

**THE UNIVERSITY OF ILLINOIS AT CHICAGO**  
**ECON 512: MACROECONOMICS II**  
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**Problem Set #1**

1. A Malthusian model. Suppose the production function is  $Y_t = A\bar{L}^\beta N_t^{1-\beta}$ , where  $Y$  is output,  $\bar{L} > 0$  is land (fixed),  $A > 0$  is the level of technology,  $N$  is population, and  $0 < \beta < 1$ . According to Malthus, population growth is given by  $n \equiv \dot{N}_t / N_t = \gamma(y_t - \bar{y})$ , where  $y \equiv Y/N$ ,  $\bar{y} > 0$  is the subsistence level of income (assume to be fixed), and  $\gamma > 0$ .

(a) Solve for the steady-state values of  $y$  and  $N$ . Investigate stability. [Hint: at the steady state  $\dot{N}_t / N_t = 0$ , and stability requires  $\partial \dot{N}_t / \partial N_t |_{ss} < 0$ ]

(b) How will an increase in  $\bar{L}$  affect the steady-state values of  $y$  and  $N$ ? An increase in  $A$ ?

2. The Solow model with land. The production function is  $Y_t = A_t (K_t)^\beta L^\gamma (N_t)^{1-\beta-\gamma}$ , where  $L$  is land (assumed to be fixed),  $0 < \beta < 1$ , and  $0 < \gamma < 1$ . Assume the technological growth rate is  $a > 0$ , the population growth rate  $n > 0$ , the saving rate  $s > 0$ , and the depreciation rate  $\delta > 0$ .

(a) Derive this model's version of the fundamental growth equation.

(b) Solve for  $g_k = (\dot{k} / k)^{ss}$ , the steady-state growth rate of the capital stock per capita. How is it related to  $g_y = (\dot{y} / y)^{ss}$ , the steady-state growth rate of income per capita? How about the aggregate growth rates  $g_K = (\dot{K} / K)^{ss}$  and  $g_Y = (\dot{Y} / Y)^{ss}$ ?

3. Suppose people supply labor inelastically and maximize  $\int_0^\infty \ln[(C/N)_t] e^{-\rho t} dt$ , with the production function given by  $Y_t = (K_t)^\beta (N_t)^{1-\beta}$ , where  $\rho$ : rate of time preference,  $C$ : consumption,  $N$ : population,  $K$ : capital, and  $0 < \beta < 1$ . The population growth rate is  $n > 0$ , and the depreciation rate is  $\delta > 0$ .

(a) Calculate the per capita steady-state capital stock ( $k$ ) and output ( $y$ ) in terms of the parameters  $\rho$ ,  $\beta$ ,  $n$ , and  $\delta$ . What is the steady-state gross saving rate,  $s$ ? [Hint:  $s \equiv (\dot{K} + \delta K) / Y$ ].

(b) Calculate the "golden rule"  $k$ ,  $y$ , and  $s$  in terms of the same parameters. Explain the difference between steady state and golden rule  $k$  and  $s$ .

(c) Phelps' rule says that economies optimize when they adhere to the following rule-of-thumb behavior: "consume labor's share of output and save capital's share." Will Phelps' rule produce the steady state or the golden rule? (Hint: capital's share is  $rK/Y$ , where the interest rate is  $r = \partial Y / \partial K$ ).