Ch 2: Newton on Space and Motion

May 16, 2008

1 Introduction

Newton is at the centre of our history for he is the first to propose a plausible theory of mechanics and a *compatible* account of motion. I will take his mechanics as given. The three laws in their modern form – especially, $\vec{F} = \mathrm{m}\vec{a}$ – are different in certain respects from the laws of the *Mathematical Principles of Natural Philosophy* (or *Principia*) (1999 – originally published in 1687), but the differences are not especially significant for our purposes.

The story in the large is quite straight-forward: space is ‘absolute’, which means *inter alia* that it is a static structure, ontologically distinct from matter; and fundamentally, or truly, motion is change of absolute place (i.e., region of absolute space). There are, however, important questions of exactly what else the ‘absoluteness’ of space amounts to, and of how Newton’s views and arguments concerning absolute space connect to his theory of mechanics, and the method by which it was derived from experiment and observation. The literature is huge, for Newton’s views are deep and complex, and we shall only touch on a few of the most relevant issues.

One thing that is clear is that motion is unique, in the sense that every body has a well-defined absolute velocity, namely its velocity with respect to absolute space; Newton’s account of motion allows for no kinematic relativity. On the other hand, as a pair of corollaries to his laws show, Newtonian mechanics does entail a principle of dynamical relativity; (closed) systems at rest are indistinguishable by mechanical experiments from identical systems in constant linear motion, and from systems in ‘free fall’ acceleration. So in particular, absolute velocity is unmeasureable, despite being a real, well-defined quantity. On the other hand, Newton also demonstrates that it is possible to measure the rate of absolute rotation – just as well, for that is a crucial task in the project to understand the motions of the planets.

2 Absolute Space

The purpose of the *Scholium* following the *Definitions* in Newton’s *Principia* (1999, 408-15) – sometimes called the ‘Scholium on Absolute Space and Time’ or the ‘Scholium to Definition 8’ – is to explain his technical, theoretical conceptions of time (which we shall put to one side), space and motion, in contrast to their more common understanding in terms of perceptable properties, such as relative position. It contains one of the two most quoted statements concerning the nature of space in the absolute-relative literature.

> Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies . . . [for instance] determined from the situation of the space with respect to the earth. Absolute and relative space are the same in species and in magnitude, but do not always remain the same numerically. For example, if the earth moves [absolutely], the space of our air, which in a relative sense and with respect to the earth remains always the same, will now be one part of the absolute space . . . now another . . . . (1999, 408-9)

At first glance, Newton’s account of absolute space here does not seem very informative: just that it is in some sense independent of external things, and that it is the same everywhere and at every time. However, the concept of ‘relative spaces’ is fairly clear, and we are told their relationship to absolute space; let’s start there.

We can most usefully think of a relative space as a system of ‘adapted co-ordinates’ (or ‘frame of reference’): Euclidean co-ordinates, often of finite extent, with a (possible) reference body at rest at the origin.[1] Note, however, that Newton does not work in co-ordinate terms in the *Principia*, preferring instead to treat space in terms of axiomatic geometry in a Euclidean space fixed relative to a planet or fixed stars, for instance. Thus trajectories are not represented by formulae, but by curves. However, to say that such spaces are ‘measures’ of absolute space is to conceive of absolute space as a space *distinct from* all the relative spaces, and to say that we use the relative spaces to assign quantitative or geometrical positions and motions. Since the relative spaces have Euclidean geometry, we naturally assume that absolute space does too (though see footnote 7). Further, since bodies move in relative spaces, they move with respect to absolute space as well; absolute motion is the subject of the next section.

Next, Newton says that at any time a relative space is some region of absolute space – the one that it measures presumably – but that if the reference body moves then the relative space becomes a different region of absolute space. In other words, absolute space is also distinct from any relative space in the sense

[1] I will give an explicit construction of such coordinates in Chapter Five based on Friedman 1983, 224-5. The reference body need only be possible because we will consider arbitrary rigid transformations of co-ordinates adapted to actual bodies (thus avoiding pointless textual qualifications). I do not see that I am thus committed to any dubious assumptions about the reality of possible bodies. It is assumed that all the frames agree, up to a choice of units and origin, on time co-ordinates.

that the same relative space can measure different regions of absolute space. The situation is something like the reverse of Descartes' distinction between body and place; for him places identified relative to reference bodies have changing compositions as space=matter moves; here relative spaces, identified with respect to reference bodies are 'composed' of different absolute regions as they move around absolute space.

And so from the *Principia* we have the idea of absolute space as a Euclidean space distinct from any relative space, and with respect to which bodies and relative spaces can move. It should be clear that such a notion is completely opposed to Cartesian conceptions of space, according to which there are only relative spaces – positions reckoned relative to reference bodies, including contiguous ones (and, in the case of OM and PM, relative to linguistic convention). And now we can understand the description of absolute space given by Newton. Absolute space is 'without reference to anything external' precisely because it is not constituted by its relations to reference bodies, as in Descartes' conception (or as a relative space), but has has an independent existence ('of its own nature'). And it is 'homogeneous and immovable' again in contrast to Descartes' hydrodynamical universe, in which space=matter is everywhere differentiated in bodies of different sizes and shapes in motion (a picture also quite different from the atomistic universe envisioned by Newton[2]). Finally, that absolute space is immovable of course also implies that it persists through time (as does the possibility that relative spaces might be numerically identical with the same region of absolute space at different times).

Of course, that characterisation leaves quite a lot unsaid about what kind of thing absolute space is – about what 'its own nature' means – and in most of the rest of this section we shall consider the question in more detail, by discussing another important source. Before we do, consider one more thing that we learn about absolute space from the *Scholium*. Newton says (1999, 409) that a body's place (absolute or relative) is the region of space (absolute or relative) that it occupies. There are two important points. First, this view also is in explicit opposition to two of Descartes' claims: (i) that 'place' principally refers to position relative to a reference point, not the region occupied (*Principles* II.14); (ii) that the concept of an 'external place' is a legitimate notion (II.13).[3] Second, according to this definition, bodies 'occupy' regions of absolute space, and so the *Scholium* says that absolute space is a given, rigid, Euclidean structure, in which extended bodies are located.

To see further what Newton thought we turn to a private essay that was not discovered and published until the 1960s – *De Gravitatione et Aequipondio Fluidorum* (Newton, 1962, II.1). While the title suggests a treatise on hydro-

[2]See Perl (1969, 510-11) for scepticism regarding Newton's atomism and acceptance of the vacuum (an essay written without reference to *De Grav*, discussed below).

