

**THE UNIVERSITY OF ILLINOIS AT CHICAGO**  
**ECON 534: Econometrics I**  
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**Homework #2: Conditional Convergence and Neoclassical Growth**

Homework #2 deals with the econometrics of "conditional" convergence in per capita income, i.e., convergence allowing for differences in the steady state across countries. This relaxes the unrealistic assumption imposed by "absolute" convergence in Homework #1.

0. THEORY. Neoclassical growth theories do not predict "absolute" convergence. Instead, they predict convergence to a steady-state value which may differ across countries. Therefore, neoclassical theory predicts convergence only after controlling for the determinants of the steady state, a concept that is called "conditional" convergence. On the contrary, endogenous growth theories predict no such convergence. Mankiw, Romer, and Weil's (QJE, May 1992) neoclassical model implies (substitute equation (12) into equation (15)):

$$\ln(y_T) - \ln(y_0) = (1 - e^{-\lambda T}) \left( \frac{\alpha}{1 - \alpha} \ln(s) - \frac{\alpha}{1 - \alpha} \ln(n) + \frac{\beta}{1 - \alpha} \ln(h) - \ln(y_0) \right), \quad (0)$$

where  $y$  is GDP per adult,  $T$  is the terminal year, 0 the initial year,  $\lambda$  is the rate of convergence,  $s$  is the fraction of income invested in physical capital,  $n$  is the population growth rate (plus the depreciation rate and the rate of technological growth),  $h$  is human capital, and  $\alpha$  and  $\beta$  are the output elasticities with respect to physical capital and human capital, respectively. Neoclassical theory predicts that  $\lambda > 0$  and thus  $(1 - e^{-\lambda T}) > 0$ .

1. DATA. Consider the data in the Appendix of MRW. Using the software of your choice, input the series *pop* (growth rate of working-age population), *s* (investment as a fraction of output), and *school* (a measure of human capital) for the 98 countries of the non-oil sample. Again, these are the countries for which the sample variable N equals 1 in the appendix. Do *not* input data for the countries for which N = 0. Assume that the sum of the depreciation rate and the rate of technological growth is 5% (= 0.05).

2. ESTIMATION. To test the predictions of the neoclassical theory, rewrite equation (0) above in linear regression form:

$$growth_i = \beta_0 + \beta_1 \ln y_{1960_i} + \beta_2 \ln s_i + \beta_3 \ln pop_i + \beta_4 \ln school_i + \varepsilon_i, \quad (1)$$

where the variables are  $growth = \ln(y_{1985}) - \ln(y_{1960})$ ,  $\ln y_{1960} = \ln(y_{1960})$ ,  $\ln s = \ln(s)$ ,  $\ln pop = \ln(pop + 0.05)$ , and  $\ln school = \ln(school)$ ; the parameters are  $\beta_0$ ,  $\beta_1 = -(1 - e^{-\lambda T})$ ,

$\beta_2 = (1 - e^{-\lambda T}) \frac{\alpha}{1 - \alpha}$ ,  $\beta_3 = -(1 - e^{-\lambda T}) \frac{\alpha}{1 - \alpha}$ ,  $\beta_4 = (1 - e^{-\lambda T}) \frac{\beta}{1 - \alpha}$ , and  $\varepsilon$  is the error term. Note that the neoclassical model's predicted signs for the parameters are  $\beta_1 < 0$ ,  $\beta_2 > 0$ ,  $\beta_3 < 0$ , and  $\beta_4 > 0$ .

(a) Using the software of your choice, construct the variables *growth*, *lny1960*, *lns*, *lnpop*, and *lnschool*. The RATS code should look like

```
set lny1960 = log(y1960)
set growth = log(y1985) - log(y1960)
set lns = log(s/100.)
set lnpop = log(pop/100.+0.05)
set lnschool = log(school/100.)
```

(b) Suppose  $\varepsilon$  satisfies the classical assumptions ( $E\varepsilon_i = 0$  and  $E\varepsilon_i^2 = \sigma^2$ , for all  $i$ ; and  $E\varepsilon_i\varepsilon_j = 0$ , for  $i \neq j$ ). Using the software of your choice, obtain the OLS estimates of  $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ , and  $\sigma^2$ .

(c) In addition assume that  $\varepsilon_i$  is normally distributed. Test the null hypothesis  $H_0 : \beta_1 = 0$  against the alternative  $H_1 : \beta_1 \neq 0$ . Also construct 95% confidence intervals for  $\beta_1, \beta_2, \beta_3$ , and  $\beta_4$ . Based on your findings, is the neoclassical growth model supported? The endogenous growth model? What is the value of  $\lambda$ , the rate of convergence, implied by the OLS estimate of  $\beta_1$ ?

3. RESTRICTED ESTIMATION. Note that  $\beta_3 = -\beta_2$ . Re-estimate the model subject to this restriction, i.e., write the regression in the restricted form:

$$growth_i = \beta_0 + \beta_1 \ln y1960_i + \beta_2 \ln spop_i + \beta_4 \ln school_i + \varepsilon_i, \quad (2)$$

where  $\ln spop \equiv \ln s - \ln pop$ .

(a) Construct the variable *lnspop*. The RATS code should look like

```
set lnspop = lns - lnpop
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(b) Suppose  $\varepsilon$  satisfies the classical assumptions ( $E\varepsilon_i = 0$  and  $E\varepsilon_i^2 = \sigma^2$ , for all  $i$ ; and  $E\varepsilon_i\varepsilon_j = 0$ , for  $i \neq j$ ). Obtain the OLS estimates of  $\beta_0, \beta_1, \beta_2, \beta_4$ , and  $\sigma^2$ .

(c) In addition assume that  $\varepsilon_i$  is normally distributed. Test the null hypothesis  $H_0 : \beta_1 = 0$  against the alternative  $H_1 : \beta_1 \neq 0$  and construct 95% confidence intervals for  $\beta_1, \beta_2$ , and  $\beta_4$ . Based on your findings, is the neoclassical growth model supported? The endogenous growth model? Compute the values of  $\lambda, \alpha$ , and  $\beta$  implied by the OLS estimates of  $\beta_1, \beta_2$ , and  $\beta_4$ .

(d) Test the imposed restriction by testing the null hypothesis  $H_0 : \beta_2 = -\beta_3$  versus the alternative  $H_1 : \beta_2 \neq -\beta_3$ .