 2(1 - z) + c, X > s]$; Bears bet, $[2(1 - z) - c : 2z + c, X < s]$. An individual who purchases one Rams bet will pay the bookie $2(1 - z) + c$ dollars and win two dollars gross, $2z - c$ dollars net, if $X > s$. If $X < s$ the Rams bettor loses his initial payment of $2(1 - z) + c$ dollars.³ Similarly, a Bears bettor wins $2(1 - z) - c$ dollars per bet if $X < s$ and loses $2z + c$ dollars per bet if $X > s$.⁴

² In practice the bookie's commission is also a choice variable whose equilibrium value is determined by the betting industry's market structure. For an interesting study in a slightly different context of how the choice of c can influence (a race track's) profit, see Gruen (1976). To partially justify the c fixed assumption used here, one might suppose a competitive betting industry in long-run equilibrium with the average cost per wager equal to c .

³ If $X = s$ all bets are off. This differs from some betting games in which the bookie wins all ties.

⁴ The odds parameter z is a transformation of what is usually referred to as "the odds." E.g., if $\bar{z} = 2/3$ and $c = 1/99$, then the Rams bet is established at $(131/99 : 67/99)$ and the Bears bet at $(65/99 : 133/99)$. In the usual parlance this represents approximately 1.96 to 1 odds and 1 to 2.05 odds, respectively, the slight asymmetry in the odds being the result of the fee c . In order that the net gains and losses in the spread-odds (s, z) game be positive, the domain of z must be slightly restricted to the interval $(c/2, 1 - c/2)$.

The bookie's choice of a spread-odds (s, z) game is restricted to the set of (s, z) which makes profit independent of the outcome of the football game. This domain for (s, z) is called the equilibrium set and is given by $V = [(s, z) \mid \Lambda_R(s, z : c) = \Lambda_B(s, z : c)]$, where Λ_R and Λ_B denote the number of Rams and Bears bets purchased at the spread s , the odds z , given the commission c .

To see that (s, z) in V makes profit independent of X , consider the betting pool (the total dollars wagered on the Rams and Bears), $[2(1 - z) + c]\Lambda_R + [2z + c]\Lambda_B$. For (s, z) in V the pool is $(1 + c)\Lambda$ where $\Lambda = \Lambda_R + \Lambda_B$. From the pool the bookie returns $2\Lambda_R$ dollars to Rams bettors if $X > s$, and at (s, z) in V this is identical to the $2\Lambda_B$ dollars returned to Bears bettors if $X < s$. The bookie's profit is therefore $c\Lambda(s, z : c)$ with certainty for each (s, z) in V .

A point-spread game is spread-odds $(s, \frac{1}{2})$; that is, z is fixed at $\frac{1}{2}$. With this restriction, wagers are made at (neglecting c) even odds on $X > s$ or $X < s$. The equilibrium point spread is \bar{s} where $(\bar{s}, \frac{1}{2})$ is in V , spread-odds $(\bar{s}, \frac{1}{2})$ is the point-spread game which the bookie may produce, and the profit from producing this game is $c\Lambda(\bar{s}, \frac{1}{2} : c)$.

An odds-betting game is spread-odds $(0, z)$ so that bets are made on the events $X > 0$ or $X < 0$, Rams win or Bears win. The equilibrium odds is denoted by \bar{z} where $(0, \bar{z})$ is in V , spread-odds $(0, \bar{z})$ is the odds game which the bookie might produce, and the profit from this game is $c\Lambda(0, \bar{z} : c)$.⁵

The profit-maximizing bookie who is permitted to produce any spread-odds (s, z) game will select an (s^*, z^*) in V so that $\Lambda(s^*, z^* : c)$ is a maximum, and spread-odds (s^*, z^*) is called an optimal betting game. If Λ is constant over all (s, z) in V then every betting game is equivalent and optimal, and the bookie's profit is identical no matter which game is produced. This situation corresponds to the initial hypothesis under which the bookie is indifferent between alternative betting games. If $\Lambda(s^*, z^* : c) > \Lambda(s, z : c)$ for some (s, z) in V , then some betting games will not be produced because they result in lower profits than an optimal game. Finally, when Λ reaches a maximum for a unique (s, z) in V , there is a unique optimal game, and profit maximizing by the bookie completely determines the spread-odds (s, z) game which is produced.