[3]Newton argues that bodies with equal volumes should have places of equal sizes. But if two such bodies have different shapes then they may have different areas. If their places are their surfaces, then such 'equal' bodies have unequal – in the obvious measure – places. (The same argument was made against Aristotle by the fifteenth century philosophers Hasdai Crescas and Joseph Albo: see Jammer, 1993, 74-8.)

statics[4], almost all of the paper is devoted to an explicit attack on Descartes' definitions of space and motion, and the development of an alternative account (although the expression 'absolute space' is not used in it). There are some controversial issues of dating, but we will take it that the paper reflects to a good approximation Newton's mature views at the time of the *Principia*. However, bear in mind that since the essay was unpublished, it can have had at best an indirect effect on his interlocutors. In particular, we have no reason to think that any of Leibniz's understanding of Newton's views was based on a knowledge of the ideas '*De Grav*'.[5]

Descartes identified space as a substance, namely the extended substance, space=matter. Newton however rejects the idea that space is a substance at all, in the sense of traditional metaphysics.[6] In the first place, in the traditional sense, something can only be a substance if it does not depend for its existence on any other thing, which space does according to Newton. He considers space to be '...a disposition of being *qua* being. No being exists or can exist which is not related to space in some way.... it follows that space is an effect arising from the first existence of being....' (1962, 136 – all remaining references in this section are to *De Grav* unless otherwise indicated). That is, Newton runs the doctrine that all things exist *somewhere* in the opposite direction from usual, to infer that space is a metaphysical consequence of the existence of anything. According to this line of thought, space is dependent for its existence on the existence of other things; in particular, Newton takes God to be the first being, and so space exists as a result of God's existence. (Note that Newton's argument does not show that absolute space is dependent on any particular thing, because it would exist in a world without God as the effect of the existence of anything at all. If it is a metaphysical necessity that God exists as the first being, then it is a metaphysical necessity that absolute space exists as an effect of God's existence, which would make it trivially true that space would not exist without God – making dependence in that sense trivial. Whether Newton's argument is valid depends crucially on how one understands what it is for one thing to

[4]The title was given by the translators, and simply comprises the first five words of the essay.

[5]Hall and Hall, in their commentary in the English translation, (Newton, 1962, 187-9) date the work as 1673-5, while Dobbs (1975, 138-46) places the essay rather later, in 1684-5, so contemporaneously with the composition of the *Principia* (see Hall, 2002, for a further discussion). The issue is at the centre of controversial issues in Newtonian scholarship. For a recent commentary on *De Grav* useful for our purposes see Stein (2002), especially on the place of Newton's metaphysics in relation to his physics. For the influences on Newton's views here by Gassendi and his teachers Isaac Barrow and Henry More see McGuire (1978), Strong (1970) and Power (1970), respectively. (While they make the case that Newton was exposed to metaphysical accounts of space akin to absolute space, they understate the extent to which Newton's conception is tied to his mechanics; as we shall see, it would be a mistake to read him as defending a pre-established philosophical account using metaphysical arguments.) Rynasiewicz (2004) also describes the influence of William Charleton. Apparently at least some of the ideas of the work were communicated by Newton, as a conversation with Locke on the subject of matter, based on the account laid out in *De Grav*, is documented; see Tamny (1979, 49).

[6]Later we shall see that in a modern sense of substance, Newton does take space to be a substance. ***Johnson

depend on another for its existence.)

Newton's second argument against the substantiality of space points to one of the mysteries about absolute space, namely the nature of the interactions it has with matter. Space, Newton claims, is inert, unable to act on matter, which is contrary to its being a substance. Now, we've already seen that Newton also claims that absolute space is immutable, so matter cannot act on space either. According to this view, absolute space seems to be tenuous stuff, unable to interact in anyway with the familiar material world, perhaps like the fictional world of a book or film – why should we think it any more real? We'll see shortly why Newton accepted absolute space as an objective reality all the same. For now it is interesting to note that he also invoked the inertness of space to explain the Cartesian law of inertia: 'there is no force of any kind [arising from absolute space] which might impede or assist or in any way change the motions of bodies. And hence projectiles describe straight lines with a uniform motion unless they meet with an impediment from some other source.' (137)

Newton also rejects the possibility that space is the property of things, since it can exist where nothing is: he believes in the existence of the vacuum (ironically, Leibniz used the same argument against Clarke's claim that space is a property in the *Correspondence*, 1956, 37). Finally, neither does he think that space is nothing at all: the vacuum is not literally nothing, but empty space; we have a 'clear idea' of uniform, infinite space, abstracted from the properties of bodies – and, in accord with Descartes' views, a clear idea must correspond to something. He is left with the view that space 'approaches more nearly to the nature of a substance' (132): in other words, it is something, but somehow more tenuous in its existence than other things. Newton's discussion on this point is important, because it shows that strictly metaphysical – and certainly non-empirical – considerations played an ineliminable role in his views on absolute space, even though, as we shall, see they are also driven by his scientific investigations of the foundations of mechanics.[7]

So much for the metaphysical status of absolute space, what more specifically is it? For our purposes we will not go too wrong if we think of it as a physical manifestation of a three-dimensional Euclidean space – in that regard like Descartes.[8] Thus, 'there are everywhere all kinds of figures, everywhere spheres, cubes, triangles, straight lines, and those of all shapes and magnitudes' (133). Any point of space is the centre of cubes, spheres, triangles, line segments and so on of any size. (Newton's space is in this regard too quite unlike Descartes', in which any point is contained by exactly one body, of determinate

[7]See McGuire (1978) and Ducheyne (2001) for far more detailed investigations of Newton's intricate theory of existence and predication and its relation both to theology and to the view of space proposed here. McGuire also discusses Gassendi, whose *De Motu* (1972, 384-5) also places space (and time) in a separate categories outside the substance/accident division, and like Newton accepted the existence of the vacuum. However, he did not have Newton's extra idea of space being a necessary consequence of being.

[8]See McGuire (1982, 149-56) for reasons to think that Newton did not take absolute space to be exactly a Euclidean space; in particular, he denies that geometric points can be physical in the *Questiones Quaedam Philosophae* in the early 1960s. It is of course true that in the *Principia* space is always treated as strictly Euclidean.

extension.) A material cube, sphere, triangle and so on could come to occupy the region around any point, and so all the correctly shaped regions need to be there in advance; the potential place of any body must be there.

But absolute space is not identical with matter: for a body to be in an absolute place is for two distinct things to be coincident – the body and the set of spatial points it occupies. Contrast this view with Descartes', according to which occupying a place involves no coincidence of body and place, just relations to some reference bodies; the difference between body and place is not ontological, but conceptual only. It is of course the ontological distinction that makes the vacuum a possibility for Newton; as we saw, the identification of space and matter makes it a conceptual impossibility for Descartes.

Moreover, while Cartesian space is a fluid that is always in motion, Newton offers a series of arguments that absolute space is rigid. First, for example, space is eternal and unchanging because it is a consequence of God's existence and He is eternal and unchanging (137). Then again, he claims that it is conceptually impossible for its parts to move (136). For Newton, motion is change of place, so to suppose the motion of one part of absolute space is to suppose its motion from one absolute place to another. Since absolute places are the relevant parts of absolute space, the hypothetically mobile part *is* its own place, and the supposed motion would require the part to move out of itself, which Newton reasonably takes to be absurd. It seems, however, that this argument can only establish the impossibility of the *absolute* motion of the parts of absolute space, since this contradiction would not follow if the parts merely changed positions relative to one another (or relative to some reference body). The idea of an incompressible, mobile fluid is not self-contradictory, and to that extent at this point Descartes' idea that space is such a fluid has not been refuted.

