Ch 1: Motion and Relativity Before Newton

May 15, 2008

Where should we begin our story? Many books start with Newton, but Newton was responding to both Galileo[1] and especially (for our purposes) Descartes. But Galileo and Descartes themselves were writing in the context of late Aristotelianism, and so were trained in and critical of that rich school of thought, so if we want to fully understand their work we would need to understand scholastic views on space and motion (see Grant, 1974, Murdoch and Sylla, 1978 and Ariew and Gabbey, 1998). But late scholasticism itself is the result of a long history tracing from Plato and Aristotle through Jewish, Arabic, Islamic and European thought. And of course Plato and Aristotle are explicitly reacting to their predecessors and contemporaries. In other words, we could start the story as early in recorded thought as we like.

However, to keep this investigation to manageable proportions, we will start with just a brief summary of some of the most relevant points of Aristotle's views concerning space and motion, then jump forward to Galileo and Descartes (and Huygens). In so doing, we ignore a great deal and risk giving the misleading impression that these are the only important contributors to the story, and even that the ideas we discuss had no precursors (for discussion of the relevant history see the references provided). The point is, however, that while Galileo and Descartes may not have been the first to have some of the kinds of ideas that we will be discussing, they were the first to have them clearly enough – and influentially enough – for the kind of discussion that we want to have. More than that, their work is indicative of the issues that were important to the development of modern physics, and of the depth and kind of analysis to which they were subjected. In particular, unless we understand Descartes sufficiently well, we will not understand how his work is a crucial background to Newton and Leibniz, and we run the risk of interpreting them from a contemporary point of view, and misunderstanding their disagreement. Indeed, it is not overstating things too much to say that one of the main points of the first three chapters of this book is that you cannot understand the relationship between Newton and Leibniz at all unless you understand how they are reacting to Descartes.

In the story of the development of mechanics that we will study, there are two strands on which we will concentrate. The first concerns 'relativity', which

---

[1] See Koyré (1965, 207-212) for a discussion of whether Newton had studied Galileo closely (see especially footnote 1 on p. 212). Note that Newton refers to him repeatedly in the *Principia*, for instance in the *Scholium* to the Laws (1999, 424), and in a number of proofs.

itself refers to *two* logically distinct, though conceptually interrelated ideas: that it is impossible to distinguish empirically one state of motion from another (which is usually called 'dynamical relativity') *or* that one state of motion is not 'privileged over' another in some way ('kinematic relativity'). The views that we will be canvassing in the next few chapters will help clarify the different senses in which some motion might be 'privileged', but a familiar example is Newton's absolute motion: motion relative to absolute space is singled out as a 'special' standard of motion – though the dynamical relativity of his laws means that the absolute speed cannot be determined. Or more generally, a number (but not all) of the definitions of motion that we will see proposed involve motion relative to some class of objects; we can then say that one definition is more kinematically relativistic than another depending on the classes. At one end of the scale, absolute motion is motion relative to one special entity, absolute space; at the other, motion relative to arbitrary reference bodies is completely relativistic, since it can be taken relative to a body in any motion whatsoever. (The matter can be put this way: if asked 'how fast does X move?', does one have to ask 'relative to what?', and if so, how wide is the class of possible answers?)

If one had a formal framework in terms of which different accounts of motion could be formulated, precise definitions of relativity could themselves be specified (e.g, Friedman 1983, §IV.5). But as I explained in the introduction, my goal here is not recapitulate technical analyses of concepts like relativity in the context of modern 'spacetime theories', but rather to investigate more closely how these concepts were understood by their originators. One can use the modern formalism to gain some insight into the logical structures that characterise the different proposals, but at the cost of losing sight of how they were arrived at and thought of in historical context. Indeed, I will suggest that certain important aspects of early modern thought are considerably distorted by the attempt to translate them into the language of spacetime theories.

One point in particular: to a contemporary way of thinking, it is natural to think that if dynamical relativity holds for two states of motion, then so should kinematic relativity – 'if you can't tell the difference, then there is no difference' – but that very idea is at stake in the next four chapters, so cannot be assumed. (A similar point holds for the converse, that kinematic relativity requires dynamical relativity – 'how can we tell the difference between two things if there is no difference?' – although that idea is made clear earlier in our story.) Hence it is important that we keep the logical distinction between kinematic and dynamical relativity clearly in mind, and resist the temptation to assimilate them, even as we see the central figures in the story struggle to develop these concepts.

The second strand of the story of mechanics concerns 'relationism', which is the idea that all spatiotemporal facts of some kind (perhaps *all* spatiotemporal facts) are just facts about the spatiotemporal relations (whatever they may be) between bodies. As is clear from the qualifications made in this characterisation, relationism is a rather broad notion. Indeed, perhaps it is more clearly defined negatively in terms of the view that space (or spacetime) is not a substance or entity over-and-above the material contents of the universe – that spatiotemporal facts are not at all facts about space itself. Again, we will come to see what relationism is by looking at the various specific forms that it has taken.

There is a particular temptation to be avoided: the conflation of kinematic relativity with relationism. They both could be glossed 'everything is relative', but they are logically distinct because there are ways to define privileged standards of motion in terms of relations: for instance, motion relative to the centre of mass of the universe. That is, there are two sense of 'relative' here: on the one hand, kinematic relativity means that frames have equal claim to be the standard *relative to which* motion should be judged; relationism about motion means that motion must be defined *in terms of the relative motions* of bodies. We will keep this distinction in mind, but it is not one that is always explicit in the readings before us; keeping the concepts distinct helps us distinguish the different contributions the authors make within a general logical framework, even if they did not clearly make the distinctions themselves.

1 Aristotle

Book IV of Aristotle's *Physics* (all quotations are from Aristotle, 1984) is an enquiry into the nature of 'place', because place is implicated in existence – 'things which exist are *somewhere*' – and because 'motion in its most general and proper sense is change of place.'[2] (*Physics* IV.1) In IV.4 he considers four proposals: *(i)* that the place of an object is its shape, *(ii)* its matter (as Plato suggested in the *Timaeus*), *(iii)* a second volume (of space, we could say) exactly coincident with it, or *(iv)* the interior surface of the bodies contiguously surrounding it (since he denies the vacuum, such a surface is guaranteed to exist). *(i)-(iii)* he rejects. For instance, the shape and matter of a body are not separate from it, while a place is separate from a body (indeed, bodies can obviously change their places without changing their shapes or material constitutions), so *(i)-(ii)* must be wrong. Distinct coincident volumes would be separate from bodies, but he rejects *(iii)* as an error caused by sloppy analysis of what happens when a body moves; certainly the body leaves its place, but it doesn't leave behind a 'bare' place with which it was previously coincident, for something else – often air – will take its place.

Thus Aristotle concludes that the place of a body is a kind of 'vessel', 'the boundary of the containing body at which it is in contact with the contained body' (*Physics* IV.4); thus the place of the soup is the internal surface of the soup can. Except, there are problems for this account when it comes to capturing our usual understanding of the motions of bodies contained by other bodies: for instance, a nail in a moving ship stays in its place according to the present definition, but there is also an important sense in which it moves. In response to similar cases, Aristotle offers a second definition: 'the place of a thing is

---

[2] 'Motion' – *kinesis* – meant change in the general sense for Aristotle and his followers, while 'local motion' ('locomotion') meant change of locale in particular. Aristotle (in *Physics* IV.4) mentions change of size as a kind of change of place, though increase and diminution fall under change of quantity, not locomotion.

the innermost *motionless* boundary of what contains it'. (*Physics* IV.4)[3] That is, a place is not just any vessel, but a 'non-portable' one. Of course, since (loco)motion is change of place, this definition makes Aristotle's account of motion viciously circular: motion is said to be change of motionless place. (There are of course other difficulties. Notably, places are usually thought to be the same size as that which they contain; the inner boundary of the containing body satisfies this desideratum, but the innermost motionless boundary won't in most cases.)

The problem for Aristotle is that he wants to respect ordinary judgements about whether bodies are in motion or not, but these are context dependent. When we say that something moves we usually mean that it moves *relative* to something in particular, where the conversational context picks out the standard of rest, the 'reference body', the 'something in particular'. Often it is simply the Earth (or the piece of it in the vicinity of the motion), as in 'Jane is moving' (relative to the countryside passing by her), but sometimes not as in 'Jane is not moving' (relative to the speeding train in which she is sitting). It depends on who is talking about her. Such context dependency is very hard (perhaps impossible) to specify in general, and Aristotle's account is thus far too thin to do the job of capturing our ordinary judgements.

That said, there is a specialised situation that Aristotle addresses in which context-dependency disappears – the problem of interpreting his theory of motion. He argues that in this context there are some privileged places. According to Aristotle's mechanics, it is in the nature of some things to move 'up' and some 'down'; the former are light (specifically, the elements fire and air, and compounds in which they predominate) while the latter are heavy (specifically, earth and water, and compounds in which they predominate) (*On the Heavens* I.2). The problem is to explicate the notions of 'up' and 'down'.

Aristotle argues that within his spherical, geocentric universe both the very centre and the inner surface of the celestial sphere (so of the lunar sphere) are motionless places: 'the one is always at rest, while the inner side of the rotating body remains always coincident with itself' (*Physics* IV.4). (The centre, of course, seems more like a point than a surface around anything, while the celestial sphere rotates, but let's not quibble here.) These two places are 'up' and 'down' and the natural motions are referred to them, making them privileged over any context dependent ups and downs (as the 14th floor of University Hall is up above the ground but down from the 28th floor). We will see this logical point running through our discussions of mechanics: physical theories may require a privileged sense of motion, especially those motions that are 'natural', and one faces the challenge of explaining how such motions are privileged in terms compatible with one's account of space and motion. In contemporary terms, any violations of dynamical relativity must be reflected in corresponding violations

---

[3] There is considerable debate concerning what exactly Aristotle's argument for this definition is (see Sorabji 1988, Chapter 11) or even whether the revision is a later interpolation (see Hussey in Aristotle, 1983, 117-8). These textual questions are interesting and challenging, but not particularly relevant here; the issue for us will be how Descartes responded to the given definitions.

of kinematic relativity.[4]

There are two more points to extract from this brief discussion of Aristotle. First, if bodies naturally move up or down then it follows that they will only move in other directions so long as mechanical forces are applied. The problem of course is that there are many things which have a horizontal component to their motion even when no obvious force is applied: for instance, a ball tossed between two children. But since all terrestrial objects are composed of the elements, up and down are the only natural motions on Earth, and some forces must be acting in such cases. Aristotle mentions (*Physics* IV.8), without enthusiastic endorsement, two possible solutions: that the hand continues to drive a column of air behind the ball after the ball leaves direct contact; and that the air pushed forward by the ball in flight follows a circular path to the rear of the ball, allowing the ball to push itself on. It's not hard to see serious problems with such ideas, and Aristotle's followers developed the alternative idea that when an external force is applied to a body it imparts an 'internal' force called 'impetus' (sometimes even thought of as a substance) which causes an unnatural projectile motion. How this account of projectile motion was supplanted by the modern conception – a body's inertia does not explain or cause its continued motion, but quantifies its natural, fundamental, primitive, resistance to changes in its motion – is an important part of our story.[5]

Finally, Aristotle's theory of natural motions is a particular instance of his theory of substantial forms. An object is to be understood as material substance (of varying degrees of specificity) coupled with form, which latter explains the changes the object undergoes. That is, (in many cases according to *Physics* II.7) the *(i)* 'final *(ii)* 'formal and *(iii)* 'efficient' causes of a change are the same: *(i)* the purpose of a change is *(ii)* to achieve an ideal state (or 'form'), and this purpose is *(iii)* what makes the change happen. Thus the form of a heavy thing – a rock, say – is the state of being situated at the centre of the universe, and the rock falls in order to (attempt to) realise that state, so its realising that state makes it fall.

