THE UNIVERSITY OF ILLINOIS AT CHICAGO

ECON 512: MACROECONOMICS II

SPRING 2008

Prof. George Karras

Problem Set #1

1. A Malthusian model. Suppose the production function is $Y_t = A\bar{L}^{\alpha}N_t^{1-\alpha}$, where Y is output, $\bar{L} > 0$ is land (fixed), $A > 0$ is the level of technology, N is population, and $0 < \alpha < 1$. According to Malthus, population growth is given by $n \equiv \dot{N}_t / N_t = \beta(y_t - \bar{y})$, where $y \equiv Y/N$, $\bar{y} > 0$ is the subsistence level of income (fixed), and $\beta > 0$.

(a) Solve for the steady-state values of y and N. Investigate stability. [Hint: at the steady state $\dot{N}_t / N_t = 0$, and stability requires $\partial \dot{N}_t / \partial N_t^*{}_{ss} < 0$]

(b) How would an increase in $\bar{L}$ affect the steady-state values of y and N? An increase in A? Explain.

2. Suppose people supply labor inelastically and maximize $\int_0^{\infty} \ln[(C/N)_t]e^{-\rho t}\,dt$, with the production function given by $Y_t = (K_t)^{\alpha}(N_t)^{1-\alpha}$, where ρ: rate of time preference, C: consumption, N: population, K: capital, and $0 < \alpha < 1$. The population growth rate is $n > 0$, and the depreciation rate is $\delta > 0$.

(a) Calculate the per capita steady-state capital stock (k) and output (y) in terms of the parameters ρ, α, n, and δ. What is the steady-state gross saving rate, s? [Hint: $s \equiv (\dot{K} + \delta K)/Y$].

(b) Calculate the "golden rule" k, y, and s in terms of the same parameters. Explain the difference between steady state and golden rule k and s.

(c) Phelps' rule says that economies optimize when they adhere to the following rule-of-thumb behavior: "consume labor's share of output and save capital's share." Will Phelps' rule produce the steady state or the golden rule? (Hint: capital's share is rK/Y, where the interest rate is $r = \partial Y/\partial K$).