THE UNIVERSITY OF ILLINOIS AT CHICAGO
ECON 534: Econometrics I
AUTUMN 2009

Prof. George Karras

Example 2: Multivariate Regression

Consider the following regression model:

$$stdgdp_i = \beta_1 + \beta_2 stdsolow_i + \beta_3 stdm1_i + \beta_4 meangov_i + \varepsilon_i\,, \quad I = 1,2,...,N, \tag{1}$$

where *stdgdp* is the standard deviation of detrended GDP (a measure of business-cycle volatility), *stdsolow* is the standard deviation of the Solow residual, *stdm1* is the standard deviation of the detrended M1 money supply, *meangov* is average government consumption as a fraction of GDP, ε is an error term, and β_1, β_2, β_3, and β_4 are parameters. The following is a RATS output which estimates (1) using the cross-sectional OECD data from G.Karras and F.Song (Fall 1996), "Sources of Business-Cycle Volatility: An Exploratory Study on a Sample of OECD Countries" *Journal of Macroeconomics*:

A. Input the data.

```
allocate 24
*
* cross section data from G.Karras and F.Song, Journal of Macroeconomics,
*                                                Fall 1996 (Appendix)
*
* Note: N=24, K=4.
*
data(unit=input,org=obs) / number stdgdp stdsolow stdm1 meangov
 1 0.0175898335346 2.3588138191884 0.0543157559544 0.1569046197688 (Australia)
 2 0.0169331673977 2.0860331691907 0.0453915686851 0.1625823916291 (Austria)
 3 0.0167926798370 1.8673502679234 0.0431795903515 0.1509336430254 (Belgium)
 4 0.0194867550979 1.6762360647194 0.0895004826866 0.1807037033556 (Canada)
 5 0.0179411133736 2.0093689346665 0.0447975353847 0.2189690791726 (Denmark)
 6 0.0248948646821 2.2531112291235 0.1226715868950 0.1670768824372 (Finland)
 7 0.0125789655760 1.6345309753864 0.0512042108332 0.1651290702207 (France)
 8 0.0183784650456 1.8362044275719 0.0417975409105 0.1789682462017 (Germany)
 9 0.0219341162869 4.0656503371884 0.0402864692620 0.151954402238 (Greece)
10 0.0419316041961 3.6793784978823 0.1038025616835 0.1478924694045 (Iceland)
11 0.0223727069730 2.0875114394318 0.0586297877174 0.1605421280493 (Ireland)
12 0.0183036147471 2.5356831042386 0.0728270014533 0.1457770768282 (Italy)
13 0.0256984255909 3.3821794920687 0.0510256704137 0.0884022813210 (Japan)
14 0.0281542447889 3.0984816295201 0.0449064361729 0.1363891290764 (Luxembourg)
15 0.0177404506142 2.1480772183521 0.0381423313581 0.1532562086739 (Netherlands)
16 0.0306835312134 3.2139640709227 0.1768667034676 0.1453466284482 (N.Zealand)
17 0.0165243320437 1.6500173929878 0.0682760106228 0.1789848701432 (Norway)
18 0.0273089906354 3.9663215803002 0.0606549403413 0.1391622409978 (Portugal)
19 0.0244528411518 2.7065317524957 0.0637879072683 0.1134299233788 (Spain)
20 0.0144892333545 1.9019994121437 0.0621086944958 0.2365157205679 (Sweden)
21 0.0274475844095 1.9450768451571 0.0539401662015 0.1178606711949 (Switzerland)
22 0.0302366336809 3.0070092375755 0.1236136549118 0.1313023449675 (Turkey)
23 0.0204076379533 1.7481745014348 0.1321026401195 0.1913590390690 (U.K.)
24 0.0202967057965 1.6960134063870 0.0309017914202 0.1785955083676 (U.S.A.)
```

B. Obtain the RATS regression output

```
linreg stdgdp
# constant stdsolow stdm1 meangov

Dependent Variable STDGDP - Estimation by Least Squares
Usable Observations     24      Degrees of Freedom    20
Centered R**2     0.646052      R Bar **2   0.592960
Uncentered R**2   0.972792      T x R**2      23.347
Mean of Dependent Variable      0.0221907707
Std Error of Dependent Variable 0.0065412907
Standard Error of Estimate      0.0041733238
Sum of Squared Residuals        0.0003483326
Regression F(3,20)                   12.1685
Significance Level of F           0.00009402
Durbin-Watson Statistic             2.217786

   Variable                     Coeff       Std Error      T-Stat     Signif
*******************************************************************************
1.  Constant                  0.015138492  0.007544442      2.00658  0.05850603
2.  STDSOLOW                  0.004136110  0.001381421      2.99410  0.00717051
3.  STDM1                     0.069799137  0.024255859      2.87762  0.00930712
4.  MEANGOV                  -0.049979664  0.032630638     -1.53168  0.14126723
```