The methodological flaw with this model of explanation is that it makes explanations too cheap and hence uninformative. We can explain any regular process simply by ascribing suitable natures: seeds grow because it is in their nature to become flowers, rocks fall because it is in their nature to be at the centre, and clocks tell time and opiates cause sleepiness because that is in their nature. But that is just to say that seeds grow because they have the power of growth; that rocks fall because they have the power of downwards motion; that clocks work because of their 'clockiness' (Leibniz's *Discourse on Metaphysics*, 1989, §10); and (following Moliére's 1673 *Le Malade Imaginaire*) that opiates cause drowsiness because of their dormitive powers, which is not to explain

---

[4] Note that Aristotle's natural/forced motion distinction is quite different from the inertial/accelerated distinction in classical mechanics, as we will see below. Insofar as the distinction maps into modern physics, it maps better into general relativity than Newtonian mechanics, see Huggett 2001.

[5] For a history of the precursors of the notion of inertia see Franklin (1976) and O'Brien (1970).

anything. One of the fundamental motivations of the mechanical philosophy, as we shall see when we turn to Descartes, was to offer a far more restrictive, and correspondingly more substantive, kind of explanation (see Nadler, 1998, for an overview).

2 Galileo

Galileo was of course a prolific and brilliant thinker and experimentalist, reshaping the way we think about motion, the universe and even science itself. He was also a superb publicist for the new physics that was coming into being, an exceptional expositor of science, and a formidable debater. The contributions that we want to consider relate to the development of the law of inertia and the understanding of relativity – specifically dynamical relativity. Consider the following exchange from *The Dialogue Concerning the Two Chief World Systems*:

> *Salviati*: But what would happen if [a ball were placed on a horizontal surface and] given an impetus in any direction?
>
> *Simplicius*: It must follow that it would move in that direction.
>
> *Salv.*: But with what sort of movement? ....
>
> *Simp.*: I cannot see any cause for acceleration or deceleration, there being no slope upward or downward.
>
> *Salv.*: ...; so how far would you have the ball continue to move?
>
> *Simp.*: As far as the extension of the surface continued without rising or falling. ....
>
> *Salv.*: Then in order for a surface to be neither downward nor upward, all its parts must be equally distant from the center. Are there are such surfaces in the world?
>
> *Simp.*: Plenty of them; such as would be the surface of our terrestrial globe if it were smooth, and not rough and mountainous as it is. But there is that of the water, when it is placid and tranquil.
>
> *Salv.*: .... Now as to [a] stone which is on top of the mast; does it not move, carried along by the ship, both of them going along the circumference of a circle about its center? And consequently is there not in it an ineradicable motion, all external impediments being removed? And is not this motion as fast as that of the ship? .... Go on and draw the final consequence ....
>
> *Simp.*: By the final conclusion you mean that the stone, moving with an indelibly impressed motion, is not going to leave the ship [when dropped] but will follow it, and finally will fall at the same place where it fell when the ship remained motionless. (2001, 172-3)[6]

---

[6] Belot (2000) begins a very nice article with this quotation, also to illustrate a connection between inertia and relativity. Here however, we have the chance to investigate how Galileo's ideas diverge from modern ones.

What is at stake here is the motion of the Earth, particularly its diurnal rotation. Aristotle argued (in *On the Heavens* II.14) that, amongst other reasons, we know the Earth to be at rest because falling bodies move perpendicularly to the ground. If the Earth were rotating, then because of its composition (predominantly of earth) it would not be moving naturally, while the falling stone would move naturally – to the 'centre' that is – since no forces act on it; the net result being that the stone would be 'left behind' and move eastward relative to the Earth as it fell. (Other terrestrial phenomena discussed in the *Two Chief World Systems* in this regard include the ability of flying birds and clouds to 'keep up' with the Earth, the absence of a continuous easterly wind, the isotropic motion of cannonballs, and the absence of centrifugal forces that might otherwise rip the Earth apart.)

Galileo mentions that this line of thought is supported by the belief that if a stone were dropped from the mast of a boat then it would land behind the mast by the amount the boat moves during the fall. Clearly this example itself does not show that the Earth moves, but is evidence for the implication of Aristotle's theory of motion that forced motions – those other than to or from the centre – are observable; any motion of the boat caused by the wind in its sails will be revealed by the stone. Galileo opposes the theory of motion, and thereby explains the unobservability of the Earth's motion, a particular instance of dynamical relativity.

Against Aristotle, Galileo gives a continuity argument in the passage quoted: if a ball speeds up as it rolls along a slope inclined down, and slows down as it rolls along a slope inclined up (and indeed, the rate at which the speed changes depends on the steepness of the slope), then on a horizontal surface the ball should preserve its motion.

What is interesting is the extent to which the picture described here falls within the Scholastic tradition. In the first place, the type of motion preserved fits within Aristotle's theory of gravity. It is sloping towards or away from the Earth's centre that leads to acceleration or deceleration, and so a horizontal – i.e., neither up nor down – surface is one perpendicular to the vertical, namely a sphere concentric with the Earth. Hence the motion preserved is along a circle concentric with the Earth's surface, and not along a straight line, for instance. Second, the analysis is presented in terms of the theory of impetus not inertia; it is 'impetus impressed forcibly' that explains why a ball would continue rolling up a slope (something hard for Aristotle to explain at all). Indeed, it is widely accepted that Galileo did not reach a modern conception of inertia (see Hooper 1998 for a recent review) although as the *Two Chief World Systems* progresses the notion of 'impetus' seems to become thinner, sometimes meaning nothing more than speed, and so approaching a modern conception. Of course, since the passage quoted is arguing towards the controversial thesis of the Earth's motion, it makes sense rhetorically that Galileo would make as many of his assumptions as possible uncontroversial to a scholastic.

How does the preservation of horizontal motion explain dynamical relativity according to Galileo? Simply, if the motion of a stone, dropped from the top of the mast of a moving boat, is compounded of its natural motion towards the centre of the Earth and a horizontal motion produced by its impressed impetus, then the net result will be that as it falls it keeps up with boat and lands at the foot of the mast – just as if the boat were at rest. Note that an important further assumption is introduced here – that the necessary impetus is impressed merely by dint of the stone's moving with the mast-top, and not by the application of any force (in Aristotle's sense). Thus, the scholastic could concede that if the ball were thrown forward then the impetus impressed would produce the horizontal component of motion necessary for the ball to land at the foot of the mast, but he denies that this would be so if the ball were simply dropped – which, though wrong, seems highly intuitive! Galileo thus has to persuade him that impetus can be impressed by the motion of a body alone, a step towards the modern conception of inertia.[7]

These considerations were a serious matter for natural philosophers of the time. For instance, in 1641 Pierre Gassendi used a fast moving trireme to perform the relevant experiments in Marseilles. The results are described in his *De Motu* of 1642 (1972, 119-25) – of course, they were only a surprise to those not already convinced by Galileo.

In fact, according to Galileo, the undetectability of the ship's (smooth) movement extends beyond the falling stone experiment: observations in a closed cabin of 'flies, butterflies, and other small flying animals', fish, 'a bottle that empties drop by drop into a narrow-mouthed vessel beneath it', a ball thrown between two people, and jumping in any direction will be exactly the same whether the boat is at rest of moving smoothly, 'nor could you tell from any of them whether the ship was moving or standing still.' (2001, 217). Galileo's reasoning is again that 'the ship's motion is common to all the things contained in it' (2001, 218), and of course preserved even as their relative motions evolve. However, the inertia of the bodies is insufficient to entail the undetectability of the ship's motion – Galileo also implicitly assumes that the mechanisms involved in the described systems are independent of the motion of the boat. For instance, that the strength of the muscles of animals are independent of the motion of the ship – otherwise the animals and ball throwers may move in rather different ways. Or again – and the same assumption was implicit even in the account of the falling rock – that the force of gravity is independent of the boat's motion, else the water might fall faster or not at all when the boat is sailing. Such assumptions are natural, but it does emphasise, in modern terms, that observational relativity is a property of the laws governing interactions; the assumption of inertia alone will only entail that a system keeps up with the boat, not that it evolves in the same way as if it were at rest.[8]

---

[7] Of course, for the stone to land exactly below the point it was dropped, its angular velocity about the Earth's surface must be preserved as it falls. But it is not according to Newtonian mechanics: conservation of angular momentum means that the stone's angular velocity increases as it falls while that of the boat remains the same, so the stone will actually land *ahead* of the mast. The effect is of course small; under plausible assumptions, the stone falls around a tenth of a millimetre ahead of where it was dropped, an amount swamped by the normal instability in a boat's motion. Interestingly, Galileo did derive a similar effect for an object dropped to Earth from the Moon (2001, 269-71).

[8] Harvey Brown stressed this point to me.

And of course, the point is that what goes for the boat goes for the Earth as a whole; the diurnal motion of the Earth means that every region of the Earth moves like the ship, because it rotates about the Earth's axis[9], and so the motion of the Earth and boat are undetectable (using terrestrial experiments) for the same reason. (This claim is, of course, untrue, since the Earth's rotation produces measureable centrifugal and Coreolis forces, observed for instance in Foucault's pendulum, the variation in the periods of pendulums at different places on the Earth's surface and the Earth's wind patterns – paradoxically, Galileo [2001, 510-13] did offer an [incorrect] explanation of the trade winds in terms of the Earth's diurnal rotation.) And so we see how the principle of dynamical relativity in Galileo's formulation has its roots in the need to explain the unobservability of the Earth's motion postulated by the new Copernican model of the solar system; how, primarily, it is about that kind of system.

3 Descartes

If there's a lesson for scientific advance in the first three chapters of this book it is that when great minds focus their attention on well-defined, specific problems (for instance Huygens on rotation or Newton on the motion of the planets) they achieve great advances, while when (like Descartes and Leibniz) they attempt 'theories of everything', reasoning from first principles, they underestimate the complexity of the problem and fail to achieve what they should. We will thus see a host of confusions in Descartes' account, which none-the-less was sufficiently compelling to beguile his contemporaries for the better part of a century, and form an important component of the ideas that Newton had to overcome.

Descartes was well aware of Galileo's achievements, but his ambition was far greater[10], though he failed to employ mathematical ideas as effectively as

---

[9] I do not believe that Galileo addresses the question of exactly what path around the spherical surface the ball in the quoted passage is supposed to follow. One might think of the great circles, but if the effects of the Earth's rotation are strictly null for the reason that Galileo gives, then the preserved motion due to a body co-moving with the Earth must be the same as that of the Earth, so *not* rotation about the Earth's centre – along a great circle – but about the Earth's axis – in the direction of a line of latitude. One might next think that a body's impetus will carry it along the circle to which its direction of motion is tangent, which the lines of latitude are for bodies moving with the Earth. But that proposal can't be right either because many circles share their tangents at any point of the sphere. And it can't be that motion is preserved along a line of latitude to which the direction of motion is tangent, because even in those case in which there is such a circle, it may not be the right one to guarantee Galileo's relativity. For instance, suppose Gassendi's trireme is following a circle that is not a line of latitude, but which is taking it due East at some moment, at which time the rock is dropped; if the rock follows a line of latitude, while the boat does not, it will not fall to the foot of the mast but to the side. Here then is a proposal that I think is sympathetic to Galilean conceptions. Instantaneous motion is not (necessarily) linearly directed, but may be circularly directed, so that a velocity is not sufficient to specify a motion, a circle must also be specified at any time. Then if body A is released from body B then, *ceteris paribus*, A will continue to move along the circle *best fitting* B's motion at the moment of release.

[10] In a letter to Mersenne in 1638, he faulted Galileo for focusing on particular effects at the expense of first causes (translated in Drake, 1978, 387-92). This 'fault' is of course exactly the virtue that my previous paragraph exalted in Huygens and Newton.