However, a third argument runs:

> ...the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted into the other. [The parts of] space are only understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered. (136)

I have quoted this argument in full because it has suggested an interesting interpretation of Newton's absolute space that I want to reject. First the argument. The idea is that regions of space have their identities in virtue of the spatial relations that they bear to other regions of space. Hence if two parts of space were to exchange positions – i.e., their spatial relations to other regions of space – they would simply become one another, and so there would in fact be no difference. Thus no motion of the parts of space is possible. To use an analogy, suppose Kai and Ivor each have $1 in their bank accounts. Then, since those dollars are not any particular pieces of paper, they have their identities simply in virtue of their relations to the bank accounts: for instance, Kai's dollar is different from Ivor's in virtue of being in Kai's account. And so, if Kai and Ivor were each to wire the other $1, we would not end up in a new situation in which Kai's dollar was now in Ivor's account, and *vice versa*, but exactly the same situation of a dollar in each account, with nothing in fact moved.

Since the criterion of identity for the parts of space=matter is a deep problem for Descartes (see Chapter One, footnote), Newton's argument here is powerful. Descartes simply has no competing notion of identity, and if there is *nothing* to the identity of a part of space other than its relations to the rest of space, then the argument clearly goes through.

Newton's account of identity prompted DiSalle to further read him as denying that 'the material universe would be intrinsically different if it existed at different spatial and temporal points' (1994, 267). One way of deriving this conclusion is as follows: imagine that the entire material universe is relocated in absolute space with all the spatial relations between bodies preserved (i.e., by rigid translations, rotations, or reflections); then by the homogeneity and isotropy of Euclidean space, the new situation differs in no purely qualitative way from the original (particularly, every body will be relocated in a region qualitatively identical with its original place). Hence, the only way in which such a relocation could produce a distinct situation would be if it produces some non-qualitative difference, specifically if bodies occupy distinct regions before and after relocation. But since each body finds itself in a new place that is qualitatively identical with its original place, in particular each new place stands in relations that are qualitatively identical to those in which the original place stood (i.e, the relations in which the original place stood to other regions of space are isomorphic to the relations that the new place bears to other regions of space); so if, as Newton proposes, the identities of places are constituted by their relations, there is no basis of attributing distinct identities to the 'original' and 'new' places, and thus the relocation of the material universe leads to no non-qualitative differences either. Therefore, since a relocation produces no qualitative or non-qualitative differences, it produces no difference at all: there would be no intrinsic difference if the material universe existed at different spatial (and *mutatis mutandis* temporal) points.[9]

This point is extremely important, in the first place for some of the arguments that Leibniz raises, and in the second for the contemporary debate over the nature of spacetime. In particular, an influential modern rendering of the

[9] A simpler system (more like time than space, an analogy that Newton mentions) with the same feature is one in which an infinite number of identical things are totally ordered by the relation $<(x, y)$, but without any other principle of individuation but that arising from the order. Then each thing is distinct from every other that it is either 'less than' or 'greater than', but each thing is also qualitatively identical to every other: every thing is greater than an infinite number of other things and less than an infinite number of things (that are themselves ordered). Since the things are not identifiable other than by their ordering, one cannot distinguish them by the relations to some particular thing for instance, for there are no principles of individuation to pick out such a reference point. Then suppose one of the things is additionally green; whichever one it is the situation is the same, an infinite ordered set of things, one of which is green. The ordering alone does not allow sufficient individuation to distinguish different colourings – there is no way to reidentify the things so that in one case one if green, and in another it is not.

idea that space is a substance is exactly that the parts of space can be reidentified independently of their material contents – that they are individuals in a suitably robust sense (see Chapter Four). On the current proposal, in terminology we shall develop later, Newton is a 'sophisticated substantivalist' (although DiSalle's treatment in full probably makes Newton no kind of substantivalist at all).

But while the passage quoted above may strictly entail that relocations of the material universe produce no intrinsic differences, we can see that that is not a conclusion which Newton would necessarily have accepted, especially if we consider the work that he wanted absolute space to do. The first point to emphasise is that Newton has *nothing like the case of relocation* in mind when he proposes his account of the identity criteria for space. He is of course arguing against the Cartesian doctrine of the mobility of regions of space, not for the impossibility of relocating systems of matter. So even if the latter follows from his theory, one should not conclude that he explicitly endorsed the conclusion – it could equally well be an unforeseen and unwelcome consequence. Indeed, his view of motion is quite incompatible with the conclusion.

For Newton, (absolute) motion is change of (absolute) place, and (absolute) place is a region of (absolute) space, but this view only makes sense if regions can be reidentified as systems of bodies change. That is, suppose that some otherwise unchanging body A, in a system of bodies, moves absolutely, then at one time it is in absolute region X and at a later time it is not; clearly such statements only make sense if X can be reidentified at different times. Moreover, that regions must be reidentifiable if there is to be a coherent notion of motion is precisely one of the reasons Newton gives for rejecting Descartes' account (129-31). For if a body's position is fixed by surrounding space=matter (a view Descartes does not actually hold) then once that space=matter has spread itself out around the universe, the place no longer exists, and there is no fact of the matter about whence the body came from, and hence how, or even whether, it has moved. It is the immobility of the parts of absolute space that solve this problem, by grounding permanent facts of the matter about which parts of space a body has occupied – providing of course that they can be reidentified.

But Newton's proposal for place identity defeats such reidentification. For the very same reasons that there is no basis to attribute a different place to the material universe as a whole after a relocation, there is no basis to attribute a new place to A after a supposed motion, if places have their identities merely in virtue of their relations to other places. The new place is qualitatively identical – in particular in its relations to other regions – to A's original place, X; thus according to Newton's proposal there is no basis to attribute distinct identities to them, so to say that A has moved at all. Of course, A may have moved relative to other bodies, but clearly Newton did not think – as Descartes did – that X's identity depends on its relations to bodies, so A's changing its positions relative to other bodies does not ground its change of absolute place. And of course, there are at every moment of the supposed motion (including the final moment) many places qualitatively distinguished by being suitable distances apart, and so non-identical on Newton's account of place identity; the problem is that without a way to reidentify these places at different times, absolute motion is not well-defined.[10] That is, it is simply inconsistent to hold that the individuation of places in Euclidean space is determined by their relations to other places *and* that absolute motion is well-defined as change of place. The most obvious and reasonable alternative is that absolute places have identities across time that are simply not determined purely qualitatively, so that there is a non-qualitative fact of the matter about whether A remains at X or not.

DiSalle apparently sees the problem, and proposes that absolute motion for Newton does not depend on '*which* parts of space are occupied or passed through' (1994, 270), but instead on '*how much* absolute space a body passes through'. However, DiSalle gives no account of what kind of space could permit distance travelled to be well-defined without also supporting a notion of 'same place'; and how could it, for a body is at the same place iff it has passed through no space. But even if a suitable geometry could be found, this reading is completely at odds with Newton's view that motion is *change* of place. A body simply cannot, in the literal sense, change its place unless it no longer occupies the one it did before, which clearly requires reidentification of the original place at the later time. At any rate, DiSalle later changed his interpretation of absolute motion and accepted that absolute motion depends on whether a body remains in '*the same* absolute place over time' (2002, 40 – his emphasis), in some non-qualitative sense of 'sameness'. (As we shall discuss momentarily, he still maintains that for Newton relocations of the material universe do not constitute distinct possibilities.)