The bookie who is constrained to either point spreads or odds will produce the former when $\Lambda(\bar{s}, \frac{1}{2} : c) > \Lambda(0, \bar{z} : c)$. When this holds we will say that point spreads beat odds. Notice that point spreads, even if it beats odds, need not be an optimal game. Finally, an extreme case to be considered in Section VII occurs when point spreads beat odds

⁵ In order that the point-spread and odds games are not identical, it is supposed that \bar{z} is not equal to zero.

and $\Lambda(0, \bar{z}; c) = 0$; that is, no one purchases the odds wager. In this case we will say that point spreads versus odds is no contest.

Remarks

1. In practice a bookie who fails to select an (s, z) in V will try to exchange some of his wagers with another bookie or will begin to alter the spread and odds parameters. Hence, all bets may not be purchased at the same (s, z) . In the model here, however, all bets are assumed to be made at an (s, z) in V . One might suppose that the bookie's skill is his ability to read the market and know the (s, z) such that $\Lambda_R = \Lambda_B$.

2. The equilibrium set corresponds to one possible measure of consensus regarding the distribution of X . If $\pi(s) = z$ for (s, z) in V , then π is a market probability distribution in the sense that the fraction of the betting pool (excluding the bookie fee) wagered on the event $X < s$ is z , and the fraction wagered on $X > s$ is $1 - z$. Bookies who fail to select (s, z) in V become bettors themselves and effectively are betting that their opinion is more accurate than this market consensus. Natural selection probably eliminates these bookies quickly either because they have become wealthy and retire or, more likely, because they have become financially or physically bankrupt, the latter bankruptcy being a likely consequence of not paying winning bettors.

3. In the standard point-spread game the bookie's commission (called the vigorish) results in spread bets being offered at 10:11 odds. With the spread-odds (s, z) notation this means that the point-spread bet is offered at odds of 20/21:22/21 and $c = 1/21$.

III

The set of potential bettors is denoted by N and remains fixed throughout. Associated with each individual i in N is a probability distribution $F_i(x)$ and a betting function; $F_i(x)$ is the i th bettor's probability assignment to the event $X < x$, and the betting function along with $F_i(x)$ determines the number of Rams or Bears bets purchased by bettor i .

Aside from the example in Section VII, beliefs will be assumed identical except for a location parameter. With this simple specification $F_i(x) = G(x - \mu_i)$, and G is further taken to be a symmetric (about zero, $G(x) = 1 - G(-x)$) distribution with continuous derivative $g > 0$. Given G , the i th bettor's beliefs are completely determined by μ_i , the bettor's opinion regarding the median-mathematical expectation of X . The distribution of the μ_i over the set of potential bettors is approximated with N large by the continuous and increasing dis-

tribution $M(y - \mu_M)$ —the proportion of potential bettors who believe the median-mathematical expectation of X is less than y . The distribution M is further assumed symmetric about zero so that μ_M is the median or average of all bettors' beliefs regarding the median-mathematical expectation of X . Market beliefs under this simple representation are therefore a location family of distributions depending on $G, M,$ and μ_M .

A potential bettor will participate in the betting market by purchasing a positive quantity of the Rams or Bears bet if and only if the EMV of the corresponding bet is positive. With this requirement the set of potential bettors may be partitioned as follows.

Given a spread-odds (s, z) game the EMV of one Rams bet for bettor i is $2z - c - 2G(s - \mu_i)$. The requirement that this be positive can be expressed directly in terms of μ_i as $\mu_i > s - Q(z - c/2)$, where $Q(p)$ is the (increasing) inverse or p th quantile of G . Making similar calculations for the Bears bet gives the partitioning, Rams bettors = $[i \text{ in } N \mid \mu_i > s - Q(z - c/2)]$, Bears bettors = $[i \text{ in } N \mid \mu_i < s - Q(z + c/2)]$; and the nonbettors are those individuals whose μ_i is the interval $[s - Q(z + c/2), s - Q(z - c/2)]$. This holds for arbitrary, not necessarily equilibrium, values for s and z . To determine V requires further assumptions which describe the quantity of wagers purchased when the EMV is positive.

