

THE UNIVERSITY OF ILLINOIS AT CHICAGO

ECON 534: Econometrics I

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Problem Set #1

1. (Exercise 6.1 in Greene, 4<sup>th</sup> edition). Production data for 22 firms in a certain industry produce the following, where  $y = \ln(\text{output})$ , and  $x = \ln(\text{labor hours input})$ :

$$\bar{y} = 20, \quad \bar{x} = 10, \quad \Sigma(y_i - \bar{y})^2 = 100, \quad \Sigma(x_i - \bar{x})^2 = 60, \quad \Sigma(x_i - \bar{x})(y_i - \bar{y}) = 30.$$

(a) Compute the OLS estimates of  $\beta_0$ ,  $\beta_1$ , and  $\sigma^2$  in the model  $y = \beta_0 + \beta_1 x + \epsilon$ , where  $\sigma^2$  is the variance of  $\epsilon$ .

(b) Test the null hypothesis  $H_0: \beta_1 = 1$ .

(c) Form a 99% confidence interval for  $\sigma^2$ .

2. (Gauss-Markov Theorem in the bivariate regression). Consider the model  $y = \beta_0 + \beta_1 x + \epsilon$ , and assume the classical assumptions are satisfied. The following proves that  $\hat{\beta}_1$ , the OLS estimate of  $\beta_1$ , is Best Linear Unbiased (BLUE).

(a) Show that  $\hat{\beta}_1$  is *linear* in  $y$ , i.e., that it can be written as  $\hat{\beta}_1 = \Sigma_i w_i y_i$ . Derive  $w_i$ .

(b) Show that  $\hat{\beta}_1$  is *unbiased* for  $\beta_1$ , i.e., that  $E(\hat{\beta}_1) = \beta_1$ .

(c) Show that  $\hat{\beta}_1$  is *best* linear unbiased. [Hint: Consider another linear unbiased estimator,  $\tilde{\beta}_1 = \Sigma_i c_i y_i$ . Show that unbiasedness for  $\tilde{\beta}_1$ , i.e.  $E(\tilde{\beta}_1) = \beta_1$ , requires  $\Sigma_i c_i = 0$  and  $\Sigma_i c_i x_i = 1$ . Calculate  $\text{Var}(\tilde{\beta}_1)$  and show that  $\text{Var}(\tilde{\beta}_1) \geq \text{Var}(\hat{\beta}_1)$ .]

3. Consider the linear regression model  $\mathbf{y} = \mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{X}_2 \boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$  where  $\mathbf{X}_1$  and  $\mathbf{X}_2$  are  $N \times K_1$  and  $N \times K_2$  matrices of explanatory variables, respectively, such that  $\mathbf{X}_1' \mathbf{X}_2 = 0$ . The error term  $\boldsymbol{\epsilon}$  satisfies the standard assumptions of the classical model. Economist A estimates the model as given, but economist B omits the variables in  $\mathbf{X}_2$ . Both use OLS.

(a) Show that A and B have the same estimator for  $\boldsymbol{\beta}_1$ .

(b) Show that, in general, A and B will have different estimators for  $\text{Var}(\hat{\boldsymbol{\beta}}_1)$ .

(c) Discuss which of the two estimators of  $\text{Var}(\hat{\boldsymbol{\beta}}_1)$  is "correct".