Surely the most reasonable understanding of the conflict between Newton's proposal for the individuation of the parts of space and his theory of absolute motion is that he failed to see the conflict. Moreover, because the account of motion is far more important to his system as a whole than a particular argument against the mobility of space (one of several), we can say that as far as his understanding of the metaphysics of space goes, the reidentifiability of places is a far more important doctrine for Newton than the relational account of the individuality of places: it is the one that is most interesting and relevant to consider. (We might even conjecture that if the contradiction had been pointed out to him, then he would have dropped the account of place identity. But let us try to avoid too much counterfactual history – especially when that counterfactual history supposes that Newton was amenable to criticism.)

Absolute motion requires the reidentifiability of places at different times, but this reidentifiability is not the same as the reidentifiability of places in different hypothetical situations. That is, it does not immediately follow that A's being

[10]Looking ahead to the examples Newton gives in the *Scholium to the Definitions*, if identity of place really is determined relations to other places, then (a) if two bodies are in relative acceleration, there is no fact of the matter about which is absolute acceleration and (b) if two spheres attached by a cord rotate in deep space, there is no fact of the matter about whether they rotate in absolute space about their centre of mass. In both cases, there are no qualitative differences – relations between bodies and between regions – on which such differences might supervene. Since Newton explicitly thinks that there are genuine differences between such states of motion, grounded in motions with respect to absolute space, it is certain that he could not have consistently maintained his account of the identity of places.

in X and A's not being in X, A's relations to all other bodies being the same, are two distinct possibilities, simply because A could be in X at one time and not in X at another. DiSalle makes use of this logical point to continue to argue that for Newton relocations of the entire material universe would not lead to a distinct situation.

However, there is no textual reason to think that Newton rejected the reidentifiability of places in different hypothetical situations – aside that is, from the account of the individuality of places which we have seen to be incompatible with Newton's core doctrines, and especially the account of absolute motion that we and DiSalle attribute to him. Thus DiSalle's claim is entirely speculative. Neither, it is true, does Newton explicitly say that places *can* be reidentified in different hypothetical situations, and so we cannot conclusively attribute the view to him – however, it is a pretty natural extension of his views concerning motion (though perhaps not one of great import to Newton).[11]

Let me make a few more observations concerning the logic of the situation. First, there is every reason to think that Newton would have accepted that the whole material universe could (various theological considerations aside) be in constant liner (absolute) motion. If so, given his account of motion, he would have been committed to the possibility that the entire universe (perhaps one rather unlike ours) could be at one place at one time, and at a different place at a later time, with the same relations between bodies. In the first place, it would seem perverse to accept the states at the two different times were distinct, and yet deny that they corresponded to two distinct hypothetical situations. In the second, Leibniz's arguments against absolute space (which we will discuss in the next chapter) arguably have as much hold over the diachronic case and the hypothetical one; so there is no escape for Newton here.

Second, any attempt to introduce a principle of diachronic identity for places, while leaving Newton's argument against the mobility of places intact, is unlikely to succeed. If X can be reidentified at different times, then there is no immediate reason that it should not bear different relations to other particular regions of space at different times (of course, we saw Newton's other arguments against the

[11]There is considerably more in DiSalle's essays than the point discussed here, much of which I am very sympathetic to indeed. In particular, he is quite right to suggest (1994, 269) that if motion is understood, not in Newtonian terms as change of absolute place, but in terms of the geometry of full 'Newtonian spacetime' (see Chapter Four), then sophisticated substantivalism (and Newton's account of the individuality of regions of space and time) is perfectly consistent. And that is because in that view absolute motion is deviation of a body's worldline with respect to a 'rigging' of spacetime. But it is entirely anachronistic to attribute anything like that to Newton, for in his account motion is with respect to parts of an enduring space, not with respect to geometrical structures of spacetime, which has no enduring parts at all. (Though of course there are those who attribute a spacetime view to Newton; see Conn 1999, and Hill 2003 for a reply.)

Note also that Nerlich (2005) reasonably argues that if the points of Euclidean space are identified only by their mutual relations, then, since they all stand in isomorphic to other points, they are not distinguished at. That Newton's views are confused in this way makes it harder to understand charitably his intentions – harder to see him unequivocally advocating the position DiSalle attributes, and easier to see the passage as an aberration in account of absolute space.

mobility of places). For this reason it would be implausible to claim on the basis of Newton's argument that he intended to deny a principle of individuation only for different hypothetical situations while maintaining it for places at different times – the latter view obviously undermines the argument. In addition, a 'principle of individuation' is, and was (see, e.g., Locke's *Enquiry* II.xxvii.3, e.g., 1964, 208-9), understood to be something that would ground reindentification at different times. To suggest that Newton had only reidentification across hypothetical situations in mind when he denied 'individuality' to places would be to attribute a deviant understanding.

None of these points proves conclusively that Newton was committed to the view that possible worlds could differ only in the absolute location of the material universe, but they do show that it is highly implausible that in the core of his views he was committed to its denial. However, before we move on, let me emphasise again that the metaphysics of absolute space is largely drawn from the unpublished *De Grav*, not the *Principia*. Leibniz's understanding of Newton's concept did not draw on the former, even though ours does.

We have now said a lot about what absolute space is, how it differs from Descartes' conception, and indeed why Newton (correctly) thought it superior to Cartesian space. But we haven't really seen why Newton introduced it in the *Principia*. In the next section we turn to that issue.

3 Absolute Motion

Corresponding to his two senses of space, Newton defines two senses of motion, absolute and relative. Absolute motion (AM) is change of absolute place, which we will take as motion relative (ironically) to absolute space, in a given direction in space at a definite absolute speed.[12] Relative motion (RM) is change of relative place, which aside from their disagreement about the nature of place, is the same notion that Descartes also called 'change of place'. More precisely, we will understand RM to be motion with respect to a relative space or reference frame: i.e., relative to an *arbitrary* (possible) reference object, in a given relative direction at a given relative speed (see footnote 1).

Some care with terminology is needed here. RM is only one kind of motion that can be defined purely in terms of the relations between bodies. In the previous chapter we discussed several relational conceptions of motion; the two offered by Aristotle, plus Descartes' PM, OM and change of place, which latter amounts to RM. (In fact, we saw that neither OM and PM have strictly relational defintions, insofar as they depend on conventions of everyday language; we shall ignore this point in the case of PM, and continue, like Descartes and

[12]We will put to one side Newton's discussion of absolute and relative time in the *Scholium*, and take time as given, so that rates of change can be defined. It seems to me that the issues involved are rather different: absolute time differs from relative time because the latter 'clock time' inherently approximates the exact time between any two events implied by the laws. Absolute space differs from relative space (also) because even perfect measurements do not reveal all its properties (specifically its relations to bodies).

Newton, to equate it with mere transference.) So there is an unfortunate ambiguity in discussions of motion, including Newton's: 'relative motion' as any conception of motion defined in terms of the relations between bodies *vs.* 'relative motion' as RM, the specific conception of motion relative to an arbitrary reference body. Here we shall use 'RM' always in the latter sense, as we defined it, and will use 'relational' to indicate the more general notion. (Another potential ambiguity, which we have noted before, is also at play; although both conceptions are relational, RM is a kinematically 'relativistic' notion, but PM is not, since a body's PM is privileged.)

