Contributions of Ricardo, Mills and Pareto

VI. Theory of Ricardian Model (Chapter 7 in MMKM, Notes from Jacob Frenkel)

A. 1. Simple model, fixed proportions, only input is labor (this is source of alleged Marxian connection). 2 economies, 2 commodities, 2 production functions, constant returns to scale mean production possibility frontier is straight line. Opportunity cost of x in terms of y = Px/Py since prices equal costs, cost equal unit labor requirements times wages, and wages are equal in each industry.

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A. Goods: x, y, x*, y*, produced in countries I and II with labor Lx, Lx*, Ly, Ly* under production functions:

I = Home	II = Abroad
$X = (1/a_{Lx})Lx$	$X^* = (1/a_{Lx^*})Lx^*$
$Y = (1/a_{Ly})Ly$	$Y^* = (1/a_{Ly^*})Ly^*$

All differences are between a_{Lx}, a_{Lx^*}, a_{Ly}, a_{Ly^*}.

Wage determination: x-firm, relative P in terms of y, so Py = 1 and Px = P, and same for P*, Max over L_x:

Max($PX - W L_x$) implies $P(dX/dLx) = W$ and $(dX/dLx) = 1/a_{Lx}$ = MPL

B. Pre-trade input-output technology

	Country I	Country II
Good X Good Y	a_{Lx} = 1 day's labor a_{Ly} = 2 days' labor	a_{Lx}^* = 3 days' labor a_{Ly^*} = 4 days' labor
P W	$a_{Lx}/a_{Ly} = ½$ $1/a_{Ly} = P/a_{Lx} = ½$	$a_{Lx^*}/a_{Ly^*} = 3/4$ $1/a_{Ly^*} = P^*/a_{Lx^*} = 1/4$

1 Country I has an absolute advantage in production of both X and Y. Why trade? It will be seen that the cheap labor argument (How can we compete with China?) Just doesn't hold. Comparative advantage is what matters.

2. What about prices? Barter economy ==> must pick numeraire: y. So P = Px/Py. In Ricardian model, we have sufficient information to determine P even without the demand side.

   A. Cost of producing X in I is ½ unit of Y. Proof: In producing X, worker gets wage W, and a_{Lx} W is days*worker*wage. Competition ==> Px $\leq$ a_{Lx} W and 1 $\leq$ a_{Ly} W
   Similarly, P* $\leq$ a_{Lx^*} W* and 1 $\leq$ a_{Ly^*} W* (Reason is, P=MC condition, but if < then X or Y won't be produced)
   Note we assume that L is perfectly mobile between X andY and perfectly immobile between I and II.

   B. Suppose we know that workers wish to consume both goods. This is the only thing we know about the demand side. (Asymptotic preferences => interior solution) So equality holds: divide to get:

   $P = a_{Lx} / a_{Ly} = ½$ And $P^* = a_{Lx^*} / a_{Ly^*} = 3/4$

3. Wages, pre-trade: $W = 1/a_{Ly} = P/a_{Lx}$ and $W^* = 1/a_{Ly}^* = P^*/a_{Lx}^*$

4. How much is produced in each country? **Factor endowments** We need to know how much labor is available. Assume labor endowments are $\underline{L}$ = 100 and $\underline{L}$* = 200. Then, assuming full employment, Lx + Ly = 100 and Lx* + Ly* = 200.

   A. What are Lx and Ly? This is a question of efficiency, determined by production functions: $Lx = a_{Lx} X$, etc. $a_{Lx} X + a_{Ly} Y = \underline{L}$ And $a_{Lx^*} X^* + a_{Ly^*} Y^* = \underline{L}^*$ are economists' statements of the budget constraints.

Exercises:

1. Draw PPC curves
2. What is the Marginal Rate of transformation?
3. Suppose when P = 1/2, home country consumers want to consume x/y = 8 and when P* = 3/4, comsumers abroad want to consume x*/y* = 2. What ratio of x/y is produced?
4. What will be the new pattern of production in the 2 countries supposing that at the opening of trade world price Pw = {P P*}?