Galileo, let alone the more complex techniques that one would expect such a project to require. His *Principles of Philosophy* (1983, to which all references are made in this section, unless otherwise indicated) aims to give an account capable of explaining all phenomena; indeed it attempts to give the explanations of a huge range of phenomena. Since it is quite impractical to try to sketch his entire philosophy here, we will concentrate on only those aspects which are most relevant to our narrative, even if the ideas taken out of their full context may seem somewhat dislocated.[11] Thus we will discuss Descartes on the nature of space and motion, especially concerning their connections to his mechanics. We shall see (here and in the next chapter) that his attempt to satisfy contrary desiderata – particularly, the needs of a physics of collisions, the constraints of ordinary language, and pressures from the Catholic church – leads to tensions that help make his system untenable.

Before we start, it will be useful to have a little of the history of *The Principles*. In the early 1630s, Descartes prepared a work, eventually published as *The World* in 1664 (1998), containing a heliocentric account of the solar system. However, after the 1633 condemnation of Galileo, Descartes withdrew the work before it was published. It was not until the publication of *The Principles* in 1644 that many of the suppressed ideas saw public light, and by that time many were considerably modified (for one thing, the later work is several times longer). There is no doubt that the condemnation was *a* cause of the changes, though, as we shall see, the extent of its effect is controversial. Lastly, in 1647 an authorised translation of *The Principles* from Latin into French by Abbé Picot was published. This work almost certainly gave Descartes the opportunity to make modifications and clarifications in some places, but it does complicate our reading sometimes, because some changes are more plausibly attributable to errors on Picot's part.[12]

Thus we turn to Descartes' views: while Galileo addressed issues of dynamical relativity, we shall see that Descartes' work is concerned far more with concepts of relationism and kinematic relativity, the former especially.

3.1 Space and Place

For Descartes, space and matter were at root the very same thing, which I'll denote 'space-matter' (pronounced 'spatter', following a suggestion of Maria Balcells) – a substance whose nature was extension (II.11). It follows immediately that a vacuum (i.e., space devoid of matter) is impossible (II.16): there can be no region of space empty of matter if every region of space *is* a volume of matter. Thus Descartes' universe is a plenum of particles of varying sizes, including those small enough to compose fluids – a 'hydrodynamical' universe, indeed one in constant, undiminishing flux.

---

[11] For a more comprehensive study of these issues in a much broader context of Cartesian thought, see Garber (1992) and Garber *et al* (1998, §II.2). For some review of the history of the issues in historical context see Ariew and Gabbey (1998, §V-VI) and Gabbey (1998).

[12] The translation used here is based on the Latin edition, with French interpolations indicated with braces.

His identification leads to a puzzle about motion: if the space that a body occupies literally is the matter of the body, then whenever the body – i.e., the matter – moves, so does the space that it occupies. But if it it doesn't change the space it occupies, it seems not to have moved after all! Descartes' response is that while the stuff of which a region of space is made does move when a body moves from that place, when we conceptualise the situation we use a non-material criterion of identity for spaces; the space remains the same, not when it remains the same piece of space-matter, but when it remains 'the same size and shape and maintains the same situation among the external bodies which determine [it]' (II.12). That is, when I wave my hand in the air, the substance of which it is made moves with it, but we can continuously identify the place where it was initially by its shape, size and position relative to my desk, say, even though the substance of that place actually changes.[13]

This general solution has a number of different realisations, which Descartes employs as he analyses the concept of motion and formulates the laws of mechanics in terms of the ontology of his space-matter universe. In fact he gives two definitions of motion and discusses several others. To understand their differences we need first to understand his account of the different senses of 'place' and 'space'. First (II.14), when we talk about a region of space-matter (of changing constitution if bodies are moving through it) as a 'place' we mean to emphasise its location relative to some reference bodies, while when we talk about it as a 'space' we emphasise its shape and size. Thus we can talk of a body 'taking the place' – but not space –of one of a different size or shape.

Second, spaces and places can be 'internal' or 'external' (II.15): the internal place is the (possibly changing) volume of space-matter taken relative to reference bodies, while the external place is a surface immediately surrounding a region of space-matter. External place sounds like Aristotle's first definition of place, but there are crucial differences.[14] Descartes explains that the surface is not literally composed of the relevant parts of the surrounding bodies, but is a geometric surface, and further specifies that this surface is determined relative to reference bodies, which of course may be distant. However, Descartes is motivated here by the same concerns that led Aristotle to his second definition of place: we often judge things – like the nail in the ship – to move when their immediate surroundings do not and to remain still when their surrounding change – like a boat anchored in a flowing river. Indeed, although he doesn't mention Aristotle by name, Descartes clearly has his second account in mind; external place is supposed to resolve the vicious circularity of Aristotle's definition by explicating 'motionless' in a non-circular way – relative to an arbitrary refer-

---

[13] This account presupposes that pieces of matter have a principle of identity in virtue of which they can be identified at different times. Since Descartes apparently understood such a principle in terms of motion, and yet understood motion in terms of changes of position relative to enduring bodies, he faces a deeply problematic circularity (see Garber 1992, 175-181). The issue is not of central importance to us, so we shall simply assume that Cartesian bodies can somehow be identified at different times (as just a primitive fact about individuals if you like).

[14] I have not always been sufficiently clear on this point (e.g., Huggett 2000, 102). I thank Pooley (2002, 7-8) for emphasising what Descartes really says.

ence body.[15] (Of course, the account also differs from Aristotle's insofar as the innermost motionless boundary of a body need not be immediately surrounding it, unlike the external place.)

3.2 Motion

Since places are defined relative to arbitrary reference bodies, it follows that change of place is a relativistic notion. For instance, consider a person sitting in a ship sailing West at the same speed as the Earth travels to the East (II.13). He 'remains in one place as far as [the ship is] concerned, for he maintains the same situation in relation to [it]', '[changes] his place as far as the shores are concerned, since he is constantly [receding from][16] from some' and finally 'he does not change his place [as determined by] points in the heavens'. However, for Descartes the correct attribution of motion to a body requires more than change of place, and so – contrary to the way Descartes is usually read – *neither* of the two concepts of motion that he explicitly defines are relativistic, as we shall now see.

First Descartes defines 'motion in the ordinary [or common] sense', which we will call 'ordinary motion', or 'OM': 'However, movement ..., as commonly interpreted, is nothing other than *the action by which some body travels from one place to another.*' (II.24) He does not specify whether he means internal or external place, suggesting that what is crucial here is that which is common to both – their relation to reference bodies. That is, OM is the action by which a body changes its relations to a reference body.

Descartes further points out (II.24) that insofar as OM is taken to be change of place, a body can be correctly said simultaneously to move and not move. However, OM is not (just) change of place, but rather the *action* which produces such a change. What does this mean? In the first place there is the Aristotelian doctrine that everything in motion – everything 'moved', that is – is moved by something (*Physics* VII.1). This principle fits nicely with his account of forced motion and the theory of impetus (though is harder to reconcile with his account of natural motions), and supports the scholastic notion that 'movement' refers as much, if not more, to the action of the mover on the moved as the change of place produced in the moved.

It is however important to remember that Descartes here is explicitly analysing the everyday, pre-scientific idea that people have of motion, and not some technical Aristotelian notion; OM is 'movement as *commonly taken*', not 'as taken by the schoolmen'. And so Descartes is claiming that even in our intuitive, pre-theoretical understanding of motion we suppose there is a force driving any

---

[15] The example of a boat on a river that Descartes cites to justify his definition is the same – though stated rather more precisely – as that found in the *Physics* motivating Aristotle's. Additionally, a number of the issues that Descartes discusses in the *Principles* – such as the size of the universe and the plurality of worlds – also arise in similar terms in *On the Heavens*. It is clear that Descartes is specifically responding to Aristotle's writings.

[16] The English translation has 'moves away from' here, for the Latin '*recedit*'. However, as I shall discuss below, it is significant that Descartes does not use '*movere*' here, and and I have correspondingly changed the translation to avoid 'move'.

moving body on. One might think that Descartes' claim is plausible with respect to his audience, educated contemporaries whose 'intuitions' would have been coloured by their scholastic education – that in fact this 'common view' was not pre-theoretical at all; but that today people with any kind of (Newtonian) physics education would not suppose such a force – they would commonly take motion to be nothing but change of place. However, there is good evidence (see Halloun and Hestenes, 1985a-b) that even today, people's physical intuitions are essentially Aristotelian (even after university-level physics!), and so Descartes may be quite right to claim that the action view of motion is the vulgar one.[17] (One might further go on to see the Aristotelian theory as embodying the prior intuitions.) And when Descartes gives examples of the kind of 'action' that he has in mind, it is clearly an intuitive, pre-theoretical one: for instance, the sitting man feels no action 'in himself' (II.24).

There is a further reason that Descartes emphasises 'action' in OM. As we shall presently see, Descartes includes a principle of inertia in his physics, in the modern sense according to which no force, no action, is required to preserve the constant, linear motion of a body. One of the things Descartes wishes to do here then is clear the ground for that discussion; our ordinary conception may involve the idea that motion requires force, but in physics there is another conception which does not. By first defining OM, he emphasises the novelty of the technical sense of motion; it also follows that there is an important sense in which the ordinary conception is just wrong.

Since OM is the action – in a pre-theoretical sense – that produces motion, it follows that our intuitive ascriptions of action determine the proper ascriptions of OM. Thus a man sitting on a ship sailing out of port is ('more properly said to be') resting in the sense of OM, because he feels no action 'in himself' (II.24). Similarly, because in everyday contexts we attribute rest to the Earth – even the fixed stars are said to move about the Earth in everyday talk – it would be 'very improper' to attribute OM to the Earth just because it changes its place relative to the fixed stars (III.29).[18] Here Descartes is explicitly analysing the rules and conventions governing the 'proper' use of ordinary language, and particularly our everyday judgements concerning motion. He finds that according to those rules not all of a body's changes of place can equally well be called its OM; indeed in the case of the Earth there is a unique correct ascription, and in the case of the sailor there are more and less correct ascriptions[19]. So it is not just that standards of rest for OM are picked out by convention, but also that the

---

[17] Descartes (II.26) attributes the view that all motion requires force to our constant experiences of the effort required to move our own bodies; he attributes that effort not to maintaining motion, but to overcoming the force of gravity as we move our limbs, a force thus opposed to motion.

[18] Pooley (2002, 11) draws attention to this claim about the Earth; his remark set me on the path to the account of OM offered here. Pooley himself only mentions II.24 in passing, and does not comment on the relation of the concept defined there to change of arbitrary place (which he does discuss), though presumably he sees that the 'ordinary sense' invoked in III.29 cannot be change of arbitrary place.

[19] In fact, Picot's French translation says that we say that the sailor is at rest, implying a unique ascription of OM.

range of possible conventions is restricted so that *not every possible standard is appropriate for judgements of OM.*

I said that my reading differed from the usual one (e.g., Garber 1992, 162-3, and Huggett 2000, 102), and it is in just this point: OM is not change of arbitrary place and thus is not fully relativistic. The ordinary reading is encouraged by Descartes' remark immediately following the definition that, because OM is defined in terms of change of place, something – indeed the man sitting on his boat – can be said simultaneously to move and not move. But, as we saw, this claim is immediately qualified by Descartes; if we pay attention to the proper use of the ordinary concept of motion – in this case, pay attention to the role of 'action' – it is more correct to say that the sailor is at rest.[20] I think that commentators usually disregard the qualification as a confusing aside, but if we take it seriously, it is immediately seen to be of a piece with the definitive statement in III.29 that the Earth has no OM. What these passages have in common is the claim that the application of OM is indeed governed by 'ordinary' use, and that proper ordinary use of 'motion' is not relativistic.

