

## Extension of Ricardian Model to many goods

AER: Dornbusch, Fischer and Samuelson: more countries

### C. Expanding the number of goods.

1. Re-rotate:  $a_{Lx} = a_x$  ,  $a_{Ly} = a_{Ly}$  , etc. So goods have  $(a_1, \dots, a_n)$  and  $(a_1^*, \dots, a_n^*)$  for labor units for production. Index commodities by comparative advantage diminishing:  $a_1^*/a_1 > a_2^*/a_2 > \dots > a_n^*/a_n$  . Is this index by absolute advantage? No, idea does not appear. The home country has a comparative advantage in good 1.
2. Can even have a continuum of commodities in  $[0, 1]$ . Let  $z$  = commodity so  $a(z)$  and  $a^*(z)$  are the input-output factors. Let  $A(z) = a^*(z)/a(z)$ , and  $A'(z) < 0$ . Good will be produced iff cost of production does not exceed price:  $w^*a^*(z)$  is cost of production in II and  $wa(z)$  is cost in I. So  $wa(z) \leq w^*a^*(z)$  holds for commodities produced at home.
  - A. Let  $w = w/w^*$  . Thus, condition is  $w \leq a^*(z)/a(z) = A(z)$ . This is the supply side, downward sloping in  $z$ .
  - B. Commodities produced at home are elements of  $0 \leq z \leq z(w)$  , where  $z$  is limiting production. But  $z$  must satisfy  $z = A^{-1}(w)$ . So abroad, commodities produced are in  $z(w) \leq z \leq 1$  .
  - C. What is the relative price of (any) 2 commodities  $z$  and  $z'$  produced at home?  $P(z)/P(z') = wa(z)/wa(z') = a(z)/a(z')$  vor all  $z, z' < z$  . And  $P(z)/P(z'') = w^*a^*(z)/w^*a^*(z'') = w [a(z)/a^*(z'')]$  for  $z''$  produced abroad. This links commodity prices to factorial terms of trade, as before.
3. For equilibrium, need demand side. Assume constant factor shares as before. Let  $b_i$  = share of expenditure on  $i$  and  $Y$  be income. Thus,  $b_i = P_i z_i / Y$  and the sum of all the  $b_i$  is 1. We assumed, in discrete space, that  $b_i = b_i^*$  (taste assumption). In continuous space,  $B(z) = P(z)C(z)/Y$ , and the integral from 0 to 1 of  $b(z)dz = 1$ , and  $b(z) = b^*(z)$ .
  - A. What fraction of  $Y$  goes to consumption of commodities produced at home? Integral from 0 to  $z$  of  $b(z)dz$  which we define as  $O(z)$  . Note that this is equally the share of world income, since tastes are the same. Note  $O(z) = b(z) > 0$
  - B. Now, the fraction spend on the rest of the commodities must be  $1 - O(z) =$  the integral from  $z$  to 1 of  $b(z)dz$

4. Characterizing the equilibrium 1:1 correspondence between goods and labor allows us to express this in terms of labor. Then total income = total expenditure on goods in equilibrium. More goods than factors  $\implies$  easier in labor market to express equilibrium. So total value of bundle of goods in I is  $WL$ . And value of demand comes from world income:  $WL + W^*L^*$ : fraction spent on home goods is  $O(z) [WL + W^*L^*]$ .

Thus, equilibrium condition is  $WL = O(z)[WL + W^*L^*]$

or  $W = \{O(z)[1 - O(z)]\}(L^*/L)$

$\equiv B(z, L^*/L)$  ; note  $dW/dz > 0$

So  $B$  is characterized by size and the border,  $z$ . Link to demand side is conditional on factor endowments, through  $L^*/L$ . Note  $dB/dz > 0$  and  $dB/d(L^*/L) > 0$ . This establishes  $z$ : note that 1 is an asymptote and  $B$  goes through 0.  $B$  summarizes equilibrium in the balance of trade as well as in the labor market, since foreign demand for our goods is  $O(z)$ . So

$O(z)W^*L^* =$  exports by II and

$\{1 - O(z)\}WL =$  imports by I. Equality here gives condition that

$dW/dz > 0$ .

Note upward slope of  $B$ :  $z$  up  $\implies$  raising wages in I, etc..

Exercises: 1) Statics: change  $L^*/L$  (double  $L^*/L$ ). This shifts  $B$  up. What about welfare effects, though? 2) Try technological progress in II.

5. Increase in  $L^*/L$  is vertical shift proportional to percentage change in  $L^*/L$ . (At fixed factoral terms of trade  $W$ ) Excess supply of labor abroad  $\implies$  rise in wage ratio. This causes loss of comparative advantage at home, so home produces *fewer* goods.

A. Welfare effects: If no distortions, wages measure welfare via efficiency. Domestic welfare per head goes up, but share of world income to I goes down.

$O = WL/(WL + W^*L^*) =$  share of income. Percent change in  $W$  is  $<$  percent change in  $L$  (note that  $z$  goes down, recall homotheticity).

- B. Consider technological progress abroad. Then  $a^*(z)$  falls; i.e.,  $A(z)$  falls. Percent decline in  $A(z)$  = percent increase in progress.  $W$  down  $\Rightarrow$   $W$  down relative to  $W^*$ .  $Z$  down also. Terms of trade:  $P(z)/P(z') = wa(z)/w^*a^*(z') = w[a(z)/a^*(z')]$ . Terms of trade “improve” for home country, i.e., the price of the thing you sell rises  $\Leftrightarrow$  terms of trade improve.

Let  $[P(z)/P(z')]^\wedge = \% \text{ change}$ . So  $W^\wedge - a^{*\wedge}(z) > 0$

1. Domestic real income increases in terms of foreign prices; in terms of domestic goods, no change. Domestic goods purchase more foreign goods now. Real income has gone up.
2. Foreign real income, though seems indeterminate. Terms of trade down, but technological improvement must more than offset this. So foreign real income up.  
 $W^*L^*/Wa(z) = \text{foreign real income in terms of our goods} = (1/w)L^*/a(z)$ . Then  $w$  down  $\Rightarrow$  foreign real income up.
3. There have been wrong analyses of this, putting too much emphasis on terms of trade. Can growth cause you to be worse off? Under different elasticities, possibly. But, like monopoly case where those elasticities won't occur, this is an illusion. In this framework, immiseration can't occur.

- C. Suppose country I gives a transfer to country II. Since B summarizes world demand, under identical tastes assumption, no change at all in picture. (Assuming income elasticity of 1) Of course, welfare-wise, country II is better off. “Transfer problem”. If marginal propensities to spend differ, though, could recipient be worse off: I give you a dollar, you spend it on shoes, so  $P$  of shoes rises. If I produce shoes, can I be better off? Correspondence principle rules this out. Even with same tastes, is there a “presumption” about the direction of change in prices?