Let λ_i^R and λ_i^B denote the number of Rams and Bears bets purchased by the i th individual. The betting rule assumed here is $\lambda_i^R = \lambda[\mu_i - s + Q(z - c/2)]$, and $\lambda_i^B = \lambda[s - \mu_i - Q(z + c/2)]$, where $\lambda(w)$ is a positive and nondecreasing function for $w > 0$ and is zero otherwise. This rule is consistent with the positive EMV requirement. If $\lambda(w) = 1$ for $w > 0$, bettors are restricted to a single bet when its EMV is positive. If $\lambda(w)$ is increasing for $w > 0$ the quantity of bets purchased is based not on EMV directly but on the difference between μ_i and $s - Q(z - c/2)$ or $s - Q(z + c/2)$, depending on which wager's EMV is positive. If $\mu_1 > \mu_2 > s - Q(z - c/2)$ the EMV of one Rams bet is greater for bettor 1 and this bettor purchases more Rams bets than bettor 2, $\lambda_1^R > \lambda_2^R > 0$. With this betting rule we will have $\lambda_i^R = \lambda_j^B > 0$ (i and j have equal but opposite opinions) when μ_i and μ_j are equidistant but on opposite sides of the interval which describes the nonbettors.

Remarks

1. The assumption of a large number of individuals with well-defined distributions defined on $X = R - B$ is an empirical proposition which is probably not unrealistic (see Winkler 1971). If X represents an arbitrary future event, however, the assumption need not be

satisfied, and the model will not be applicable.⁶ For example, if individuals do not possess a subjective distribution on the electoral (or popular) vote point spread, then the model cannot explain why odds beats point spreads in wagering on the outcome of a presidential election.

2. Under the football game interpretation, each potential bettor knows that $X = R - B$ is an integer-valued random variable. Therefore, there is the assumption that the outcomes of $R - B$ are sufficiently numerous that $F_i(x)$ may be approximated by a continuous distribution. In other cases this may be far from adequate, for example, baseball and soccer games. In the limit where the winner scores one and the loser zero, no point-spreads versus odds question will arise.

3. The location family of distributions is a very restrictive assumption which undoubtedly does not allow for the full diversity of actual beliefs. The assumption is adopted because it permits easy calculation of optimal game conditions and thereby will illustrate, albeit in a restrictive setting, how one betting game can be favored over another.

4. The assumed betting criterion is also adopted because it is convenient. With a location family of market beliefs, neither the equilibrium set nor the optimal game conditions will be found to depend on λ . But the criterion might also be fairly realistic since bettors often rationalize their wagers by appealing to the difference between the quoted spread and the spread at which they would decide not to wager.

IV

Before examining the conditions for an optimal game given an arbitrary location family of market beliefs, it is instructive to consider the following special case.

Suppose G is a uniform distribution on the interval $(-10, 10)$; $G(x) = .5 + .05x$. Let $c = .1$, $\mu_M = 3$, and let M be an arbitrary distribution satisfying the previous assumptions.

A Rams bettor is anyone who finds the *EMV* of one Rams bet to be positive or, on making the calculations, $\mu_i > s - 20z + 11$. Under the assumed betting criterion such a bettor purchases $\lambda_i^R = \lambda(\mu_i - T - 4)$ Rams bets, where we let T denote $s - 20z + 7$. Summing (integrating) over all such purchases gives

⁶The assumption can fail in either of two ways. Beliefs might be so vague that individuals feel completely incapable of assigning probabilities to future events. Alternatively, beliefs might be inconsistent with a probability distribution, e.g., if one assigned probabilities of .3 and .8 to the respective events $X < 0$ and $X > 1$.

$$\begin{aligned} \Lambda^R &= n \int_{\mu > T+4} \lambda(\mu - T - 4) dM(\mu - 3) \\ &= n \int_{w > T+1} \lambda(w - T - 1) dM(w), \end{aligned}$$

where n is the number of potential bettors. Making similar calculations gives

$$\Lambda^B = n \int_{w < T-1} \lambda(-w + T - 1) dM(w).$$

This expresses the total quantity of Rams and Bears bets purchased given an arbitrary spread-odds (s, z) game.