Indeed, that our ordinary ascriptions of motion are not relativistic is also an important point in the discussion of that follows his second definition motion. II.29-30 says that ordinary usage does not attribute motion to the Earth, something quite incompatible with a fully relativistic notion. As we shall see, he explicitly distorts his second, technical or 'philosophical' definition of motion in order to respect this aspect of ordinary use – and it because of this lack of relativity in ordinary usage, that he can say that the Earth is at rest in the technical sense.

So while Descartes observes a loose use of 'motion' in everyday discourse that is fully relativistic, relative to entirely arbitrary reference bodies, there is ample evidence that he denied that this relativistic use obeyed the rules that govern proper everyday motion discourse. It is in this way that OM is not after all relativistic. (Analogously, one might in some situations stretch the meaning of 'classic movie' to include *The Karate Kid*, even though it falls outside the ordinary use.) If you think about it, this view is quite plausible; perhaps there are some things that are never the standard of rest for a given body in any conversational context. In short, Descartes believes that there is sense of 'x moves' in ordinary language distinct from mere change of arbitrary relative position; a sense in which we might argue with someone, 'the seated passenger is not actually moving, even though he is getting nearer to Dover.' Of course, this concept is not a clean one, but its murkiness is only that of ordinary language, whose sense Descartes wishes to contrast with a technical notion.

---

[20] Descartes' Latin text prefaces this qualification with a misleading '*quin etiam*'/'moreover', which suggests a *simple* clarification; but Descartes' 'clarification' explicitly restricts the ordinary use. (Picot's translation uses a more emphatic '*toutefois*'/'however'.) Recall too from footnote 16 that in the similar discussion of change of place of II.13 Descartes does not use the term '*motus*' as he does when he defines ordinary 'motion'; instead he uses terms such as 'recede from' that explicitly involve reference bodies. The careful usage suggests that something could, for instance, recede from shore without thereby *moving* in the sense of OM, which is what I propose here.

Finally, although as far as I am aware, none of Descartes' writings discuss OM in the context of Galileo's relativity principle[21], it will help us understand their different contributions if we do so. Their ideas seem to pull in opposite directions: the different motions of a boat are indiscernible according to Galileo, but being at rest in harbour *vs.* being blown across the water are different OMs according to Descartes. But the idea that if two motions are indiscernible then neither is 'special' – that dynamical relativity should entail kinematic relativity – is an idea that we won't encounter explicitly until we discuss Leibniz's work. There is in fact nothing in Galileo's discussion to suggest that he thought that the different states of motion were equals. Indeed, the whole point is of course to show that even if the Earth were moving, we couldn't tell, *not* that it makes no difference whether or not we count the Earth as moving. And Descartes says nothing that would lead one to think that the motion or rest (in the ordinary sense) of the ship could be experimentally determined by someone in a cabin. After all, the sitting sailor experiences the same 'action' (*viz.*, none) however the ship moves. The point is that Galileo's 'relativity' is of the dynamical kind and Descartes' of the kinematic kind. (In fact, because Galileo's impetus drives bodies in circles, while – as we shall see – Descartes' inertia is rectilinear, they do disagree about the observability of circular motion in general.)

The second Cartesian definition of movement ('movement properly speaking') we shall refer to as 'proper motion' (PM):

> If, however, we consider what should be understood by movement, according to the truth of the matter rather than in accordance with common usage: we can say that it is *the transference of one part of matter or of one body, from the vicinity of those bodies immediately contiguous to it and considered as at rest, into the vicinity of others* (II.25).

To modern ears this 'container view' of motion sounds bizarre: according to this way of thinking, the soup does not have PM when the can is tossed into the shopping trolly, but when it pours out into a pan it does. One wonders what use this notion could really be. But of course the account has a pedigree: it is (to a good approximation) change of place in the sense of Aristotle's first definition – motion from the innermost surface of the bodies in contact with it.[22] Looking

---

[21] In *The World* (51) Descartes offers an explanation of the unobservability of the Earth's motion in the cases that Galileo discusses which is rather different from Galileo's: for instance, the bodies do not fall vertically because the Earth's rotational motion is preserved in them, but because everything on Earth is pushed around by the celestial vortex (see the discussion later in this section) at the same rate. This idea is related to Descartes' confusion over the conservation of angular momentum; see III.144 and the interpolations in IV.22, found in Picot's translation.

[22] The 'place' in question is not exactly Aristotle's: for instance, it is motion relative to the surrounding bodies themselves that counts, not motion relative to their inner surface. Thus Descartes is consistent with his view from *Regulae* of the 1620s that changes in the shape of that surface does not constitute motion of the body (see Gabbey, 1998, 654). However, for our purposes the difference is unimportant. Pooley (2002, 9) correctly points out (contrary to Garber, 1992, 164) that it would be completely wrong to understand PM as motion from

forward, we shall see that Newton attacked this account at length when setting up his own view of motion.

Descartes makes some important interpretative remarks about PM. First, PM is explicitly the 'transference' or change in relative distance, and neither 'the force or action which transfers' nor 'a substance' (II.25): it is distinguished both from the everyday notion of motion, OM, and from technical impetus theories. Second, the PM of a body is at any time uniquely attributable – not this or that speed and direction, relative to this or that reference body, but one particular motion 'because at any given time, only a certain number of bodies can be contiguous to it' (II.28). Finally, and most puzzlingly, PM is not just motion from contiguous surroundings, but only from contiguous surroundings that are 'considered at rest'.

When he explains this qualification, Descartes makes clear that it is motivated by the fact that 'mere' transference is symmetric: A can't separate from contiguous body B, without B being simultaneously separated from contiguous body A. Thus one is tempted to take him as saying that PM is kinematically relativistic in the sense that the PM of any body is either its motion relative to its contiguous surroundings 'taken at rest', or rest, since it is taken at rest, and its contiguous surroundings are thus in PM. But such a reading cannot be correct, because, as we saw, Descartes is explicit that PM is not a relativistic notion at all – it is uniquely attributable. A more careful reading of his discussion of symmetry makes clear that the qualification concerns a rather different issue.

What concerns Descartes in particular is our ordinary attribution of rest to the Earth, even though it is in motion relative to many things with which it is contiguous: for instance, rivers, winds, animals and, as we'll discuss later, particles of celestial space=matter (II.28). The alleged reason that we ordinarily attribute rest to the Earth is that it is transfered in different directions (and presumably different speeds) from different contiguous bodies: imagine, for instance, a Southerly wind blowing up the Mississippi valley. Thus if we treated each transference from a different contiguous body as *the* motion of the Earth we obtain a contradiction (II.30): the Earth moves both South and North because of its transferences from the wind and Mississippi, respectively. So in the case of the Earth, 'lest we deviate too far from the customary manner of speaking' (II.30), Descartes qualifies the definition of PM in conformity with ordinary usage, so that the Earth is at rest, properly speaking, while the bodies in contact with it move. Explicitly, in the terms of the definition, we ordinarily attribute rest to the Earth, while *we consider its surroundings to be in motion*, not rest – hence the Earth has no PM, despite its transference from them. (The same applies to the other planets for exactly the same reasons [III.28], and so presumably also to other bodies that we ordinarily take to be at rest because of their contiguous surroundings move in different directions.) That is, the qualifi-

---

Descartes' *external place*, a notion that corresponds far more closely to Aristotle's second definition, because external places are fixed by arbitrary reference bodies, not the surrounding bodies themselves. That is, while OM is motion from place, internal and external, PM is *not* for Descartes.

cation 'considered at rest' is introduced to make PM, unlike mere transference, non-symmetric.

It may be objected that we have not removed the relativity of PM, merely located its source more precisely. If the definition states that a body is only in PM if *both* it moves from its contiguous surroundings *and* our ordinary use of language attributes rest to those surroundings, then whether a body has PM or not depends on our ordinary attributions of motion. But, as we have seen, such attributions are relativistic (to some degree); it seems we might ordinary attribute rest or motion to some of the things moving across the Earth, and hence either proper rest or motion to the Earth. That result, of course, is contrary to Descartes' clear claim that PM is uniquely attributable. However, I understand Descartes' suggestion to be that in the context of bodies like the Earth and its surroundings, the conventions of ordinary language use are *unequivocal*: it would be against the rules of ordinary language ever to attribute rest to the Earth's surroundings (or motion to the Earth). The force of II.30 is to explain why our everyday use of 'motion' is in fact completely non-relativistic in that special (kind of) case. And since PM is to follow ordinary use, if our ordinary use unequivocally holds a body's contiguous surroundings to be at rest then, despite that body's transference from those surroundings, it has no PM. Only in the case in which it is ordinarily permissible to attribute rest to a body's surroundings, is it possible to attribute PM that body. Or more precisely, Descartes' definition should be understood to state that a body is in PM if *both* it moves from its contiguous surroundings *and* our ordinary use of language permits the attribution of rest to those surroundings. And thus PM is non-relativistic, for there is always a fact of the matter about whether a body's surroundings are unequivocally at rest ordinarily speaking, or whether we may take them to be at rest.[23]

In his discussion of the Earth's PM, Descartes indirectly addresses a problem for his definition of PM, which he wishes to take a unique value for any body at any time. If a body's contiguous surroundings are in some kind of turbu-

---

[23] Garber (1992, 162-72) also argues against a relativistic conception of proper motion in Descartes, but he argues that PM is in fact unqualified, mere transference. According to his view, what the definition means is 'if a body has motion in the frame of its contiguous surroundings (i.e., when they are 'regarded at rest' in the sense of being taken as a reference body) then the body has PM' (see p.169) – but that is just to say that it has PM if it is transferred from its contiguous surroundings. Motion in this sense is symmetric, but not relativistic: a body's transference from its surroundings is not removed by regarding it as a reference body. However, Garber's understanding cannot be right. According to Garber's reading, we are free to consider the Earth's surroundings at rest, and so should attribute proper motion to the Earth. But II.29-30 turns on the claim that despite being transferred from its contiguous surroundings, the Earth is not in motion properly speaking, only the surroundings are (the point is made again at III.28). That is, Descartes is quite clear that PM, unlike transference, is not symmetric – hence they just cannot be the same thing.

Garber wants PM to be transference, because, as we shall discuss in the next section, he believes PM to be the sense of motion implicit in Descartes laws of motion, and because, as we shall discuss immediately below, he believes that only transference can play that role. I essentially follow him on these two points, but I don't agree that PM has to be mere transference in order to satisfy them. In particular I think that Descartes thinks that they are compatible with the definition as I have explained it.

lent motion amongst themselves, then the body has different motions relative to different parts of the surroundings, and so *the* transference is apparently a collection of velocities. One can imagine various solutions to the difficulty, such as a rule for picking out a particular part of the surroundings, or defining PM as motion relative to the centre of mass of the surroundings, but instead Descartes ties this definition also to conventions of ordinary usage.

This move, however, is at odds with his program of mechanical explanation. Such explanations are supposed to be given in terms of the 'modes' of bodies alone (roughly, their objective but non-essential properties), but while 'mere' transference from contiguous bodies is a mode, PM is not. The problem is not that PM is kinematically relativistic (it isn't), but that PM is in part a relation to us and our use of language – rather than an objective property of bodies alone. Descartes is quite clear on this aspect of PM, saying that 'we must remember that all the real and positive properties which are in moving bodies, and by virtue of which we say that they move, are also found in those contiguous to them, even though we consider the second group to be at rest' (II.30). That is, the Earth is objectively in motion, even though it is has no PM; more generally, transference is, and PM is not, a mode.[24]

So now Descartes is faced with the difficulty that PM, not being a mode, is not a mechanical conception, despite the fact that he describes it as 'the truth of the matter', and hence, *prima facie*, proposes that the definition of PM is the definition of the 'term' motion as it appears in his laws.