C. Replicate using Matrix Algebra

C1. Input the vector y and the matrix X. Display X.

```
declare vector y(24)
declare rectangular x(24,4)
do i=1,24
(01.0023) compute y(i)   = stdgdp(i)
(01.0045) compute x(i,1) = 1
(01.0064) compute x(i,2) = stdsolow(i)
(01.0088) compute x(i,3) = stdm1(i)
(01.0112) compute x(i,4) = meangov(i)
(01.0136) end do i
write x
      1.0000        2.3588        0.0543        0.1569
      1.0000        2.0860        0.0454        0.1626
      1.0000        1.8674        0.0432        0.1509
      1.0000        1.6762        0.0895        0.1807
      1.0000        2.0094        0.0448        0.2190
      1.0000        2.2531        0.1227        0.1671
      1.0000        1.6345        0.0512        0.1651
      1.0000        1.8362        0.0418        0.1790
      1.0000        4.0657        0.0403        0.1520
      1.0000        3.6794        0.1038        0.1479
      1.0000        2.0875        0.0586        0.1605
      1.0000        2.5357        0.0728        0.1458
      1.0000        3.3822        0.0510        0.0884
      1.0000        3.0985        0.0449        0.1364
      1.0000        2.1481        0.0381        0.1533
      1.0000        3.2140        0.1769        0.1453
      1.0000        1.6500        0.0683        0.1790
      1.0000        3.9663        0.0607        0.1392
      1.0000        2.7065        0.0638        0.1134
      1.0000        1.9020        0.0621        0.2365
      1.0000        1.9451        0.0539        0.1179
      1.0000        3.0070        0.1236        0.1313
      1.0000        1.7482        0.1321        0.1914
      1.0000        1.6960        0.0309        0.1786
```

C2. Compute X'X and invert to get $(X'X)^{-1}$.

```
compute xprimex = tr(x)*x
write xprimex
     24.0000        58.5537         1.6747         3.7980
     58.5537       156.3632         4.1992         8.9593
      1.6747         4.1992         0.1477         0.2645
      3.7980         8.9593         0.2645         0.6245


compute xprimexinv = inv(tr(x)*x)
write xprimexinv
      3.2681        -0.4663        -0.7730       -12.8578
     -0.4663         0.1096        -0.3782         1.4242
     -0.7730        -0.3782        33.7807        -4.1795
    -12.8578         1.4242        -4.1795        61.1346
```

C3. Compute $\hat{\beta}=(X'X)^{-1}X'y$, $\hat{y}=X\hat{\beta}$, and $\hat{\varepsilon}=y-\hat{y}$.

```
declare vector betahat(4)
compute betahat = inv(tr(x)*x)*tr(x)*y
write betahat
      0.0151    4.1361e-003         0.0698        -0.0500


declare vector yhat(24)
declare vector epsilonhat(24)
compute yhat = x*betahat
compute epsilonhat = y - yhat
do i=1,24
(01.0023) display y(i) yhat(i) epsilonhat(i)
(01.0066) end do i
      0.01759       0.02084      -0.00325
      0.01693       0.01881      -0.00188
      0.01679       0.01833      -0.00154
      0.01949       0.01929  1.99621e-004
      0.01794       0.01563       0.00231
      0.02489       0.02467  2.25334e-004
      0.01258       0.01722      -0.00464
      0.01838       0.01671       0.00167
      0.02193       0.02717      -0.00524
      0.04193       0.03021       0.01172
      0.02237       0.01984       0.00253
      0.01830       0.02342      -0.00512
      0.02570       0.02827      -0.00257
      0.02815       0.02427       0.00388
      0.01774       0.01903      -0.00129
      0.03068       0.03351      -0.00283
      0.01652       0.01778      -0.00126
      0.02731       0.02882      -0.00151
      0.02445       0.02512 -6.63314e-004
      0.01449       0.01552      -0.00103
      0.02745       0.02106       0.00639
      0.03024       0.02964  5.95143e-004
      0.02041       0.02203      -0.00162
      0.02030       0.01538       0.00491
```

C4. Demonstrate that $X'\hat{\varepsilon}=0$, $\Sigma_i\hat{\varepsilon}_i=0$, and $\hat{\sigma}^2 \equiv s^2=\Sigma_i\hat{\varepsilon}_i^2/(N-K)$.

```
compute xprimeepsilonhat = tr(x)*epsilonhat
write xprimeepsilonhat
-1.2681e-015
-2.6221e-015
-8.2206e-017
-1.9284e-016


compute mean = 0.
compute sigmahatsq = 0.
do i=1,24
(01.0023) compute mean = mean + epsilonhat(i)/24.
(01.0049) compute sigmahatsq = sigmahatsq + (epsilonhat(i)**2)/(24.-4.)
(01.0084) end do i
compute sigmahat = sqrt(sigmahatsq)
display mean sigmahat
-5.28820e-017       0.00417
```

C5. Derive $Var(\hat{\beta})=s^2(X'X)^{-1}$.

```
declare rectangular varbetahat(4,4)
compute varbetahat = sigmahatsq*inv(tr(x)*x)
write varbetahat
 5.6919e-005   -8.1216e-006   -1.3464e-005   -2.2394e-004
-8.1216e-006    1.9083e-006   -6.5876e-006    2.4806e-005
-1.3464e-005   -6.5876e-006    5.8835e-004   -7.2793e-005
-2.2394e-004    2.4806e-005   -7.2793e-005    1.0648e-003
```

C6. Derive R^2 and $\bar{R}^2$.

```
compute Rsquare = (%corr(y,yhat))**2
compute Rsquarebar = 1. - (24.-1.)*(1.-Rsquare)/(24.-4.)
display Rsquare Rsquarebar
      0.64605       0.59296
*
end


Normal Completion
```