The main work of the *Scholium* is to demonstrate that relational conceptions of motion, specifically RM and PM, are inadequate for defining motion. PM is, we saw defined to be the kind of motion involved in Descartes' mechanics, so the *Scholium* is a direct attack on the foundations of his theory. Then, since AM is adequate in the ways that Newton demands, and since he considers no other possibilities, he concludes that AM is a correct analysis of motion and, by implication, that absolute space is an objective reality.[13] In so doing, Newton does not deny the intelligibility or utility of relative space and RM: of course we observe and describe positions and motions in relative terms perfectly satisfactorily all the time – in Newton's phrase, they are the 'sensible measures'

[13]This section follows the interpretation – as I understand it – proposed by Rynasiewicz (1995a, b), central to which is the contention that Newton and his interlocutors all shared the conviction that there was a privileged, fundamental – 'true' – conception of motion: e.g., PM or AM. This interpretation is in opposition to an earlier tradition, according to which Newton argued in the *Scholium* for true motion on the basis of the empirical criterion of the tendency of a rotating body to recede from its axis of rotation; clearly no such argument is required if all hands already accept the existence of true motion. We will not argue in detail against this interpretation here, since I view Rynasiewicz's arguments against it as decisive (see also Rynasiewicz, 2004). However, at the end of the section I will make some remarks to clarify how the interpretation offered here differs from others found in the literature; since they are closely bound up with Twentieth Century arguments concerning absolute space and motion, they will also be discussed in Chapter Four.

However, it is important to acknowledge that although Rynasiewicz differs in his reading of the bucket argument from Stein (1967), it was Stein's ground-breaking work on Newton that paved the way for all that has followed in terms of our increased understanding of Newton and (perhaps to Stein's dismay) the philosophical debate. Instead of taking true motion as a given, distinguished from PM and RM by its properties, causes and effects, Stein understood Newton to take absolute space and motion as theoretical postulates of his system, like mass, force and the rest, and the bucket argument to establish their empirical content. Stein has shown that in many ways Newton was a master of scientific argument (e.g., elsewhere in 1967, 1970, 1977, 2002), but the logic of the *Scholium* is not quite as subtle as Stein's view contends.

The historical part of this book is importantly influenced by Rynasiewicz's work, for the question of a privileged sense of motion is one of the main themes that we been following – precisely to understand Newton's assumptions. (1995a) contains Rynasiewicz's reading of the *Principia*, and this section is in substantial agreement with it, aside from some small (and one slightly larger – footnote 21) disagreement over the details of Newton's arguments. (1995b) sets Newton's work in historical context, like the first part of this work. Whilst I'm largely in agreement with this paper, we have been able to take a detailed look at a number of issues not discussed by Rynasiewicz.

I am grateful to Rynasiewicz's for his work, and for chastising me for not paying it the attention it deserves. There of course remains the possibility that I will misrepresent his views here: insofar as I do, the discussion should be taken as giving my reading of Newton, inspired by his.

of spatial quantities. Moreover, at the end of the *Scholium* Newton says that the essential point of the *Principia* is to show how to determine AM from data that is entirely in the form of relative motions (of the planets for instance).

I say that Newton aims to 'demonstrate' the inadequacy of RM and PM, though at points it seems more accurate to say that he simply wishes to enumerate ways in which AM is distinct from them. Indeed, he starts the discussion by saying that 'absolute and relative rest and motion are *distinguished* from each other by their properties, causes and motions' (1999, 411, my emphasis – all references in this section are to the *Scholium*). However, it is clear that Newton is mainly offering arguments against what he takes Descartes' views to be, and so we will read him in this spirit.

Since it is certain that Descartes is the target of the *Scholium*, it is tempting to think that Newton (incorrectly) identifies RM with OM, so that in arguing against RM and PM he takes himself to argue against Descartes' two explicit definitions of motion. This suspicion is fostered by the fact that Newton refers to RM as motion in 'ordinary' usage, and by his explicit discussion of Descartes' 'vulgar' motion in *De Grav.* But it is impossible to settle the issue definitively, for he may just as easily of consciously disagreed with Descartes about what common usage is like. So in what follows, it is RM that we consider; not exactly OM but close. And either way, clearly RM is an obvious way in which one might understand motion, so it is important for Newton to explain why it is not a suitable notion for mechanics.

The general form of Newton's arguments is that certain states of motion – *properly conceived* – are privileged in ways incompatible with either RM or PM. To keep things straight let us call this proper conception 'true motion' (TM), motion defined 'according to the truth of the matter', whichever of the possible definitions (some of which we have seen) that is. Newton himself uses the term 'true motion' in the Scholium, often in the context of arguing that it is not RM or PM, without prejudging the issue of whether it is AM. However, he sometimes seems to use 'absolute motion' when he means TM, and perhaps *vice versa*; our more careful use thus diverges from Newton's to some extent. Our use is supported by the fact that, as we saw in the previous chapter, the idea that there is some fundamentally correct sense of motion, which had to be explicated for the new physics, was common to Newton's contemporaries. Descartes certainly took most seriously the question of what motion is in the truest sense: in his view TM was PM. In the next chapter we shall consider Leibniz's account.

I have not yet assumed anything about the kinematic relativity of TM, so it is compatible with the definition given that TMs can be arbitrarily ascribed to bodies – that TM is RM, for instance. However, like Descartes, like Huygens (though see §5), and like Leibniz (as we shall see), Newton believes TM to be privileged, so not generally relativistic in the kinematic sense: not all motions can be truly ascribed. Nonetheless, he does not explicitly assume that TM is privileged in his arguments. If he did then RM could be immediately dismissed as the true conception, for every continuous motion is simultaneously attributable to a body in the sense of RM. Instead, Newton implicitly assumes that TM is privileged, by ascribing certain properties to TM or by assuming correlations with TM, all of which are incompatible with general kinematic relativity. Then he shows that because it is kinematically relativistic, RM cannot have the relevant properties or be correlated in the appropriate ways, and hence is not TM. We will understand TM in Newton's sense for the purposes of this section, but afterwards will simply take it to be privileged *by definition*. Indeed, we will, with Descartes, Newton and Leibniz take it to be *uniquely* attributable, in the sense that each body has exactly one TM: PM and AM are obviously candidates in this sense, and we shall see Leibniz propose a third such conception. Equivalently, in subsequent sections we shall understand 'x moves-truly' to be a 'complete' or one-place predicate, unlike 'x moves-relative-to y'.[14]

With the analytical framework at last in place, we turn to Newton's arguments that absolute and relative motions are distinguished: (i)-(iii) in terms of the *properties* of motion, (iv)-(v) in terms of its *causes* and (vi)-(vii) in terms of its *effects*.[15]

(i) The first argument from *properties* is the least clear and convincing of the seven. The property that Newton relies on is that 'bodies truly at rest are at rest in relation to one another' (411). The argument then seems to be an indirect proof. It is possible, as Descartes says (*Principles*, III.29), that there is a body at true rest in the distant reaches of the universe; let's call it 'alpha'

[14]Both 'x moves-properly-speaking' and 'x moves-absolutely' are complete predicates, even though they are defined in terms of relations. (Formally, x moves-properly-speaking $\equiv_{df}$ x moves-relative-to immediate-surroundings(x), while x moves-absolutely $\equiv_{df}$ x moves-relative-to absolute-space.) Completeness refers to the number of arguments of a predicate, not its analysis.