A charitable reading of the situation is to take Descartes' explanation at face value: he defines PM so that the general readership of the *Principles* will not be confused by the overly deviant usage of familiar terms. Such a strategy does not mean that he capitulates rigour to pre-scientific conceptions, but it can only be judged successful to the extent that in the cases discussed in the book, the difference between PM and mere transference is irrelevant. It seems to me that the explanations offered by Descartes indeed satisfy this demand. One kind of case is that in which we can ordinarily attribute rest to the surroundings; the difference between PM and transference effectively collapses, and Descartes can give 'explanations' in terms of PM, even though we really know that it is the transference that does the explanatory work. The other kind of case is that in which the motion of the surrounding bodies does all the explanatory work, and that can be taken as PM or transference again. For instance, Descartes' explanation of gravity (IV.20-7, especially IV.23) refers only to the transferences/PMs of the parts of the celestial vortex surrounding the Earth. (I don't mean to say that Descartes' explanations are acceptable, even in the terms of his own system. In particular, as we shall discuss below and in the next chapter, he

---

[24] In the passages under discussion, Descartes is pretty clear that PM is not a mode, for the reasons given: '. . . if we wish to attribute to movement a nature which is absolutely its own, without referring it to any other thing . . . ' then motion should be taken as mere transference (II.29). However, we should note that Descartes does at one point say that the Earth 'has the mode of a body at rest' (IV.22). Since it is at rest in the sense of PM but not transference (because of the motions of the contiguous bodies), he suggests that PM is a mode after all. I attribute this passage to a slip in terminology.

attributes orbital rotation to the planets; a motion which cannot be understood to be PM – the have none – and which is due to the rotation of the vortex, not its turbulence.)

A less charitable view has been defended by Pooley (2002, 9-18). In short, he argues that Descartes qualified the definition of motion simply to avoid conflict with the Church. While this argument has plausibility, (we know *The World* was suppressed for this reason) it does indict Descartes of dissembling in regard to his true views about motion, whereas my proposal does not. We shall return to this question below.[25]

In what follows we shall follow what I take to be Descartes' strategy and equate PM with transference for all practical purposes; we shall assume that we are always considering cases in which we can attribute rest in the ordinary sense to a body's surroundings. Unlike Descartes, I shall assume that my reader can handle the concept of motion as 'mere' transference, even if it does shock her intuitions about the Earth's rest! (This procedure will continue in the next chapter, where we shall see that Newton indeed took Descartes' PM to be nothing but transference.)

And so we find in the *Principles four* significant concepts of motion: change of arbitrary place (i.e., motion relation to an arbitrary reference body), OM, PM and now transference from immediate surroundings.[26] Change of place and transference are 'purely relative' in the sense that we will use that concept, since they are defined in terms of the spatial relations alone. OM and PM are not purely relational, for they are in part relations to us and our linguistic conventions. Only change of place is fully kinematically relativistic, amounting to motion relative to entirely arbitrary reference bodies. OM is relativistic to some degree – the degree permitted by our linguistic conventions (which in cases such as the Earth make our attributions in fact unique). Finally, although PM is strictly distinct from transference (the Earth, for instance, is transferred but not moved properly speaking) both are uniquely attributable, and hence not at all relativistic.

It seems reasonable to assume that PM (or rather mere transference) is the sense operative in his mechanics, but we will now investigate this issue more closely.

---

[25] Pooley's insistence that Descartes' qualification that the surroundings be 'considered at rest' is a substantive part of the definition of PM changed my mind on the issue. I am less convinced about Pooley's account of Descartes' motivations, and I have a different understanding of what Descartes meant by the qualification; in the first place, because I read Descartes as saying that the choice of what is to be considered at rest is not arbitrary, but governed by the rules of ordinary discourse. Pooley is far from being the first to point to such theological motivations; as he notes, Henry More suggested the same in 1662, and Koyré, (1978, 265) has a similar argument.

[26] There is in fact at least one further sense of motion to be found: motion in virtue of being inside a moving body (II.35). One of Newton's criticisms of Descartes is that he does not properly integrate this kind of motion into his conceptual scheme; see the next chapter.

3.3 Mechanics

We now turn to Descartes' theory of motion. It seems reasonable to assume that he intends 'motion' in the sense of PM (or rather, mere transference) but is that correct? To address this issue – *the* main question of this book is what sorts of motion theories of mechanics require – we first need to understand some of Descartes' theory. He begins with some general principles and definitions, and then proceeds to a series of laws governing specific interactions. Here lies one of Descartes' most enduring specific contributions to modern scientific thought: not the content of the laws, which, other than the law of inertia and 'Rule 1', were quickly rejected, but the model of how laws – rather than forms – could play a role in scientific accounts of nature.[27]

The most general principle is (II.36) that God is the 'universal and primary' cause of motion, creating matter with motion or rest, and conserving 'in the total sum of matter, by His normal participation, the same quantity of motion and rest'. The 'quantity of motion' conserved is such that 'when one part of matter moves twice as fast as another twice as large, there is as much motion in the smaller as the larger' (II.36): it is proportional to size and speed.[28] In Newtonian (and indeed relativistic and quantum) mechanics momentum – mass × velocity – is conserved, but Descartes' conserved quantity, while similar, differs in two important ways. First, instead of mass we find 'size' or volume (note that according to Descartes most objects have pores that do not contribute to their volume). Second, the quantity depends on speed (the undirected rate of change of position), not velocity (motion in some direction at a given speed). This definition means that Descartes' conservation principle is false; mass × velocity is conserved, but mass × speed is not. (Note that Descartes was aware of the notion of velocity: e.g., II.44.) As we shall see in Chapter 3, Leibniz offered a convincing refutation of the conservation of Descartes' quantity of motion, and proposed mass $\times$velocity^2 – essentially kinetic energy – in its place.

Descartes proposes two further conservation principles: his first law states that if a body is at rest,

> . . . we do not believe that it will ever begin to move unless driven to do so by some external cause. Nor if it is moving, is there any significant reason to think that it will ever cease to move of its own accord and without some other thing which impedes it. (II.37)

This law – which is a special case of a more general principle of immutability – also follows from God's nature. It does not however follow immediately from the conservation principle, since one could imagine a system of bodies that conserved their total quantity of motion, even as their individual speeds were changed by internal causes.

The second law elaborates on the first:

---

[27] See Milton, (1998), for a discussion of the development of the idea of a law of nature in this historical context. See Gabbey (1998, §V) for some earlier attempts at collision rules.

[28] Descartes was not original in quantifying motion in this way. In the Fourteenth Century, Buridan had quantified 'impetus' similarly (see Franklin, 1976, 42-50).

> . . . each part of matter, considered individually, tends to continue in its movement only along straight lines, and never along curved ones' (II.39).

And so in the important respects Descartes proposed the modern conception of inertia; in contrast to Galileo the preserved motion is along straight lines, not horizontal, and in contrast to impetus theory (which Galileo did not explicitly abandon) motion is preserved naturally, not by any kind of internal cause.[29]

The third law concerns collisions:

> . . . when a moving body meets another, if it has less force to continue to move in a straight line than the other has to resist it, it is turned aside in another direction, retaining its quantity of motion and changing only the direction of that motion. If, however, it has more force; it moves the other body with it, and loses as much of its motion as it gives to that other.' (II.40)[30]

Since this law makes the outcome of a collision depend on a competition between bodies that is decided by the 'forces' they possess, it is usually referred to as the 'contest model of impact'.[31]

Of course this law is only useful if the 'force to continue' is defined: 'this force must be measured not only by the size of the body in which it is, and by the area of the surface which separates this body from those around it; but

---

[29] Descartes gave the law in *The World* (1998, 25-30), though he published it for the first time in *The Principles*.

It is also true that Aristotle's *Physics* IV.8 says that in a void there would be no reason for a body to stop, 'for why here rather than there?', and so 'it will either stand still or it must be carried to infinity, unless something stronger gets in its way', which sounds something like the law. But this claim is crucially different from Descartes'. In the first place, Aristotle argues here that the void is impossible because such a motion is incoherent: he holds that natural motion must be understood in terms of privileged destinations, but since the void is homogeneous there are no privileged destinations in the void, and natural motion is impossible. That is, the apparently inertial motion mentioned here is only introduced hypothetically in order to be to be rejected. Secondly, even taken at face value, Aristotle's statement is very different from Descartes' law of inertia: that a body would continue in a certain way if *per impossibile* it were in a vacuum has in itself no bearing on how it actually behaves. That it always has a *tendency* to move in a certain way whether in a vacuum or not makes a great deal of difference to how it actually behaves. See footnote 5 for references concerning precursors to the concept of inertia. Also see Prendergast (1975) for an analysis of Descartes on tendencies in mechanics.

It is often said that Gassendi, in the *De Motu* of 1642, was the first to publish the correct law (e.g., Koyré, 1965, 185-7). This claim is overstated to my mind: Gassendi's discussion of terrestrial inertia is the same as Galileo's, in that bodies are taken to move naturally along circles concentric with the Earth (1972, 130-2); and Gassendi's additional claim that bodies will continue to travel in the same direction naturally in a vacuum, doesn't obviously differ from Aristotle's in any way, except insofar as Gassendi accepted the vacuum. It seems to me that the prize can only go the person who is clear that even on Earth, bodies tend to constant linear motion.

[30] Note that the first and third laws guarantee the conservation of the quantity of motion in the absence of forces and in collisions respectively.

[31] Garber (1992, Chapter 8) describes Descartes' changing thought – away from this simple model – on the question of how the outcome of a collision is decided.

also by the speed and nature of its movement, and by the different ways in which bodies come in contact with one another.' (II.43) It's not clear that this definition is very helpful: we are told that it depends on a body's size, area, speed, the 'nature of its movement', and the kind of collision it undergoes. The first three are at least precise quantities, modulo the outstanding issue of what concept of motion is involved (though Descartes does not tell us what mathematical dependence force has on them) – but it is hard to know how to quantify the kind of motion and collision in a way that might render force a precise quantity. We can however understand how Descartes was led to such an awkward conception, if we remember that bodies are supposed to be moving in a plenum, not empty space. Then, for instance, we expect that a body would be slowed down by friction with the matter flowing over its surface, plausibly as a function of area, and that such an effect would affect the outcomes of its collisions. Descartes' approach seems to build such considerations into the laws of mechanics via the notion of force.[32]

The plenum assumption goes a long way to explaining the ultimate failure of Cartesian mechanics, despite the efforts of some of the greatest minds of the day: problems involving the huge numbers of bodies conceived as comprising the Cartesian world are fiendishly difficult. Part of Newton's success can be attributed to that fact that the solar system can be treated as a problem involving bodies in empty space to great accuracy. Now, Descartes in fact appreciated the difficulties of the plenum to some extent, and was therefore content to restrict to almost nil the scope of his quantitative work. In particular, when he turns to the application of his laws, he makes an idealisation in order to make the problem of colliding bodies tractable: he considers what would happen according to these laws if 'two bodies were to come into contact, and if they were perfectly solid, and separated from all others {both solid and fluid} in such a way that their movements would be neither impeded nor aided by any of the surrounding bodies' (II.45). Of course, such a situation is impossible, since a Cartesian vacuum is impossible, so Descartes is describing a strictly ideal situation to isolate certain features of his mechanics – something that one does in physics all the time, for instance in considering what would happen to the moon if no forces acted on it. (In IV.21 he says that 'empty space' should be understood to be a region of matter which 'in no way either helped or hindered the movement of other bodies'. So maybe a better way to understand his idealisation is not so much in terms of the vacuum, but in terms of frictionless contact.)

Given the idealisation, force becomes nothing but size × speed; we can neglect surface friction, while the assumption of solidity (and that collisions occur along a straight line) means that the 'kind of collision' is a constant factor. Under these assumptions, Descartes derives a series of seven 'rules of collision', describing various possible situations, of which we will consider two representatives. (Most of) them are indeed idealisations in the counterfactual sense, since they are not even approximately true in realistic situations; Rule 3,

---

[32] See Slowik, 2002, Chapter 3 for an extensive and useful analysis of Descartes' notion of force; although see Huggett 2004 for some criticisms.

below, is a case in point.

