

THE UNIVERSITY OF ILLINOIS AT CHICAGO

ECON 534: Econometrics I

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Problem Set #1

1. (Exercise 6.1 in Greene, 4th edition). Production data for 22 firms in a certain industry produce the following, where $y = \ln(\text{output})$, and $x = \ln(\text{labor hours input})$:

$$\bar{y} = 20, \quad \bar{x} = 10, \quad \Sigma(y_i - \bar{y})^2 = 100, \quad \Sigma(x_i - \bar{x})^2 = 60, \quad \Sigma(x_i - \bar{x})(y_i - \bar{y}) = 30.$$

(a) Compute the OLS estimates of β_0 , β_1 , and σ^2 in the model $y = \beta_0 + \beta_1 x + \epsilon$, where σ^2 is the variance of ϵ .

(b) Test the null hypothesis $H_0: \beta_1 = 1$.

(c) Form a 99% confidence interval for σ^2 .

2. (Gauss-Markov Theorem in the bivariate regression). Consider the model $y = \beta_0 + \beta_1 x + \epsilon$, and assume the classical assumptions are satisfied. The following proves that $\hat{\beta}_1$, the OLS estimate of β_1 , is Best Linear Unbiased (BLUE).

(a) Show that $\hat{\beta}_1$ is *linear* in y , i.e., that it can be written as $\hat{\beta}_1 = \Sigma_i w_i y_i$. Derive w_i .

(b) Show that $\hat{\beta}_1$ is *unbiased* for β_1 , i.e., that $E(\hat{\beta}_1) = \beta_1$.

(c) Show that $\hat{\beta}_1$ is *best* linear unbiased. [Hint: Consider another linear unbiased estimator, $\tilde{\beta}_1 = \Sigma_i c_i y_i$. Show that unbiasedness for $\tilde{\beta}_1$, i.e. $E(\tilde{\beta}_1) = \beta_1$, requires $\Sigma_i c_i = 0$ and $\Sigma_i c_i x_i = 1$. Calculate $\text{Var}(\tilde{\beta}_1)$ and show that $\text{Var}(\tilde{\beta}_1) \geq \text{Var}(\hat{\beta}_1)$.]

3. Consider the linear regression model $\mathbf{y} = \mathbf{X}_1 \boldsymbol{\beta}_1 + \mathbf{X}_2 \boldsymbol{\beta}_2 + \boldsymbol{\epsilon}$ where \mathbf{X}_1 and \mathbf{X}_2 are $N \times K_1$ and $N \times K_2$ matrices of explanatory variables, respectively, such that $\mathbf{X}_1' \mathbf{X}_2 = 0$. The error term $\boldsymbol{\epsilon}$ satisfies the standard assumptions of the classical model. Economist A estimates the model as given, but economist B omits the variables in \mathbf{X}_2 . Both use OLS.

(a) Show that A and B have the same estimator for $\boldsymbol{\beta}_1$.

(b) Show that, in general, A and B will have different estimators for $\text{Var}(\hat{\boldsymbol{\beta}}_1)$.

(c) Discuss which of the two estimators of $\text{Var}(\hat{\boldsymbol{\beta}}_1)$ is "correct".