Effect of Change in P on W and r (Stolper-Samuelson)

Formal analysis: effect of change in P on w and r. Y is numeraire.

A. Equilibrium conditions (P=MC conditions)

(1) a_{Lx} W + a_{Kx} r = P (cost of producing X)

(2) a_{Ly} w + a_{Ky} r = 1 (cost of producing Y)

B. Totally differentiate the 2 above conditions:

a_{Lx} dw + a_{Kx} dr = dP - ((wd a_{Lx} + r a_{Kx})/P).

Dividing by P, etc. gives the elasticity form:
S_{lx} dw/w + S_{Kx} dr/r = dP/P - ((wd a_{Lx} + rd a_{Kx})/P) .

These a's are not constants: a_{Lx} = a_{Lx} (X, r/w) = K*/L* where * is equilibrium level. W and r are functions, implicitly, of X* and Y* and s_{Lx} (Elasticity form) = w a_{Lx} /P

Similarly for Y:
W a_{Ly} dw/w + r a_{Ky} dr/r = 0 - (wd a_{Ly} + rd a_{Ky}).

Now a_{Lx} and a_{Kx} change only due to technology or w/r ratio. If w/r ratio rises, a_{Lx} falls and a_{Kx} rises.

C. What is slope of isoquant? Remember, CRS is assumed. Otherwise, a change in P has a scale effect as well as a substitution effect on s. That is, a_{Lx} is not only a function of $a_{K.x}$.

This is the equation of the isoquant. In equilibrium (under cost minimization for the firm, which is given P, w and r) the isoquant is tangent to the isocost line. Min cost over a_{Lx} ,a_{Kx} = min a_{Lx} (a_{Kx})w + a_{Kx} r over $a_{K x}$

==>(d a_{Lx} /d a_{Kx}) w + r = 0

Now dL/dK given X=X0 corresponds to d a_{Lx} /d a_{Kx} = -r/w

Around equilibrium. Envelope theorem. Thus,

$$r d\, a_{Kx} + w d\, a_{Lx} = 0$$

(Hence usefulness of this for distortions, when you're not around the equilibrium and during growth.) Thus $s_{Lx}\, dw/w + s_{kx}\, dr/r = dP/P$, or

Define $Q_{lx} \equiv s_{Lx} = w\, a_{Lx}/P$. Then

(3) $Q_{lx}\, w\hat{} + Q_{Kx}\, r\hat{} = P\hat{}$ and

(4) $Q_{Ly}\, w\hat{} + Q_{Ky}\, r\hat{} = 0$

Or, in matrix notation,

$$\begin{matrix} Q_{Lx} & Q_{Kx} \\ Q_{Ly} & Q_{Ky} \end{matrix} \quad \begin{matrix} W\hat{} \\ R\hat{} \end{matrix} \quad = \quad \begin{matrix} P \\ 0 \end{matrix}$$

C. Solve matrix: $w\hat{} = (Q_{ky}\,'/\det Q)P\hat{}$ and

$R\hat{} = (a_{Lx}\,'/\det(Q))P\hat{}$

$\mathrm{Det}(Q) = Q_{Lx} Q_{Ky} - Q_{Kx} Q_{ly}$ and Q is the matrix of Qij 's.

D. Since Y is K-intensive by definition, denominator is positive. That is, P rising means w rises and r falls. Denominator is a fraction < numerator; in elasticity form, $w\hat{}/P\hat{} > 1$. **Magnification effect**: wage rate goes up more than price. And $r\hat{}/P\hat{} < 0$. So

$W\hat{} > P\hat{} > r\hat{}$

The change in price is a weighted average of changes in factor prices.

E. What happens to w/r? $(W/r)\hat{} = \{1/\det(Q)\}P\hat{}$ This also reflects the magnification effect via the factor intensities. If Q_{kx} and Q_{ky} are close together, large change in w due to small change in P. (3) and (4) give this result also.

F. There is a 1:1 relationship between the factor rentals ratio and commodity prices. This is the **Stolper-Samuelson Theorem**. Suppose we impose a tariff on an imported good.

What happens to the MPK/MPL? Should laborers favor the tariff? Previously, producers of that good were assumed to benefit and consumers of it to lose out. Thus, there were no unambiguous answers. But Stolper-Samuelson gives unambiguous answer.