In the simplest, completely symmetrical case (Rule 1, II.46), two equal bodies move together at equal speeds along a straight line. Denoting their size by m and their speeds by $u_1 = u_2 = u$, the initial quantity of motion of the system is $2mu$; conservation implies that their final speeds satisfy $v_1 + v_2 = 2u$. Since they have the same sizes and initial speeds, in the given circumstances, they have equal forces, a situation not explicitly covered in the third law. Likely motivated by the symmetry of the situation, Descartes treats the collision as a case of the first part of the third law: the directions of the motions reverse, but each body retains its quantity of motion. In such a collision the two bodies bounce off each other and move apart at speeds $v_1 = v_2 = u$ along the line of collision.

Rule 3 (II.48) covers the case in which two equal bodies, of size m, collide along a straight line with different speeds, $u_1 > u_2$. Again, conservation means that the final speeds, v_1 and v_2 must satisfy $v_1 + v_2 = u_1 + u_2$. This time the first body has the greatest force ($mu_1 > mu_2$) and so 'wins' the collision: according to the third law, after the collision the bodies must move together in the original direction of the first body, with speed v. Since the first 'loses as much motion as it gives to' the second, we find $v = (u_1 + u_2)/2$.

Before we discuss these laws in terms of Descartes' account of motion, this is a good point to emphasise that Descartes intends to replace the Scholastic theory of substantial forms with 'mechanical explanation'. Instead of permissive, potentially uninformative explanations he claims that 'all the properties which we clearly perceive in [the universe] are reducible to the sole fact that it is divisible and its parts movable; and that it is therefore capable of all the dispositions which we perceive can result from the movement of its parts.' (II.23) Thus, all physical explanations terminate in the geometry of matter.[33] The laws of motion and the rules – their more specific (though hypothetical) manifestations – govern how bodies move, and specifically require collisions for non-inertial motions; they are where physical explanations end, *not* in a multitude of Aristotelian potentialities.[34] Of course, the laws themselves are grounded in God.

Rules 1 and 3 illustrate how to apply the laws of Cartesian mechanics in idealised cases, but they also present problems with his system. In the first place, as Leibniz shows in his *Specimen of Dynamics* (1989, 133-4), the situation of Rule 1 can be approached continuously from the case of Rule 3, by taking $u_2 \to$

---

[33] Note that Descartes specifically says that everything is reduced to motion because he holds that the individuation of bodies, and hence their instantaneous arrangement, depends on motion: pieces of matter are pieces of the same body if they move together. *Cf.* footnote 13.

It should also be mentioned that Descartes' official argument against the theory of forms is that the notion of change that it involves, 'the act of a being which is in potency, insofar as it is in potency' (1998, 26) is impenetrably obscure in contrast with the notion of geometrical, or spatial motion.

[34] It should be noted in practice that in practice Laws 1 and 2, and not Law 3 and the rules, do almost all of the actual work in Descartes' explanations. For instance, as we will discuss below, centrifugal effects are crucial, but Descartes understands them largely in terms of the tendency of bodies to continue along tangential paths.

u_1, while the outcomes cannot: in the limit the bodies suddenly rebound instead of moving off together. If one demands a principle of continuity in physical laws, then these outcomes are not possible. Further, these laws and rules are in conflict with relativistic conceptions of motion; exploring this issue can help us understand how laws of mechanics might require privileged quantities of motion.

First, the rules simply make no sense if the motions involved are understood relativistically (we shall see below how Huygens developed a properly relativistic set of laws to replace Descartes'). For instance, consider what happens if we understand speed in terms of rate of change of arbitrary (internal or external) place, so that any ascription of relative motion to a body is equally legitimate. Suppose that two identical snooker balls move towards each other at equal and opposite speeds relative to the table that they roll across: if speed is taken as the rate of change of place relative to the table, then Rule 1 applies and the balls will bounce apart after they collide. But suppose that during the collision someone is walking by the table at half the (initial) speed of one of the balls. If speed is taken as the rate of change of (internal or external) place relative to her, then the other ball has three times the speed and so Rule 3 applies: the balls move off together after the collision – the very same collision. But it is a contradiction for the balls to bounce apart (Rule 1) and not bounce apart (Rule 3), and so the laws themselves are logically inconsistent if speed is interpreted as change of arbitrary place.

The same example casts doubt on the idea that motion in the laws is to be taken in the sense of OM. True, proper use of the ordinary sense of motion is not fully relativistic, but our discussion left open the possibility that it was not uniquely ascribable according to Descartes. If so, then it seems likely that situations like that just described also arise if we take speed in the laws as quantifying OM – the rate of ordinarily attributed relative motion. After all, the contradiction requires only that colliding bodies be legitimately attributed two different pairs of motions, first positive equal speeds, and second positive unequal speeds. Then the two rules would apply simultaneously, which is impossible. Indeed, Rule 1 and Rule 3 are also inconsistent with Rule 6 (II.51) governing the collision of equal bodies, one of which is stationary, if they apply simultaneously, and so any two legitimate attributions of speeds to the same collision will lead to a contradiction. And so unless the conversational rules governing the proper use of OM only ever allow a unique attribution to any body, Descartes' theory of mechanics is inconsistent if speed is interpreted in the 'ordinary' sense. Aside from the special case of the Earth, it is hard to say definitively what Descartes' opinion about the uniqueness of OM was, but it seems to me unlikely either that our ordinary attributions are unique, or that Descartes held them to be so.[35]

---

[35] Of course, the way that I described OM, emphasising the role of ordinary language in its understanding, already made it seem an unlikely candidate for a technical, scientific notion of motion. Change of arbitrary place, however, does seem like a more plausible candidate for interpreting Descartes' mechanics (until one starts to worry about relativity); and so commentators who (mistakenly) equate the 'ordinary' sense of motion with change of arbitrary place will speak *as if* scientific motion could be OM.

But what kind of motion could be attributed uniquely, as a consistent interpretation of the laws and rules requires? The first possibility is that there is some special universal frame of reference to which all motions are to be referred. If motion is to be change of place as Descartes envisioned in the *Principles*, then there would have to be some special body which was *the* reference body for the universe.[36] The Earth seems like the only possible candidate for such a body in the *Principles*, but that is only counted as a local standard of rest – covering the fixed stars in III.29 – while the laws presumably hold everywhere in the infinite universe (matter is the same everywhere [II.22]). Further, the Earth's rest is only ever 'in common use', and so is unlikely to play a precise, technical role in physics. Thus it seems unlikely that Descartes has a fixed reference body in mind.[37]

The final possibility in Cartesian thought for interpreting the rules is PM (which we are assuming amounts to transference in the cases in hand), which according to Descartes can be attributed uniquely. Supposing that he is right, PM thus permits – barring other difficulties – a logically consistent interpretation of the laws.[38] That is, whether a collision two snooker balls constitute an instance of Rule 1 or Rule 3 depends on how the balls move relative to their immediate surroundings. If we count the table as the most significant of the surrounding bodies and they have equal and opposite velocities relative to the table, then they have the same speeds and Rule 1 unambiguously applies to their collision.

If motion is understood as PM then Galileo's specific principle of relativity should hold. For the interpretation implies that the outcomes of mechanical experiments depend on the motion of the experimental set-up relative to the boat – the immediate surroundings – which is the same whether the boat moves or not. On the other hand, there is also a clear *failure* of dynamical relativity,

---

[36] Some readers (Barbour 1989, §8.7) claim that in his earlier *The World* Descartes held a view of space akin to Newton's absolute space (to be discussed in the next chapter) and understood motion relative to it to be a preferred, universal standard without recourse to a preferred material reference body. Alternatively, Garber (1992, 158) claims that Descartes' concept of motion in *The World* is mere change of place, and thus fully relativistic. The first view seems more plausible to me, although Descartes is nowhere near as explicit about these issues in the earlier work, so it may be hard to say whether he had a very clear idea in mind at all.

[37] Note that if there were a universal frame of reference, then Galileo's principle of relativity would be false. Let a pair of billiard balls collide at equal and opposite velocities relative to the cabin: if they bounce apart with unchanged speeds then Rule 1 is in effect, the balls have equal speeds in the universal frame as well, and the ship is at rest in the universal frame; if they move off together then Rule 3 must be in effect, one of the balls is moving faster than the other in the universal frame, and the ship (relative to which they have the same speeds) must itself be moving in that frame. I do not suggest that such reasoning motivated Descartes, but mention it simply to demonstrate the logical inter-relations of the various ideas under discussion.

[38] As we discussed above in footnote **??**, it's not quite clear that PM is uniquely attributable, because some bodies moving from their surroundings may be 'considered at rest'. Thus in full generality it is mere transference from immediate surroundings that we should invoke to interpret the rules, not PM, which is a more informal variant. However, in the circumstances that we shall discuss the difference is not significant and, like Descartes, we shall continue to refer to PM.

in the sense that seeing whether Rule 1 or 3 holds, shows whether or not the two balls are 'really' travelling at the same speed. We have to conclude that Galileo's principle of observational relativity, as formulated in the *Dialogue*, does not imply the dynamical relativity of the laws of mechanics – Descartes' system is relativistic in Galileo's sense but not dynamically. Part of the point is that Galileo's principle is not intended as an abstract constraint on laws – as it will become in Huygens and Leibniz – but has its source in a concern with the undetectability of the Earth's motion. Descartes can explain the absence of the effects of the Earth's motion expected by the scholastics, even though his laws are not relativistic. (Note that 'Galilean relativity' typically refers to the dynamical principle that *no* systems in constant relative motion can be observationally distinguished; my point is that that principle is a generalisation of the principle described by Galileo.)

Of course, if one wants to interpret the laws in terms of PM, then one faces several daunting problems. For instance, how do a body's possibly different motions relative to the possibly many bodies surrounding it determine a unique speed? If there is a breeze across the table, so that the balls have different speeds relative to the air around them, do they now move off together? (Of course, Rule 3 is not true outside of Descartes' idealisation, so if the balls now had different speeds, we should expect some third outcome, different from that prescribed by Rules 1 and 3.) Or again, there's nothing to guarantee in general that two colliding bodies have surroundings that are themselves mutually at rest. In the most extreme case, two bodies at rest 'properly speaking' could collide if their surroundings collided. And finally, it seems that the idealisation of the rules that the bodies collide *in vacuo* now means that motions cannot be properly attributed to them at all. We will not pursue these difficulties however, but instead discuss later (in the next chapter) the set of objections raised by Newton.

Despite these difficulties, since relativistic notions of motion rather obviously make a nonsense of the laws, it seems plausible that Descartes himself intended the only non-relativistic, technical conception that he explicitly defines – PM (or rather, mere transference) – as the operative sense of motion in his mechanics.[39] However, it is worth understanding how other concerns than the requirements of his mechanics apparently played a role in the genesis of the idea of PM (of course, we've already discussed the influence of ordinary language on the definition) – they help explain how Descartes might have been led to such a problematic notion.

3.4 The Earth's Motion

In Book III of *The Principles*, Descartes develops his account of the motions observed in the heavens – of the planets, fixed stars, comets and so on. Descartes'

---

[39] For an extended argument for this view, see Garber 1992, Chapters 6-8 – particularly (247-8), invoking Descartes' letter to Clerselier of February 17 1645 (260-2) in which he explicitly invokes transference to interpret the rules. See Slowik (1999 and 2002, Chapter 6) for objections to Garber.

explanation of the motions of planets is that the matter between heavenly bodies forms a fluid, and that the portion of the fluid around a sun – ours in particular – rotates about it, carrying any planets with it (III.24-6). Thus the Earth and the other planets are swept around the Sun in a vortex of inter-stellar matter, relative to which they are at rest, much as a rubber duck might be swept around the whirlpool formed as the water drains from the tub – though not 'down' any cosmic plug hole. (The planets are also surrounded by smaller vortices which carry the moons around them [III.33].) This apparently heliocentric view of the solar system certainly presented a problem for Descartes, since he was writing in the immediate aftermath of Galileo's condemnation by the church for advocating the Copernican heresy that the Earth moved.