[15]Newton says that 'absolute and relative rest and motion are *distinguished* from each other by their properties, causes and effects' (1999, 411 – my emphasis), but there is an important ambiguity in what Newton might mean by the word 'distinguish': (a) that the properties, causes and effects demonstrate that TM is not RM or PM – that they are distinct conceptions of motion, that a body might possess one and not the other; or (b), that the properties, causes and effects provide independent empirical criteria by which one might actually tell whether a given body has TM – they distinguish those bodies that merely posses RM or PM, from those that possess TM. But while the causes and especially effects do provide such empirical criteria, the properties (all quoted below in the main text) certainly do not. In the first place, as stated by Newton, they are not properties that are possessed by a body iff it is in TM, but are desiderata for an account of TM – schema that must be made true by any adequate definition of TM. In the second place, even where they entail criteria for the TM of a body, they are circular, requiring first that some other body be identified as being in a certain state of TM – they do not entail independent criteria. (E.g., if extended, body A is at true rest, and A's parts are at relative rest, then part B is at true rest.) Either way, it would be nonsense to say that a body observed to possess one of the properties must be in TM. This point is rather important, because if one only looks at the arguments from causes and effects – or just from effects – then one might be tempted to think that Newton's goal is just to provide criteria by which we can tell when a body is in TM – to 'distinguish' in the sense of (b). But taking all the arguments together as Newton requests, and thus taking them all to have the same end, means that since the arguments from properties do not seek to provide empirical criteria for TM, *neither do the arguments from causes or effects* – though such criteria are implicit in them. That is, the arguments do not seek to show that TM has empirical content (still less that it is required to explain the phenomena, and less yet that it is implied by the phenomena), but just to show that TM is not PM or RM – to 'distinguish' in the sense of (a). We will return to this point at the end of the section.

(after Neumann's 'body alpha' – see Chapter 4). Suppose that the true rest of local bodies is definable in terms of the relative positions of local bodies; then if there were a local body at true rest, we could know it to be at true rest, and hence – by the property stated – know that it was at rest relative to alpha. However, 'it cannot be known from the position of bodies in relation to one another in our regions whether or not any of these maintains a given position with relation to that distant body'. Therefore, the supposition is false, and true rest cannot be defined in terms of the relative position of local bodies.

Of course, it's not much of a surprise that true rest cannot be defined in terms of relative positions; one would expect that it would require relative *rest* at least. Certainly, the notions that Descartes proposes would only say that a body was at rest if it was at rest relative to something, not just because it had a certain position relative to something. It is possible that Newton in fact meant to include change of position when he said 'position' here (which is how Rynasiewicz, 1995, 144, reads him). Alternatively, it could be that the three different arguments from properties are intended to distinguish true motions from three different relational concepts: first, any definition of rest/motion in terms of (local) relative positions, and second and third, as we shall see, PM and RM respectively. Then (i) is just given for completeness.

The question of whether 'position' includes 'change of position' makes a difference to the issue of the grounds on which Newton supposes that we cannot know that a local body is at rest relative to alpha. If 'position' is to be taken literally, then likely the point is that instantaneous positions cannot determine any relative motions, which seems a rather trivial point. If 'position' includes relative motions, then the argument seems to be a *non sequitur*: for instance, suppose that TM were motion relative to some particular body, which happened to be in our solar system. Then we could know that some local body was at true rest and, via the stated property, that it was a rest relative to any distant body at true rest. The mere distance of alpha – which prevents us from observing any relative motions directly – does not prevent such an inference. Newton rejected any such a view of TM, but if he is simply assuming that local relations cannot define true motion, then he is begging the question as he states it.

(ii) The second argument from properties is considerably clearer and convincing. '. . . parts which keep given positions in relation to wholes participate in the [true] motions of such wholes' (411): if the can is truly moving then so is the soup at rest in it. This might seem like common sense, but of course it would not be true if TM were PM. So to support this property Newton points out that when a rotating body recedes from its axis of rotation, so does every part of it, indicating that every part also rotates.[16] Of course this claim comes directly from the *Principles*, where Descartes tells us that although the planets are at rest in the vortex, because they are swept around by it they have a tendency to recede from the Sun (see the previous chapter). Then, given the tacit assumption that if a body has a tendency to recede then it is in TM, it follows

[16]Newton also makes the similar point that the 'impetus' (i.e., mass$\times\vec{v}$) of a body is the sum of the impetuses of its parts, which it would not be if the internal parts were in fact at true rest.

that TM 'cannot be [defined] by means of change of position from the vicinity of bodies that are regarded as being at rest'[17] (411) – unless those surroundings are themselves at true rest. Of course here we have Descartes' definition of PM almost verbatim. In summary, since objects at rest with respect to their immediate surroundings have the same TM as their surroundings but not (if those surroundings themselves move) the same PM, TM is not PM.

(iii) The third argument from properties is the closest that Newton comes in the *Scholium* to a direct argument for absolute space (as opposed to a disjunctive syllogism). Consider mobile places, such as relative places but not absolute places. Then it is a property of TM that '. . . when a place moves, whatever is placed in it moves along with it, and therefore a body moving away from a place that moves participates also in the motion of its place.' But the motion of the place is itself a motion from a place, and so on, so '. . . every whole motion is compounded of the motion of a body away from its initial place, and the motion of this place away from its place, and so on . . . ' (411). Newton assumes that the regress must be terminated, though the only reason that the text suggests for this is so that a unique motion can be attributed: again, the privileged nature of TM is apparent. Clearly unmoving places are needed to do the job, but, he claims, the only unmoving places are immobile places; not relative places but those places that 'constitute the space that I call immovable' (412). In other words, A may have one motion relative to B, but that motion can be compounded with the motion of B relative to C, and that motion can be compounded with the motion of C relative to D, and that . . . (yielding successively the motions of A relative to B, C, D . . .). The regress means that we will never be able to determine *the* motion of A – unless it is terminated with some places that are truly at rest. Absolute places do that work.

The *causes* that distinguish TM from RM are impressed forces. Newton makes the assumption that the TM of a body is changed if and only if net forces act on it: he assumes that TM is the sense of motion operative in Descartes' principle of inertia (*modulo* their differences concerning the nature of forces). Since there is a unique fact about whether the impressed force is zero or not, it immediately follows that there is a unique fact about whether TM is changing or not, which is what Newton exploits in the arguments. Note that he has moved from a discussion of different states of constant TM to a discussion of changes in TM. The arguments from properties sought to show that the distinction between rest and motion could not be properly captured by RM or PM, now he gives two arguments that the difference between constant and accelerated motion cannot either.

(iv) First, consider a body A, say a rocket in deep space, and let some reference body B, say a nearby space station, be 'regarded at rest'. Then if a force is exerted on B but not A, B will accelerate (truly) but not A, so A's RM (with respect to B) will change without a change in its TM. (v) Second,

[17] I have followed Rynasiewicz's (1995a, 151-3) suggestion and exchanged Cohen's translation of '*defineri*' as 'determined' for 'defined'. This exchange makes it clearer that at this point Newton is concerned with distinguishing TM from RM, not explaining how TM might be observed from RMs.

if suitable forces are applied to both A and B, so that they have the same accelerations, then A's TM must be changing, but not its RM (with respect to B). In either case, A's RM (with respect to B) cannot be its TM, since one changes but not the other.