To solve for the (s, z) in V use the symmetry of M and obtain

$$\begin{aligned} \Lambda^R - \Lambda^B &= n \int_{w > T+1} \lambda(w - T - 1) dM(w) \\ &\quad - n \int_{w > -T+1} \lambda(w + T - 1) dM(w). \end{aligned}$$

At $T = 0$ this is zero and, since M is increasing, $\Lambda^R - \Lambda^B = 0$ only if $T = 0$. Hence, $T = 0$ or $s(z) = 20z - 7$ gives the equilibrium set; at any $[s(z), z]$ the bookie's profit will not depend on the realization of X . The equilibrium point spread would be $s(1/2) = 3$, and the equilibrium odds is a solution to $s(z) = 0$ or $z = .35$.

Substituting $s(z)$ for s in the expressions for Λ^R and Λ^B gives the representation of profits for (s, z) in V as

$$c(\Lambda_R + \Lambda_B) = 2cn \int_{w > 1} \lambda(w - 1) dM(w),$$

a term which does not depend on (s, z) . So why does the bookie produce one game, say point spreads, over another? No (deterministic) answer is to be found in this example. The bookie's profit is identical in all spread-odds (s, z) games, every game is optimal, and disparate beliefs as represented here do not offer an explanation for the production of a particular betting game. In a world where the assumptions of this example are satisfied, the existence of point-spread betting would merely represent the bookie's random choice from the set of all spread-odds (s, z) games.

To put this result differently, consider the point-spread game with $\bar{s} = 3$. Instead of computing the equilibrium odds, mimic the example in the introduction and ask each bettor to compute the odds game at which his behavior would be identical to behavior under point spreads. For example, a Rams bettor with point spreads will purchase $\lambda(v_i)$ bets where $\mu_i - 4 = v_i > 0$. An odds game in which the same

number of Rams bets would be purchased is found by solving $\mu_i - 11 - 20z = v_i$, or $z = .35$. Every Rams and, with similar calculations, every Bears bettor will obtain this same solution, and at $z = .35$ every nonbettor under point spreads remains a nonbettor under odds. The behavior of each individual is therefore invariant under the transformation from $(s, z) = (3, \frac{1}{2})$ to $(0, .35)$. Since $(3, \frac{1}{2})$ is in V and because no behavior is altered at $(0, .35)$, it follows that $.35$ is the equilibrium odds. The point-spread and odds games are therefore equivalent.

In this example any individual can find an equalizing odds game, and this is generally true as long as bettors use an *EMV*-based decision criterion. What makes this example special and misleading is that every individual arrives at the same equalizing odds of $z = .35$. When G is not uniform we will see that not all bettors compute the same equalizing odds, and the invariance of betting behavior as (s, z) varies over V will no longer hold.

V

For the spread-odds (s, z) game with arbitrary (s, z) , the *EMV*-based betting criterion provides a straightforward characterization of Λ^R and Λ^B as a function of G, M , and μ_M . The equilibrium set can then be obtained by solving for the (s, z) which satisfy the equilibrium condition. The betting pool and bookie profits for the spread-odds (s, z) game with (s, z) in V then follows directly.

Proposition 1: For arbitrary (not necessarily equilibrium) (s, z) ,

$$\Lambda^R = n \int_{w > T+A} \lambda(w - T - A) dM(w),$$

$$\Lambda^B = n \int_{w < T-A} \lambda(-w + T - A) dM(w),$$

where $T = s - \mu_M - [Q(z + c/2) + Q(z - c/2)]/2$ and $A = [Q(z + c/2) - Q(z - c/2)]/2$.

Proof: This follows from the assumed betting criterion, the representation of market beliefs, and the direct substitution $w = \mu - \mu_M$.

Proposition 2: The equilibrium set is $V = \{(s, z) | s(z) = \mu_M + [Q(z + c/2) + Q(z - c/2)]/2\}$.

