

**THE UNIVERSITY OF ILLINOIS AT CHICAGO**  
**ECON 534: Econometrics I**  
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**Homework #4: Heteroskedasticity Tests in the Neoclassical Growth Model**

Homework #4 uses the White and Breusch-Pagan procedures to test for the presence of heteroskedasticity in the empirical Neoclassical growth model, and to correct using Weighted Least Squares (WLS).

0. THEORY. As in the previous Homeworks, the theoretical model is

$$\ln(y_T) - \ln(y_0) = (1 - e^{-\lambda T}) \left( \frac{\alpha}{1 - \alpha} \ln(s) - \frac{\alpha}{1 - \alpha} \ln(n) + \frac{\beta}{1 - \alpha} \ln(h) - \ln(y_0) \right), \quad (0)$$

where variables and parameters are defined as in Homework #2.

1. DATA. As in the previous Homeworks, consider the MRW data for the 98 countries of the non-oil sample.

2. OLS ESTIMATION. As in the last two Homeworks, rewrite equation (0) above in linear regression form:

$$growth_i = \beta_0 + \beta_1 \ln y1960_i + \beta_2 \ln s_i + \beta_3 \ln pop_i + \beta_4 \ln school_i + \varepsilon_i, \quad (1)$$

where the variables and parameters are defined as in Homework #2. Using OLS, estimate equation (1) and recover the squared estimated residuals. In RATS, writing

```
linreg growth / resid  
# constant lny1960 lns lnpop lnschool
```

will run the regression and save the residuals  $\hat{\varepsilon}$  as the new variable *resid*. Create the squared residuals series, *residsq*, with the instruction

```
set residsq = resid**2
```

3. DIAGNOSIS OF HETEROSKEDASTICITY. We want to investigate whether the error term  $\varepsilon$  is homoskedastic. Implement the following two tests.

(a) *The White Test*. First, construct the products and cross-products of the explanatory variables. In RATS, specify

```
set lny1960sq = lny1960**2  
set lnsq = lns**2
```

```

set lnpop2 = lnpop**2
set lnschool2 = lnschool**2
set lny1960lns = lny1960*lns
set lny1960lnpop = lny1960*lnpop
set lny1960lnschool = lny1960*lnschool
set lnlnpop = lns*lnpop
set lnlnschool = lns*lnschool
set lnpoplnschool = lnpop*lnschool

```

Next, regress  $\hat{\varepsilon}^2$  on these products and cross-products (including those of the constant term):

$$\begin{aligned}
\hat{\varepsilon}_i^2 = & \delta_0 + \delta_1 \ln y1960_i + \delta_2 \ln s_i + \delta_3 \ln pop_i + \delta_4 \ln school_i \\
& + \delta_5 \ln y1960_i^2 + \delta_6 \ln s_i^2 + \delta_7 \ln pop_i^2 + \delta_8 \ln school_i^2 \\
& + \delta_9 \ln y1960_i \ln s_i + \delta_{10} \ln y1960_i \ln pop_i + \dots + \delta_{14} \ln pop_i \ln school_i + v_i,
\end{aligned} \tag{2}$$

Using the  $R^2$  from this auxiliary regression calculate White's statistic and test the null hypothesis of homoskedasticity.

(b) *The Breusch-Pagan Test.* You somehow obtain the information that  $\sigma_i^2 = E(\varepsilon_i^2)$ , the variance of  $\varepsilon_i$ , may have the following exact structure:

$$\sigma_i^2 = h(\alpha_0 + \alpha_1 (\ln pop_i)^2), \tag{3}$$

where the function  $h$  takes only positive values. Estimate the auxiliary regression of  $\hat{\varepsilon}_i^2$  on a constant and  $(\ln pop_i)^2$ , and use the regression's  $R^2$  to calculate the Breusch-Pagan statistic for the null hypothesis of  $\alpha_1 = 0$  (i.e., there is homoskedasticity).

4. WEIGHTED LEAST SQUARES (WLS) ESTIMATION. Suppose  $\sigma_i^2$  is indeed proportional to  $(\ln pop_i)^2$ . Apply WLS estimation as follows. First transform the variables by dividing by  $\ln pop_i$ . In RATS, write

```

set trgrowth = growth/lnpop
set trconstant = 1/lnpop
set trlny1960 = lny1960/lnpop
set trlns = lns/lnpop
set trlnpop = lnpop/lnpop
set trlnschool = lnschool/lnpop

```

Next, implement WLS by estimating the transformed model with OLS. Compare the OLS and WLS results. Is the neoclassical model supported by the WLS results?