Newton claims that his argument applies whatever body is the reference object: 'Therefore every relative motion can be changed while the true motion is preserved, and can be preserved while the true one is changed ....' (412) So, for instance, he assumes that any reference body at all could be put into the same TM as A, without changing A's TM – so that A's TM cannot be it's motion relative to any reference body. In general a body's TM is not RM and, implicitly, in particular a body's TM is not its motion relative to its contiguous surroundings – its PM – either. (Note the strong assumption that any reference body can be accelerated – including, presumably the fixed stars, since they define the reference frame of much of his work. The assumption is of course stronger than is needed; that some reference bodies can be accelerated suffices.)

The *effects* that distinguish TM from RM are 'the forces of receding from the axis of circular motion' (412). Of course, as we saw, such tendencies were central to Descartes' natural philosophy, and it is only reasonable to assume, with Newton, that such effects are associated with *true* rotations. In his arguments, Newton draws on his famous 'bucket experiment': (a) a bucket containing water and hanging from a cord is set spinning – since initially the water is at rest, its surface is flat; (b) gradually, however, friction between the water and the sides of the bucket causes the water also to spin and, as it speeds up, it starts to rise up the sides of the bucket and a concave surface forms; (c) eventually the water reaches a maximum height when it rotates at the same rate as the bucket.[18] In terms Descartes should accept, 'the rise of the water reveals its endeavor to recede from the axis of motion, and from such an endeavor one can find out and measure the [TM] of the water ... ' (413). Notice now that the discussion has shifted again, from different states of acceleration in general, to different states of true rotation; in the following arguments, Newton shows that neither PM nor RM can capture the distinction between truly rotating and non-rotating systems.

(vi) Newton's first argument from effects establishes that TM is not PM. At the beginning of the experiment, stage (a), the water has the maximum motion relative to the bucket, but it is flat, showing no endeavor to recede, and so, as all of Newton's contemporaries can be expected to agree, it has no TM. But the sides of the bucket are of course the 'bodies immediately contiguous' to the water, and so according Descartes' definition the water has the maximum

[18]In Chapter Five we will discuss several different kinds of explanations of such 'inertial effects', all of which are reasonable answers to the question of why the surface of the rotating water is curved. A useful one for now is that the circular motion of the water requires that a force directed towards the axis of rotation act on every part of the water. When the water is concave, the increasing height of water produces an outwards-increasing pressure gradient, since the pressure at a point is proportional to the weight of water above it. An increasing pressure gradient away from the axis means a net force towards the axis at any point, and so the surface is curved because system is in an equilibrium state; the centripetal force is caused by the deformation is exactly that required to keep the water rotating in place.

PM. At the last stage of the experiment (c) however, the water has no motion relative to the bucket, but has the maximum curvature; by analogous reasoning, it has the most TM but no PM at all. And so it is clear that TM cannot be PM, for in the bucket the TM is 'the direct opposite' of/is anti-correlated with PM. 'Therefore, that endeavor does not depend on the change of position of the water with respect to surrounding bodies, and thus true circular motion cannot be [defined] by means of such changes of position.' (413)

The phenomena of rotation show that it would be unequivocally wrong to think that PM is the true sense of motion. The water in the bucket is of course the celestial vortex in miniature, and since almost every cosmological explanation Descartes' offers depends on the tendency of bodies to recede being a result of their rotation in that case, there is no way he can deny that the recession of water in the bucket is a result of its rotation here, without abandoning the larger part of his system.[19]

In *De Grav* Newton criticises the notion of PM by referring to passages in Descartes' *Principles* in which the planets experience centrifugal effects even though they possess no PM (1962, 123).[20] Newton's bucket simply provides a more homely example of the very same phenomenon. The assumption that does the work in the argument is that TM is the kind of motion referred to in the mechanical explanation of centrifugal effects – since PM is anti-correlated with such effects, it cannot be the sense of motion used in mechanics. Or put another way, Descartes may have tried to define the sense of motion to be used in his mechanics, but when he applied his theory, he used 'motion' (and especially 'rotation') in a sense at odds with that definition. Newton wants to give a different definition, that is applicable to the phenomena, so that his mechanics (and Descartes' to the extent that they overlap) can be applied. It is hard to see how Descartes could resist the force of this objection: from a logical point of view, he committed the same error that Euclid would have if he had counted figures with four equal sides as falling under the definition of a triangle.

(vii) The second argument from effects is not very clearly distinguished from the first by Newton, but since it is intended to show that TM is not RM, it is worth pointing out.[21] The argument is that 'the truly circular motion of each revolving body is unique, corresponding to a unique endeavor ... while relative motions are innumerable in accordance with their varied relations to external bodies ....' (413). That is, the height to which the water climbs is determined by the rate at which it truly rotates. Hence, since the water can only reach a unique height at a time, the water can only have a unique true rotation at

[19]Quibblers can always point out that the tendency to recede *is* correlated with motion of the water relative to the surrounding bodies contiguous from above: the air touching it. It would be straight forward to modify the experiment to avoid this objection – would a flexible covering on the water really remove the tendency to recede? Further, this objection doesn't get around the tendency to recede of particles of water that are not on the upper surface.

[20]Of course they are transferred from their surroundings, because of the turbulence of the parts of the vortex. However, it is not that motion, but the orbital motion that the planets have in common with the vortex that gives rise to the effects.

[21]Overlooking this argument in his discussion of the bucket (1995, 146-51) is the one lacuna in Rynasiewicz's definitive treatment of the *Scholium*.

any time. But of course relative to different things – the bucket, the Earth, the Moon, the Sun, our galaxy, the fixed stars and so on – the water has different motions, and hence many RMs. Therefore, TM is not RM (or even Descartes' OM, as we have distinguished it from RM). The point here is similar to that made in the arguments from causes; the dynamical laws of physics imply a priviledged kind of motion – the one produced by forces or revealed by the endeavor to recede – but RM is too promiscuous to priviledge motion in this way. (Newton points to a further argument at this point: that the motion of one body relative to another cannot, by itself, have any effects at all.)

It's worth emphasising that the bucket arguments are of a piece with the others in refuting the claims of PM or RM to be TM, assuming reasonable or widely accepted features of TM. In particular, contrary to much of the literature, there is no argument that the tendency to recede is a criterion of motion, instead that is simply assumed – and why not, since Descartes (and, for instance, Huygens) said as much? The assumption was just part of the shared background of Newton and his contemporaries.

And once again, presumably the overall form of the argument is – implicitly – a disjunctive syllogism: TM is either RM or PM or AM, and since for various reasons it is neither RM of PM, while none of these reasons speaks against AM, it must be AM.