Proof: By the symmetry of M we have

$$\Lambda^R - \Lambda^B = n \int_{w > A+T} \lambda(w - T - A) dM - n \int_{w > A-T} \lambda(w + T - A) dM.$$

This expression is zero at $T = 0$ and, using the assumption that M is increasing, it may be verified that $\Lambda^R - \Lambda^B < 0$ for $T > 0$ and $\Lambda^R - \Lambda^B$

> 0 for $t < 0$. Hence $\Lambda^R - \Lambda^B = 0$ if and only if $T = 0$, and this is the proposition.

Proposition 3: Bookie profits for $[s(z), z]$ in V are

$$c(\Lambda^R + \Lambda^B) = cn \int_{w>A} \lambda(w - A) dM(w).$$

An optimal game is therefore an $[s^*(z^*), z^*]$ such that $A(z^*:c) = \min_z A(z:c)$.

Proof: The first part of the proposition follows on substituting $s(z)$ for s in proposition 1 and then using the symmetry of M . The second part follows on noting that with M increasing the expression for profits increases as A decreases.

Substituting $s(z)$ for s in the previous expression for nonbettors, we find that as (s,z) varies over V , the nonbettors are those individuals whose μ_i fall in the interval $[\mu_M - A(z:c), \mu_M + A(z:c)]$. Since the μ_i are symmetric about μ_M the proportion of potential bettors who purchase no bets is $1 - 2M[A(z:c)]$. Since M is increasing this proportion is minimized at $\min_z A(z:c)$, which is the same as the maximum-profit condition. The objectives of maximizing profits and maximizing the participation of bettors in the market are therefore equivalent, given our assumptions on beliefs and betting behavior.

When $A(z:c)$ is a constant function of z , all betting games will yield equal profits and an equal number of bettors. This occurs when Q is linear (for $0 < p < 1$), and this in turn implies that G is uniform, the example of the previous section. Conversely, when G is not uniform A is not constant, and some betting games will not be produced because they generate lower profits. The bookie fee in effect drives a wedge, consisting of the nonbettors, between Rams and Bears bettors, and the size of this wedge depends on which spread-odds (s,z) game is produced.

Finally, consider the point-spread game with the equilibrium spread μ_M . If Rams bettors compute an odds game at which their behavior is identical with the point-spread game, they will find $z_R = G\{Q[(1 - c)/2] - \mu_M\} + c/2$. The Bears bettors will compute $z_B = G\{Q[(1 + c)/2] - \mu_M\} - c/2$. If G is not uniform the equilibrium odds are not equal to z_R or z_B so that the odds game with $z = z_R$ or z_B is not a feasible game for the bookie. The existence of equalizing odds for any given bettor does not therefore imply that all betting games are equivalent. The point-spreads versus odds question depends on betting behavior not at equalizing odds but at the equilibrium odds, and unless G is uniform these two odds will not be equal.

Given the assumed beliefs and betting criterion, proposition 3 gives the optimality condition for spread-odds (s,z) ; namely, the bookie

produces an $[s(z), z]$ game such that $A(z:c)$ is a minimum. The next section translates this condition into simple conditions on G .

Remark

Let the (median) consensus probability of the event $X < s$ be denoted by $\pi_M(s)$ where half of the individuals in N believe $P(X < s) < \pi_M(s)$ and half believe $P(X < s) > \pi_M(s)$. Suppose further that this consensus is correct: If $\pi_M(s) = z$ for a large number of football games, then z is the frequency of $X < s$. It may be verified that for the present model $\pi_M(s) = G(s - \mu_M)$. Furthermore, if $c = 0$ everyone in N purchases a wager (the set of nonbettors has probability zero), and π_M is revealed by the equilibrium set since $V = [(s,z) | \pi_M(s) = z]$. However, from proposition 2 we see that with $c > 0$ there will exist discrepancies between $\pi(s) = z$, (s,z) in V and the true measure of the market's opinion $\pi_M(s)$. This discrepancy arises because with $c > 0$ some potential bettors do not wager, and their opinions are not fully represented by the equilibrium set. For example, if $c = .2$, G is a mean zero, variance 49 Gaussian distribution and $\mu_M = 7$, then one finds the equilibrium odds is $\bar{z} = .179$, and this is a biased estimate of the potential market's belief regarding $X < 0$ since $\pi_M(0) = .159$. Hence \bar{z} will not reflect the frequency of $X < 0$ even if the consensus beliefs of potential bettors are correct. Such discrepancies might be a reason for the observed difference between the probabilities implied by the odds in horse race betting and the frequency with which horses win (see Ali [1977, p. 810] for a similar result; see also Seligman [1975]).