First, we have seen that Descartes repeatedly claims that according to the ordinary sense of the term, the Earth is at rest. And in III.29 he is explicit that he intends this claim to be taken in the formal sense of OM. (So clearly one thing we should not think is that Descartes introduced PM with the purpose of saying that the Earth was at rest in some sense.) In this passage he explains the reason that we attribute rest to the Earth, which is our pervasive experience of its apparent rest.

This view is awkward for Descartes' presentation, for he certainly wants to be able to talk, for instance, about the motion of the Earth in the vortex about the Sun, on a par with the other planets. (We do ordinarily attribute motion to them of course.) But he maintains his position that when he attributes motion to the Earth, he is 'speaking improperly' – in the terminology I have developed, he should really say that it changes its place. Once again we see very clearly that OM, unlike change of place, is not a generally relativistic notion.[40]

Second, what of PM? According to Descartes', the Earth has no PM because it is:

> ...not transported from the vicinity of the parts of the heaven immediately contiguous to the [Earth] [i.e., of the bodies immediately surrounding it] inasmuch as we consider those parts of the heaven to be at rest. For to be thus transported [the Earth] would have to be simultaneously separated from all the contiguous surroundings, which does not happen. (III.28)

It is very tempting to take Descartes as saying that it is because the Earth is swept with the surrounding vortex that it does not have PM. That is, we understood his definition state that a body has PM if *both* it moves from its contiguous surroundings *and* our ordinary use of language permits the attribution of rest to those surroundings. The obvious reading of the quoted passage,

---

[40] Descartes actually considers a state of affairs in which the relativity of OM would be expanded to permit us to say that the Earth has OM. If the fixed stars were at the extremities of the universe, so that there were no bodies relative to which they moved, then ordinary use would justify attributing rest to them – and hence OM to the Earth. (The picture that Descartes has is something like the Aristotelian universe, in which motions are taken relative to the celestial sphere.) However, he rejects such a situation as implausible – he thinks it move likely that there are further bodies relative to which the stars move (and, oddly, that the Earth is at rest).

given Descartes' model of the solar system, is that the Earth has no PM because it has no transference from its surroundings – since it is swept around with them. Indeed, such a reading is almost always given. In a moment we shall see that that is not the explanation given by Descartes at all. First though, consider another issue.

As with Galileo, the influence of the theological issue of the Earth's motion in the development of mechanics and our understanding of motion is unmistakable; but how great was this influence on the view presented in *The Principles*? Some attribute all (or the larger part) of Descartes' discussion of motion to his fear of the Inquisition. Barbour (1989, Chapter 8) suggests that in the *The Principles* both the relativistic sense of motion – mere change of arbitrary place – and PM are intended to avoid difficulties with the Church; and that both are quite distinct from the notion of motion implicit in the rules of collision (Barbour apparently assimilates the ordinary sense of motion of II.24 with change of arbitrary place). The relativistic notion simply renders meaningless the question of whether the Earth moves 'really' (or at least puts the onus on asking what the operative sense of motion is); and, as we just saw, the Earth does not have PM. In particular, Barbour seems to think that Descartes sees no other use for PM than solving the problem of the Earth's movement. That the Galilean condemnation caused Descartes to rethink the nature of motion far more carefully is hard to doubt, but to view his mature view as merely an elaborate attempt to say that the Earth is at rest seems uncharitable. Given the detail with which Descartes picks apart the various notions, it also seems implausible that he considered them to have no real role to play in his system. (See Garber, 1992, 185-8, for criticisms of other versions of the view that Barbour expresses.)

Pooley (2002) also sees the determining influence of the Church on the qualification that for PM a body's surroundings must be 'considered at rest' – and clarifies Descartes' grounds for claiming that the Earth has no PM. He points out that Descartes continues in the passage quoted above:

> However, because the matter of the heaven is a fluid, sometimes some of its particles, and sometimes others, move away from the Planet to which they are contiguous, and this by a movement which must be attributed solely to them and not to the Planet: in the same way as the partial transferences of water and air which occur on the earth's surface are usually attributed, not to the earth, but to those portions of water and air which are transported.(III.28)

(In addition, we should note that according to IV.22, the surrounding particles of the vortex have to move faster – with greater 'agitation' – than the Earth in order to account for gravity.) It seems perfectly reasonable, as Pooley suggests, to see here an explicit echo of the discussion of II.29-30, concerning the qualification. He goes on to propose that Descartes so defines PM simply to be able to assent to the Church view that the Earth is at rest – even though the turbulent motion of the Earth's surroundings mean that it is transferred from them (2002, 10-11, 17).

This suggestion also seems implausible to me. The motion referred to here is the bouncing off the Earth of particles of the vortex in random directions, and so the reciprocal transference of the Earth would not be understood as its annual or diurnal motion. If all that Descartes was saying was that some things bounce off the Earth, it is hard to see what he would have had to fear from the Inquisition – clearly there would be no more heresy involved than in noting winds and currents.

Instead the quoted passage can be easily read as explaining why the Earth has no PM according to Descartes definition. Perhaps surprisingly, this is *not* because it is swept around by the vortex, but because ordinary usage will not support the attribution of rest to its turbulent surroundings. That is, according to Descartes, the Earth has no PM because it violates the second condition of the definition not the first – as we have now seen repeatedly, it is transfered from its surroundings, since they are turbulent, but the definition is crafted so that bodies in such surroundings are not moved in the sense of PM. Again, one may see the influence of the Church, but we should also give credence to Descartes' claim that the definition is crafted in order to conform to the ordinary language of his readers.[41]

Of course, none of this is to say that Descartes' account of PM can be made to do all that it needs to; I share the scepticism of almost all of his commentators about the adequacy of such a conception, even for his system. But I am on the side of those who take PM as a sincere – if misbegotten – account of motion, not a mere sop to the Church, and indeed that it is the sense of motion that he intends in his mechanics. However, even if the issue remains contentious, we can put it aside now, for what is most important for our purposes is that Newton took PM (or rather mere transference) seriously as Descartes' scientific analysis of motion, and that we understand something of the collision mechanics from which Huygens and Leibniz's theories developed.

One final issue concerns an important theoretical issue of the period – how to explain rotation. This kind of motion was extremely important for Descartes because he repeatedly invokes rotation to explain various phenomena: the shape of stars (III.61), the transmission of light (III.64), the motions of comets (III. 126-8), the positions of the planets (III.140), and gravity (IV.20-7), for instance. In particular, whether a body entering the vortex from the outside becomes a

---

[41] Note that Descartes says (III.38) in Tycho Brahe's model of the solar system – in which the planets rotate about the Sun, while the Sun rotates about the Earth – the Earth has PM. He assumes that the motions of the planets are again explained by vortices, now rotating about Earth and Sun, and thus argues that the rotation of the Earth relative to its vortex means that it is transferred from its contiguous surroundings. One might think that this argument shows that after all it is the gross motion of the Earth relative to the vortex that counts in determining PM, and that any motion due to turbulence is irrelevant. But a closer reading shows that this is not so. Descartes argues that if the whole heavens are to rotate about the Earth daily, then 'all parts of the Earth are transferred from all parts' of the vortex (III.38). That is, he argues that there effectively is no turbulent motion of the Earth's surroundings in Brahe's model (I read him to say that because the vortex moves so quickly, any turbulence can be ignored) – consequently, presumably, there is no block to an ordinary attribution of rest. Descartes does not ignore the second clause of the definition, he argues (perhaps unconvincingly) that in Brahe's model it is satisfied.

planet or comet depends on how ‘solid’ it is (its solidity depends in part on how much matter of the ‘third’ [terrestrial] element it is composed of [III.52, 121-2]); for the more solid a body is the greater its centrifugal tendency away from the centre of the vortex. Thus if a body is too solid, its tendency to recede will be greater than that of the matter of the vortex that moves with it, and it will leave the vortex, becoming a comet (Descartes did not know that comets are periodic). If a body is less solid, and so becomes a planet, its position also depends on its tendency to recede and hence its solidity relative to part of the vortex that carries it around. The vortex thus attains equilibrium when the solidity of matter in it increases with distance from the centre, and each planet finds its proper place according to its solidity. Descartes correctly identified tangential inertial motion as the explanation for centrifugal ‘forces’ but failed to give a clear analysis of the situation (III.56-9). The problem is largely that he had no workable mathematical analysis of the situation, and so simply equated inertia – the tendency to move along a tangent – with centripetal forces – the ‘tendency’ to move towards the centre. Be that as it may, Descartes physics depends on bodies’ tendency to recede from the centre of rotation as a real, observable effect; Newton used this point to devastating effect.

4 Huygens

Descartes was not the only Cartesian whose work is relevant to the development of the concept of motion in classical physics. In particular, it is through the work of Christiaan Huygens[42] that Descartes’ framework made its most lasting contributions to physics (the law of inertia aside). Huygens was a natural philosopher whose mathematical and physical talents placed him in the same league as Newton (who was 13 years his junior), and the two were well aware of each others work. Huygens was, however, even more reluctant to publish his results than Newton, and his work is spread through correspondence and notes, which makes his effect on the thought of the period hard to pin down in detail.[43] His more obvious contributions were very well known: especially his discovery (in 1655) of the first moon around Saturn; his invention (in 1656-9) of clocks powered by weights or springs and with cycloidal pendulums (which, unlike the

---

[42] The best approximation for English speakers to the correct pronunciation is ‘how-gens’ (with a hard ‘g’), *not* the ‘hoy-gens’ that you will commonly hear; if you can work up a spit-ball in the back of your throat on the ‘g’, so much the better.

[43] For a comparison of Huygens and Newton, and some discussion of their mutual influence, see Hall, 1976. Yoder (1988) is a longer treatment of some of Huygens’ work and influence. Thinking about Huygens one wonders two things: first, why did he fail to discover universal gravitation, and second, would modern mechanics been developed earlier if he had published more of his results? Regarding the first, Hall mentions Huygens’ attachment to the mathematical rigour of Euclidean geometry preventing him from using the mathematical techniques that Newton used. Regarding the second question, Hall suggests that had Huygens published more of his work, Newton might been even less inclined to publish his work on gravitation for fear of competition, and the development of classical physics would have been retarded. At any rate, it would be simplistic to think that Huygens failed to ‘be Newton’ because of his tentative approach to publication.

simple circular pendulum clock of Galileo, has a period that is independent of its amplitude – crucial to accurate time keeping)[44]; his laws of collision (published by the Royal Society in London in 1669); and his theorems on bodies in circular motion (published without proofs in 1673). It is the last two that will concern us here.

Huygens' account of collision is found in *The Motion of Colliding Bodies* first published in 1703 (1977). What makes Huygens' work so striking are the beautiful relativity arguments he uses to derive the laws.[45] For instance, consider the first proposition, covering a special case of the kind of collision covered by Descartes' incorrect Rule 3 (Huygens' second proposition covers the general case), in which a body moving with speed u collides with an identical body at rest. The derivation is a classic, still found in physics text books (Feynman, 1963, §10.3), since it employs one of the most useful techniques in physics – figure out what will happen in some frame in which an interaction is particularly simple, then transform the solution to the frame of interest.

The first step of the proof assumes Descartes' (correct) Rule 1 concerning equal bodies colliding with equal speeds, and the following principle of dynamical relativity:

> ...when two bodies collide, then even if each of them is simultaneously subject to some other additional motion, they will in no way act on each other with respect to the common motion by which they are each moved. It is as if that common motion were totally absent. (1977, 575)

Consider a collision between two identical balls with equal speeds u relative to boat smoothly moving down a river. Relative to the boat the balls will bounce apart with reversed speeds; for according to Rule 1, that is what they would do if they had those motions relative to a stationary boat, and according to the relativity principle, their common motion – that of the boat – makes no difference to the collision.