B. $Y = -(a_{Lx} / a_{Ly}) X + \underline{L}/a_{Ly}$ and $Y^* = -(a_{Lx^*} / a_{Ly^*}) X^* + \underline{L}^*/a_{Ly^*}$

MRT = ½ And MRT* = 3/4 (Draw in xy space, y-intercept is 50 in both cases, x intercept for -P slope is 100 and x intercept for -P* slope is 60.)

Demand as in exercises: x = 80 , y = 10, x*=40, y*=20

The suppply prices differ: horizontal S @P= ½ for I, P*=3/4 for II, dnwrd D, x/y axis, intersection @8; x*/y* axis, intersection @ 2

The usual trade condition: PX + Y = PCx + Cy (C=consumption) is not strong enough: it is also true that X=Cx and Y=Cy.

4. Open the economy to trade. If price abroad is > 1/2, producers will want to export; foreigners will import if Pw < 3/4 . Trade if $P \leq Pw \leq P^*$. But we will not yet discuss how Pw is determined (Mill does this). Given trade, what will be the pattern of production?

A. Assume Pw = 5/8 . Must move to complete specialization to maximize national income. (I specializes in X, II in Y, see in XY space because ½ = MRT(production) < MRT(exchange)= 5/8 for I.) Production choice is *separated* from consumption choice; producers maximize national income.

B. What does economy do with the goods produced? We have 100 X's and 50 Y's to distribute. The value of the exports must equal the value of the imports of the other country. In country I, there is now a higher relativ eprice of X, but there is also an income effect. (There is always one conflict). So look at y: at a lower relative price there is a reinforcing income effect: *more* Y must be consumed. Similarly, in the other country, more X* must be consumed.

C. Assume domestic consumption of Y goes up (from Cy=10) to Cy = 31.25 and assume Cx* goes up (from Cx*=40) to Cx*=50. Then Cx=50, and Cy*=18.75, algebraically, but also the 2 "trade triangles" are equal in area (shaded area in xy space; country I bc is from X=100 to Y=62.5, II is from Y=50; both slopes = 5/8, triangles shaded are 50 in X's and 31.25 in Y's). Moreover, the value of consumption at home is greater than (or equal to) the pre-trade value of consumption under any weighted index of values. E.g., now consumption is 50X and 31.25Y, and 50(5/8) + 31.25 = 62.5 > 80(1/2) + 10 = 50' evem 50(1/2) + 31.25 = 56.25 > 50. Similarly in II: 50(5/8) + 18.75 =50=40(3/4) + 20 = 50, and 50(3/4) + 18.75 = 56.25 > 50. (Note that under the 5/8 index country II does not actually "gain".)

D. Price must equal the cost of production, so $W = Pw/a_{Lx}$ (Only x is produced in I) and $W^* = 1/a_{Ly^*}$ (Only Y is produced in II.) Thus,

$W/W^* = (1/a_{Lx})/(1/a_{Ly^*})Pw$

The measure W/W* is called the **factoral terms of trade**. It is linked the equilibrium prices. Below a world price of P, no trade occurs, and $W/W^* = (1/a_{Ly})/(1/a_{Ly^*})$. Moreover, below a price of P, no X is even produced at home. Similarly, above P*, $W^* = Pw^*/a_{Lx^*}$, since no Y* is produced, so $W/W^* = (P/a_{Lx})/(P/a_{Lx^*}) = (1/a_{Lx})/(1/a_{Lx^*})$.