VI

Suppose in addition to previous assumptions that G is strictly unimodal; that is, $g(0) > g(x)$ for all x and $g(x)$ increasing (decreasing) as x is negative (positive). With this assumption μ_i is the expectation, median, and mode or most likely outcome of X . In such a world we have:

Theorem 1: If G is strictly unimodal, then point spreads is the unique optimal betting game.

Proof: The first-order condition for the minimum of A is $q(z + 1/2c) - q(z - c/2) = 0$, or $g[Q(z + c/2)]^{-1} - g[Q(z - c/2)]^{-1} = 0$. With G symmetric and strictly unimodal this holds if and only if $Q(z + c/2) = -Q(z - c/2) = Q(1 - z + c/2)$, or $z = 1/2$. One may now verify that A is decreasing for $z < 1/2$ and increasing for $z > 1/2$ so that at $z = 1/2$, A is at its unique minimum value. By proposition 3, $z = 1/2$, or the point-spread game is the unique optimal betting game.

To obtain simple necessary and sufficient conditions which characterize the optimal game, suppose c is small and use a linear

approximation for $Q(z + c/2)$. Expanding around $c = 0$ and suppressing quadratic and higher-order terms gives $Q(z + c/2) = Q(z) + q(z)c/2$. Up to this order of approximation $s(z) = \mu_M + Q(z)$, $\bar{z} = G(-\mu_M) = 1 - G(\mu_M)$, and $A(z:c) = q(z) = g[Q(z)]^{-1}$.

Theorem 2: For c small, spread-odds $[s(z^*), z^*]$ is an optimal game if and only if $Q(z^*)$ is a global mode of $G(x)$.⁷

Proof: Minimizing $A(z:c)$ with c small is approximately the same as minimizing $q(z) = g[Q(z)]^{-1}$, and by definition this minimum is achieved at a mode of $G(x)$. (Notice that if $Q[z^*]$ is a mode, then by symmetry so is $-Q[z^*] = Q[1 - z^*]$, so that $[s(1 - z^*), 1 - z^*]$ is also an optimal game.)

Theorem 3: For c small, point spreads beat odds if and only if $g(0) > g(\mu_M)$.

Proof: With c small, $A(\bar{z} : c) = g(\mu_M)^{-1}$ (since $g[\mu_M] = g[-\mu_M]$) and $A(1/2:c) = g(0)^{-1}$ so that the theorem follows from proposition 3.

It should be emphasized that the simple optimality conditions in this section are a consequence of the simple specification of beliefs and betting behavior. Under less restrictive assumptions there will undoubtedly exist different configurations of beliefs, betting behavior, and preferences which are revealed by the bookie's production of point spreads.

The conditions illustrate how market beliefs and betting behavior can be mapped via the bookie into a particular betting game even when betting games are considered equally thrilling. For example, under the assumptions of theorem 2, the existence of point-spread betting does not reveal bettor preferences but instead reveals an aspect of a bettor's beliefs, namely, that $0 = Q(1/2)$ is the mode of G . There is not a one-to-one correspondence between point spreads and the previously described bettor-preferences explanation, and such an explanation therefore is not logically necessary in order to understand the existence of point-spread betting.