Next, Huygens transforms this result to the frame of someone standing on the river bank, as the boat moves by, also with speed u, from left to right.

---

[44] 'Cycloidal' and 'circular' refer to the path followed by the pendulum bob. Huygens' theoretical work on his pendulum clock was published in 1673 as *The Pendulum Clock* (Huygens, 1986).

[45] Wallis (1668), Wren (1668) and Huygens (1669) published laws of motion in response to a general invitation by the Royal Society (Huygens later complained about Wren apparently receiving priority, but his submission was received last – see Oldenburg 1968, letters 1135 and 1139). Both Wren and Huygens treated elastic collisions, and gave essentially the same laws; neither gave detailed proofs of their laws. Wren claimed to have 'entertain'd' the laws 'divers years' previously. According to Blackwell (Huygens 1977, footnote 1) Huygens likely had the proofs and laws by 1656, and *The Motion of Colliding Bodies* was composed by 1673. Bennett (1982, 71-3) describes a meeting between Wren and Huygens in 1661 in which Huygens revealed, without stating them, that he had laws covering Wren's experimental work on collision; Bennett proposes that Wren developed his theory later in that year. While Huygens' derivation is theoretical, based on considerations of relativity, Wren's was essentially experimental (Bennett, 1982, 119). Note that because of their rejection of Descartes' conservation law for mass × speed, these results were not immediately accepted by the members of the Royal Society; see Hall 1966).

Supposing that the balls collide along the line in which the boat moves, relative to the bank observer before the collision the left ball has speed $u+u=2u$ while the right ball has speed $u-u=0$. Afterwards, the left ball has speed $u-u=0$ and the right ball speed $u+u=2u$. Relative to the river bank observer, a moving ball collides with an identical stationary one and they swap velocities, moving apart; Huygens concludes that this result – not Descartes' Rule 3 – is the law for such collisions.

We need to be rather careful to understand just how Huygens is in disagreement with Descartes here. In the first place, Descartes would have to agree with the algebra of the argument. If the balls behave as described relative to the boat, then as a simple matter of kinematics they behave as described relative to the river bank, since the calculation merely assumes the additivity of velocities (which everyone assumed before relativity theory). But as we discussed earlier, this conclusion need not lead to a contradiction in Descartes account, if one interprets the rules in terms of a privileged standard of speed: for instance, PM. Then one could interpret Huygens' example as one in which speeds of the balls relative to the boat are equal to their privileged speeds; because – interpreting the laws in terms of PM, for example – the boat is at rest relative to the immediately contiguous surroundings of the balls. Thus Rule 1 would apply, and describe the collision relative to the boat, and Huygens' calculation could be understood merely to redescribe Rule 1 relative to the bank. On this understanding, since the bank-relative speeds do not equal the privileged speeds of the balls, that the collision in the bank frame is between equals with different speeds does not mean that Rule 3 applies. Hence there is no contradiction.

Thus the consistent application of Descartes' rules requires unique privileged speeds, which in turn determine which rule uniquely applies correctly to each collision; the rules have the form, 'if two bodies of such and such masses collide with so and so *privileged* speeds then the outcome in terms of privileged motions will be this'. In contrast, Huygens derives a system of rules of collision which refer only to speeds in arbitrary frames (that are themselves in constant linear relative motion): rules of the form, 'if two bodies of such and such masses collide with so and so speeds in a frame, then the outcome in the frame will be this'. With such rules there is no uniquely correct rule for each collision, only a unique rule relative to any given frame; and no privileged standard of velocity is needed for consistency, since the different rules in two frames are derived as the appropriate transforms of each other.

The difference between the forms of the laws can be seen keenly by considering Huygens' principle of relativity, which, as we already know, is incompatible with Descartes' mechanics (interpreted, as consistency requires, in terms of a privileged standard of motion). That is, suppose equals collide with equal and opposite privileged motions so that Rule 1 applies and they bounce apart. Now consider a situation that differs only by a common motion (the addition of the same velocity to both); now we have a collision of equals at different speeds, and so the qualitative and observably different Rule 3 applies. Or to put things in Huygens' terms, in Descartes' competition model of collision, the balls *do* 'act on each other with respect to the common motion' because their common motion changes the Cartesian forces of the balls. It is not far-fetched to think that Huygens had exactly the kind of non-relativity of Cartesian mechanics in mind when he formulated the principle.

Huygens' principle entails that there is no way, on the basis of collision experiments, to observe the common (i.e., shared, not ordinary) motion of colliding bodies, because it requires the laws to be independent of such a quantity. And so he proposes a form of dynamical relativity – the unobservability of a privileged standard of motion – which is abstracted from particular cases of (supposed) unobservable motions, such as that of the Earth or a ship. That is, with Huygens, dynamical relativity became a formal constraint on theories and not merely a statement of some particular empirical facts, such as the (supposed) unobservability of the Earth's motion. This point is no criticism of Galileo, for, as we have noted, the kind of laws of motion to which such a principle might be applied did not even properly exist until Descartes' work. What Huygens did was reformulated Galileo's dynamical relativity in the context provided by Descartes; the generalisation of this idea, that laws reflect symmetries, is one of the most profound and important insights in physics, and the bedrock of most fundamental physics to the present.[46]

Now, Huygens' dynamical relativity is logically compatible with the denial of kinematic relativity. Just because one cannot observe a privileged standard of motion, just because the laws do not require one for their application, it does not follow that there is none (e.g., Newton held Huygens' dynamical relativity but denied kinematic relativity). But in *The Motion of Colliding Bodies* there are intimations of kinematic relativity too. For instance, in his argument and the figures accompanying them, Huygens has the balls hanging from lengths of cord, one in the left hand of a man on the boat and the right hand of the man on the bank, another in their other hands; the balls come together because each observer brings his hands together. Why do *both* of them hold the strings, when either one alone could produce the collision in the argument? Surely the answer is that Huygens wanted to avoid privileging either observer, and hence either description. If for instance the man on the boat alone held the strings, then one might conclude that 'really' the balls have equal speeds, even though the speeds in any frame are sufficient to determine the outcome. Because the man on the bank also holds the strings, no reasoning of this kind is possible; each frame has equal claim to be the correct description, and so there only are frame-relative motions. As Huygens puts is, both can be said to be describing a collision that they 'made' happen (indeed, explaining that such is the case, occupies the bulk of the proof of the first proposition).

One final result from the paper is worth noting, since it was an important influence on an idea of Leibniz's, which will be central to a later chapter. Proposition 11 of *The Motion of Colliding Bodies* states the conservation of $mass \times speed^2$, essentially the conservation of kinetic energy. For Huygens this result is simply a consequence of Descartes' Rule 1, while Leibniz (to whom

---

[46] See Gabbey, 1998, §IV, for a discussion of Huygens' relativity in historical context. See Brading and Castellani, 2003, for a contemporary discussion of symmetry in physics.

Huygens was a mentor) elevates it to a law of mechanics (indeed, well beyond, as we shall see).

The second of Huygens' achievements that are relevant to our discussion is his analysis of circular motion. This work was carried out as part of his investigation of gravity from 1659 (as Yoder, 1988, discusses, he uses the technique of numerically equating forces due to gravity with 'centrifugal' forces). However, that work on rotation, *De Vi Centrifua* was not published until 1703 (and then with substantial alterations); all that was published during his lifetime was a list of theorems without proofs, printed as an appendix to *The Pendulum Clock* (1986, 176-8). The most familiar result is (in modern terms) the formula for the force on a body of mass m, rotating with a speed v, around a circle of radius r: $\mathit{force} \propto \mathrm{m}v^2/r$.

If one considers, as Huygens did, systems in which rotating bodies are held in their paths by strings attached to the centres of rotation, it immediately follows from this formula that the centrifugal force – and so the tension in the string – is a measure of the rate of rotation. Huygens, realising this situation, noted (in 1668) that 'circular motion ... has its criterion, that straight motion does not possess at all' (Dugas, 1958, 306): such effects reveal a difference between non-rotating and rotating frames. So Huygens did not (in 1668) consider his principle of relativity to be a principle of *general* relativity; common rotational motions do have observable effects. But such effects show that rotation cannot be understood in a merely frame-relative way, because in a frame in which a system experiencing centrifugal forces is at rest the frame-relative motion is manifestly wrong. As we shall see in the next chapter, when he read Newton's argument that it should be understood in terms of motion relative to absolute space, Huygens changed his view on the 'criterion' of circular motion.

5 Conclusion

We are now in a position to understand Newton's famous views concerning absolute space and motion in historical context; and later Leibniz's too. In the discussion of this chapter we have highlighted the development of ideas of the nature of space and motion, and seen how they related to theories of motion, especially via the law of inertia, and concepts of kinematic and dynamical relativity. We have also seen how such ideas developed in the context of Aristotelian philosophy, the pre-theoretical use of scientific terms and religious controversy. Let us review the main outline of the story.

Galileo developed the idea of dynamical relativity – the indistinguishability of different states of motion – in the quite specific context of the Earth's motion, and somewhat more generally terrestrial phenomena. His thinking did not fully shake off Aristotelian and scholastic thought, and so his explanation of relativity was grounded in a concept of circular inertia.

Aristotle and the question of the Earth's motion also loomed large for Descartes, but in quite different ways. First, he aimed to construct a system of thought that would ground the mechanical philosophy which was then achieving ascendancy over scholastic thought. He denied powers to matter to such an extent that he took the only essential property of matter to be extension, and thus identified space and matter. His geometrization of nature further led him to view motion in purely relational terms, at least as far as as motion is an objective property: motion as motion relative to arbitrary reference bodies or immediate surroundings. The former kind of motion is relativistic in the kinematic sense, the latter is privileged; both are relational concepts. However, Descartes was also (overly) concerned with the pre-scientific concept of motion and so was led to give explicit definitions of motion that are subjective; ordinary motion depends on our ascriptions of 'action' and even his technical – 'proper' – conception was modified to avoid clashing with the vulgar attribution of rest to the Earth.

Then there is the question of how these different conceptions of motion relate to Descartes laws of mechanics: the modern (more-or-less) law of inertia and the 'contest' model of collision. The collision laws themselves are empirically flawed, but the model of a system of mechanical laws was extremely influential. The rules 'derived' from the laws are not dynamically relativistic, which suggests that Descartes intended to interpret them with a conception of motion that was not kinematically relativistic, for it obviously makes no sense that the laws should depend on a privileged sense of motion if there is none. Arguably then, Descartes intended to interpret his laws in terms of proper motions (or rather, mere transference from immediate surroundings). This reading is also attractive because it avoids the uncharitable claim that Descartes introduced proper motion merely to say, without sincerity, that the Earth was at rest – because it is at rest in its immediate surroundings (as a whole). On our reading he can say it *and* really mean it.

Descartes' motivations and conclusions concerning motion are indeed convoluted, but it is well-worth looking at them in some depth to see how he tried to judge just what would be important for understanding the nature of motion in the new science that was developing.

Finally we discussed Huygens' theory of collision and rotation. He said very little about the nature of motion, but succeeded in applying the notion of dynamical relativity to the laws of mechanics (between frames in constant relative linear motion), thereby wedding some of the most important insights of Galileo and Descartes. This work also has intimations of kinematic relativity. His work on rotation led him to deny full dynamical relativity, for he, like Descartes, saw that centrifugal tendencies provide an empirical criterion for a privileged sense of rotation. As we shall see, he only connected this denial of full dynamical relativity to a denial of full kinematic relativity later, after reading Newton's arguments in the *Principia*. When he realised that making rotations observable pointed in the direction of absolute space, he tried unsuccessfully to revise his views on the 'criterion' for motion.