P=MC and $Px = a_{Lx} W$. If W = 5/8 and W* = 1/4, W/W* = 20/8=5/2
$Pw < a_{Lx}/a_{Ly}$ ==> W/W* = 2
$Pw > a_{Lx^*}/a_{Ly^*}$ ==> W/W* = 3

Thus, no configuration leads to an equalization of domestic and foreign wages (i.e., W/W* = 1), since W/W* in [2,3]. And only alon the sloped stretch can both goods be produced. At the corners, the patterns of specialization are

Lower: (X,Y) and (Y*)
Upper: (X) and (X*,Y*)

Note that with 2 goods, at least n-1 countries must be completely specialized. (Except if initial home price were the same for several of the countries *and* corner solution obtained.)
(Draw vertical: W/W* axis, horizontal Pw, vertical intercept = $(1/a_{Ly})/1/a_{Ly^*}$) horizontal to $P=a_{Lx}/a_{Ly}$, sloping up (slope = a_{Ly^*}/a_{Lx}) till $P^*=a_{Lx^*}/a_{Ly^*}$ then horizontal)

5. From the point of view of the world market, we wish to add X+X* and Y+Y*. Diverting all resources to the Y industry gives $\underline{L}/a_{Ly} + \underline{L^*}/a_{Ly^*}$ = 100 units of Y. World transformation schedule is achieved by following most efficient production. (Draw in Y+Y* vs X+X* space, Y-intercept=100, slope-1/2 to A at 100X[in this half

X,Y,Y*produced],kink[specialization: X,Y*], slope = -3/4 to 166 X-intercept[X,X*,Y*]Point A is called the **Ricardian point.**) Profit maximization implies that world will be *on* the boundary. Note that a point on this curve (A) corresponds to a line segment on the factoral curve.

A. Suppose factors were mobile internationally. Then everyone will go to I. This is the first time that absolute advantage comes into play. And world wages are equalized. (In XY space, slope=-1/2 and linear transf)

B. Derivation of world supply from world PPC: when Pw < a_{Lx}/a_{Ly} = 1/2, both countries produce only Y. At Pw = 1/2, country II specializes (alone) in Y*. At Pw > 1/2, indeterminacy, until Pw = a_{Lx^*}/a_{Ly^*} = 3/4. (Draw step supply fcn in Pw, x/y space) . Depending on demand, price will be in [1/2,3/4]. Temporarily fix demand so it intersects the vertical segment.

What determines the details of the supply side? Note that the horizontal coordinate here (at vertical segment of step) is $(L/a_{Lx})/(L^*/a_{Ly^*})$

1. Doubling the quantity of L moves this point to the right, causing intersection of D to drop Pw to 1/2. Can have a country well-endowed with labor which is still inefficient. "Effective labor units"

2. Doubling L increases supply of X. This should drive its price down. Similarly, L up => increase in demand for foreign goods, i.e. Py up.

3. Doubling L ---> the after-trade price for I is the same as the pre-trade price. Small countries gain more; big countries have little gain from trade. A large country probably can't import enough to satisfy it -- thus it cannot take full advantage of specialization.

6. Relative advantage: Key feature here is relationship between a_{Lx}/a_{Ly} and $a_{Lx}{}^*/a_{Ly^*}$. If $a_{Lx}/a_{Ly} < a_{Lx^*}/a_{Ly^*}$, then home country has relative or comparative advantage in X. Ricardian analysis contributed pattern of trade, rather than what determines equilibrium.

7. Increase in number of countries, keeping number of goods fixed at 2 gives stepwise supply schedule and PPC.(Draw)

B. John Stuart Mill

1. Only complete specialization in each country ==> full gains. What are the conditions for $P < Pw < P^*$? Need to know more about the demand side. Classicists don't look at utility though: they go to numerical examples.

   A. Mill assumes 2 commodities, the spending on which is independent of relative prices and income. For example, a Cobb-Douglas utility function. Assume ½ income is spent on X and ½ income on Y, etc.

      I. Let $U(Cx,Cy) = CxCy$. Max $U(Cx,Cy)$ s.t. $Pcx + Cy \leq I$ --> FOC: $P = (dU/dCx)/(dU/dCy)$ so therefore $P = Cy/Cx$ or $Pcx = Cy$, i.e., values of consumption are equal.

      Ii. Or $Cx = (1/P)(I/2)$ and $Cy = I/2$, i.e., ½ of income spend on each good.