VII

In this section the location-family-of-market-beliefs assumption is dropped, individual betting is supposed consistent with the positive *EMV* requirement but is otherwise arbitrary, and a very simple example is presented in which point spreads versus odds is no contest. Only the point-spread and odds games are compared, and there is no attempt to obtain simple or general optimality conditions. The example is presented to illustrate how point spreads can be favored over odds when bettors use an arbitrary *EMV*-based decision criterion. The

⁷ x^* is a global mode of G if $g(x^*) \geq g(x)$ for all x .

more specific criterion used in the previous sections, while influencing the particular optimality conditions, is not therefore crucial for the bookie to favor one betting game over another.

Suppose N consists of just two potential bettors. Let $c = 1/9$ and suppose beliefs are given by the uniform distributions,

$$F_1(x) = \frac{5}{18}, \quad \frac{5}{3} \leq x \leq \frac{13}{3},$$

$$F_2(x) = \frac{x}{12} + \frac{2}{9}, \quad -\frac{8}{3} \leq x \leq \frac{28}{3}.$$

These distributions are depicted in figure 1. Observe that the distributions do not differ by just location and are not therefore covered by the previous sections. Also note that there are disparate beliefs both in the sense that $F_1 \neq F_2$ and also in the sense that beliefs regarding the event $X < 0$ are much less dispersed than beliefs regarding, say, $X < 2$.

The *EMV*s of the Rams and Bears bets for both individuals given the point-spread game are graphed in figure 2 as a function of s . From the figure one sees that when $4/3 \leq s \leq 24/9$, individual 1 is a Bears bettor and individual 2 is a Rams bettor. The equilibrium point spread will therefore be somewhere in this interval, with the particular spread being dependent on each individual's betting criterion. Under the criterion of Section III the equilibrium spread is 2; if individuals purchase a quantity of wagers which is proportional to the *EMV* of the wager, then $\bar{s} = 16/9$. But in any event both potential

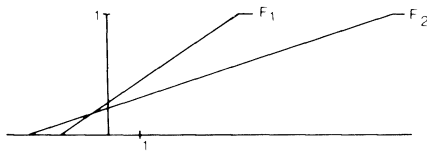


FIG. 1.—Beliefs of potential bettors 1 and 2

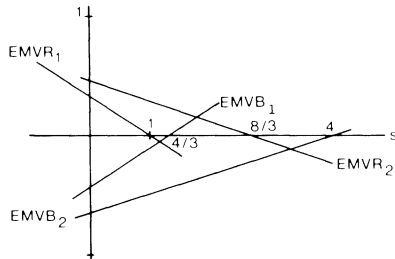


FIG. 2.—*EMV*s of Bears and Rams bets for potential bettors given point-spread game

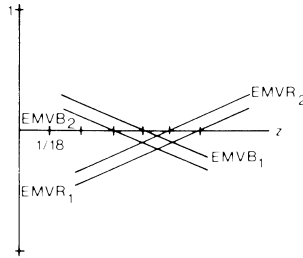


FIG. 3.—*EMV*s of Bears and Rams bets for potential bettors given odds game

bettors will participate in the betting market by purchasing some point-spread bets.

The *EMV*s of the Rams and Bears bets given the odds game are graphed in figure 3 as a function of z . For $z < 2/9$ the *EMV* of a Bears bet is positive for both individuals and for $z > 5/18$ the *EMV* of a Rams bet is positive for both individuals; hence, the equilibrium odds cannot be in either of these intervals. The only way to satisfy the equilibrium condition is to let $2/9 \leq z \leq 5/18$, and with any such odds both individuals purchase no bets. Therefore point spreads versus odds is no contest. If the point-spread device had not been invented and wagers could only be made on the winner of the game, this football betting market would not exist.

VIII

So what is the answer to the point-spreads versus odds question? Perhaps the answer is that point spreads are more fun than odds. Since this, however, is not the only logical possibility, the answer ultimately must be based on empirical evidence. In a world where point spreads are not considered more fun than odds, point spreads can exist as the game produced by a profit-maximizing bookie. Difference of opinion, besides leading to the formation of a betting market, can also have implications for the particular betting game produced by the bookie. Finally, to profit from these findings you should look for an informal setting in a location-family-of-beliefs world where bettors use a criterion as described in Section III. If a profit-maximizing bookie is producing the point-spread game, you might then wager that your fellow bettors have expectations equal to their modes.

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