   B. Determinants of the equilibrium price

      I. Let $a = L/a_{Lx}$ = maximal amount of X producible; $b = L/a_{Ly}$; $a^* = L/a_{Lx^*}$; $b^* = L/a_{Ly^*}$. Then $b/a = a_{Lx} /a_{Ly}$ and $b^*/a^* = a_{Lx^*} /a_{Ly^*}$

      Ii. Comparative advantage, by assumption, => $b/a < b^*/a^*$, I.e., slopes on PPC differ. We search for conditions that $b/a < Pw < b^*/a^*$

   C. Mill assumptions => $Pw = b^*/a$ (see PPC diagram)

      I. At A, there are b* units of Y, and a units of X. Thus, $Y/X = b^*/a$. We will show that b^*/a is the equilibrium price: ie.e., we will show that $b/a < P < b^*/a^* \iff b/a < b^*/a < b^*/a^*$.

      Ii. We know the trade triangles must be equal. The expports of II must be units of Y: ½ of II's income. Now II's income is L^*/a_{Ly^*} , thuse exports by II are $(1/2)L^*/a_{Ly^*}$. I's income is $Pw(L/a_{Lx})$: so I imports $Pw (1/2)(L/a_{Lx})$. Thus

$(1/2)(L^*/a_{Ly^*}) = (1/2)Pw(L/a_{Lx})$

So Pw = b*/a. This is the point of complete specialization. No more indeterminacy.

D. In our example, b* = 50 and a = 100. So the equilibrium price is 1/2. But then $½ \leq ½ < 3/4$: home price = world price. Thus, 5/8 was inconsistent. A large country does not usually enjoy specialization fruits -- but in our example L < L*. I's labor was less effective: what matters is L/a_{Lx} and L^*/a_{Ly^*} .

E. Meaning of b/a < b*/a < b*/a*

I. If home country is very small, b/a < b*/a* <b*/a, foreign country will not be specialized. So fix size of foreign country. Increasing size of I shifts lines out. Utility curve touches on sloped portion.

Ii. Condition is equivalent to b/b* < 1 < a/a*. That is, this is the condition for full gains from trade.

Iii. Absolute advantage: b < b* *and* a > a* . Home country must be *able* to produce more X and less Y than foreign country.

Iv **Absolute advantage** refers to totals, i.e., note that L and L* do *not* cancel out when the equation is in this form.

F. Pareto: How do I know there are gains from trade? Can only say clearly that gains exist if production of *both* X *and* Y increase. Compensation potential must exist.

I. Before trade, each country produces both goods, spending (1/2)I on each good. Production was a/2 and a*/2 for X; (a+a*)/2 in world. After trade, there are a X's and b* Y's.

Ii. Pareto's condition is

$(a + a^*)/2 \leq a$ And $(b + b^*)/2 \leq b$; i.e., $a \geq a^*$ and $b^* \geq b$.

Same as Mill's condition.

Iii. Complete specialization and welfare gains when $a \geq a^*$ and $b^* > b$. In our example, for home country, Cx = 50, Cy = 25; for foreign country, Cy* = 25, Cx* = 33.3, pre-trade. Sum of world output post-trade is $\geq$ pre-trade. (I,e., forms parallelogram: vector gains)

G. How does country size apply? $(L/a_{Ly})/(L/a_{Lx}) < (L^*/a_{Ly^*})/(L/a_{Ly}) < (L^*/a_{Ly^*})/(L^*/a_{Lx^*})$ Is the condition. Thus, $a_{Ly^*}/a_{Ly} < L^*/L < a_{L^*x}/a_{Lx}$, i.e., factor endowment condition: L/L* must be between the cost ratios. (Note that this is a necessary but not a sufficient condition. And all this depends on eta (income elasticity) = 1.

--> $a_{Lx}/a_{Ly} < (L^*/L)(a_{Lx}/a_{Ly^*}) < a_{Lx^*}/a_{Ly^*}$

(Because of tastes)

References:
Chipman's has the math
Baghwati has empirical results
Stern's is more empirical