Newton's arguments end with a comment on the controversy concerning the Earth's motion. In his view, while AM is motion in the true sense used in mechanics, motion in the sense of ordinary usage is RM – the observable measure of motion. According to Newton, the meaning of 'motion' in the bible is clearly not the scientific one, but the ordinary one and 'those who there interpret these words as referring to [AM] do violence to the scriptures' (414) – they take the bible to be a physics text. That is, according to Newton, the question of the Earth's motion – instead of playing the central role it obviously did for Galileo and Descartes (though of course Descartes also says that the Earth is at rest in the ordinary sense) – should be cleanly separated from the question of motion in mechanics. Fortunately so, because, as we'll discuss at the beginning of the next section, the whole point of the *Principia* according to Newton was to settle the question of the Earth's motion against geocentric hypotheses (though not exactly in favour of the Copernican hypothesis, for it is at best the centre of mass of the solar system, not Sun, that is at rest). Newton's quick rejection of this issue was of course easier in Protestant England than Catholic Europe. We should also bear in mind that, as we shall see in the next chapter, other theological issues – such as the infinity of space and the nature of miracles – were relevant to Newtonian physics, under the heading of 'natural religion'; and also that Newton had his own theological conflicts with the Church of England (see, e.g., Westfall, 1980, especially Chapter 8).

Before we move on let us clarify what Newton's view are by showing *what Newton's arguments are not* (again, see Rynasiewicz 1995a, b for additional arguments). First, as the arguments of the *Scholium* make clear – especially the second concerning the properties of motion, and the arguments from causes and effects – the considerations Newton advances are closely tied to the demands of mechanics: both of his and Descartes' (and Huygens'). Thus the interpretation that Newton abandoned his empirical, scientific method in the *Scholium* in favour of fanciful metaphysics simply does not fit the text. At its crudest, the charge is that Newton was perniciously influenced by a theological point of view learned from Henry More, and gratuitously shoe-horned it in to his physics, arguing for it with a blatant *non sequitur*: the water with a convex surface doesn't rotate relative to the bucket therefore it rotates relative to absolute space (therefore the metaphysics is vindicated). (This interpretation is especially associated with Mach, 1960, Section II.vi and Reichenbach, 1959, Chapter II, but was common wisdom until the 1970s; see also Barbour, 1989, Chapter 11.) On the contrary, Newton is absolutely concerned with the scientific question of what kind of conception of motion is necessary to ground the kind of mechanical principles that he and his contemporaries sought; PM and RM cannot. (Additionally, the idea that Newton abandons his scientific methodology in the *Scholium* depends on a far too narrow and crude view of Newton's empiricism in the *Principia*, as Stein, 1967, argued.)

Next, it is tempting to swing too far in the other direction, and read Newton as attempting to provide *nothing but* an analysis of the spatial, temporal and motive concepts implicit in his mechanics. According to DiSalle (2002, 2006, developing Stein, 1967), the arguments from properties work rather as we have suggested, showing that TM, as generally conceived, could not be as Descartes defined it, but could be AM. However he takes Newton's arguments from causes and effects to be *definitions* of absolute motion in terms of the law of inertia (p.42) and absolute rotation in terms of centrifugal forces (p.44-5): a body has absolute motion $\equiv_{\text{df}}$ net forces act on it, and a body rotates absolutely $\equiv_{\text{df}}$ its parts tend to recede from some axis. (DiSalle then takes the first definition to fail, since forces only determine the accelerations of bodies, not their velocities; at best it is a definition of absolute acceleration.) The idea is that Newton sought to define the undefined theoretical quantities in his theory – true quantities of motion – empirically and in terms of the consequences of his mechanics (indeed, largely in terms of that portion of it that overlaps with the physics of Descartes and Huygens). On this reading, there is no *non sequitur* in inferring absolute motion from the convex surface of the water, for it follows by definition (and absolute motion on this understanding no longer has any implications concerning absolute space).

This reading is more sympathetic to Newton's scientific than the previous one, but it sits with the text poorly. (a) Newton never says anything in the arguments that could construed as giving such a definition, only that TM cannot be defined as PM or RM. (b) For him to propose such a definition would be bizarre, because at the very start of the *Scholium* he defines absolute motion as change of absolute place, and so already implicitly defines absolute velocity, acceleration and rotation: e.g., at what rate and in what direction a body changes absolute place. (And there is no textual evidence that Newton thought this definition inadequate, in need of further refinement in empirical terms.) So DiSalle has, in effect, Newton offering a second set of definitions that are not only gratuitous but which need not, as a point of logic, agree with the original ones. (c) Drawing such a distinction between the logic of the arguments from properties and those from causes and effects does considerable violence to the text. Newton makes no such distinction explicit, and as we have seen they can be naturally read as being of similar form. Now it is true in that Newton's arguments from causes and effects (and the second argument from properties) do rely on the way in which accelerations are distinguished within mechanics, and in particular how centrifugal effects provide a criterion for rotation. But one should not (as we noted footnote 15) be thus misled into thinking that the purpose of those arguments is to show how such motions can be experimentally distinguished – that question is already more-or-less agreed on by Newton and his interlocutors. Instead the arguments show, *from* the fact that bodies in TMs can be distinguished from those that are not, *that* TM is distinguished from PM or RM. Broadly, the criteria determine that a body is/is not in TM when according to their definitions it is not/is in PM or RM; but that form of argument in no way involves defining AM. (d) Finally, this reading attributes a rather obvious and major gaffe to Newton, since it proposes that he thought differences in forces were sufficient to distinguish different absolute velocities. On the reading I have presented there is no such blunder – Newton establishes just what he wanted, and what he wanted was quite sufficient for his purposes.

DiSalle reads Newton as attempting to define true quantities of motion including velocity with nothing but the resources of the laws of mechanics. For instance: 'Thus [Newton] has defined a theoretical quantity, absolute rotation, by exhibiting how it is detected and measured by inertial effects' (p.45). But as we have seen, *De Grav* shows that absolute space as Newton conceived it certainly involves more than the notion of motion implicit in the laws. It is true that metaphysics and theology do not make their presence felt very strongly in the *Scholium*, but even in that context it is implausible, as we have just seen, to read Newton's arguments as definitions (supposedly) requiring nothing more than the laws and phenomena. We do much better to take Newton to postulate absolute space (as we discussed it in the context of the *Scholium*), to define AM to be change of absolute place, and to interpret the concept of 'motion' appearing in the laws – i.e., TM – as AM. Then Newton's arguments show that his interpretation of TM as AM is the only one possible out of any that were available to him. That demonstration alone garners sufficient praise for Newton, and certainly explains and justifies its inclusion in a work of science, without the need to attribute Twentieth Century sophistication.[22]

[22] I do not mean to suggest that Newton's views on absolute space are *a priori* either logically or historically. Regarding the former possibility, Stein (2002) makes a convincing case that Newton's metaphysics is not an *a priori* one of 'clear and distinct ideas', like even Locke's at root. Though note that Newton's *a posteriori* grounds include historical and textual analysis of the 'revealed truths' of theology, and so are not grounded in experimental physics alone. And historically, I don't know of any evidence to suggest that Newton first came to his view of absolute space via contemplation of either physics or non-physical metaphysics alone. Further, I should also add that while I disagree with DiSalle's reading of Newton, I am very sympathetic with it as an interpretation of Newtonian mechanics (see Brown, 2006, for a similar interpretation of special relativity). Indeed I believe it complements the interpretation that I propose in Chapter Five.

And neither does another view, tracing back to Earman (1970) do justice to Newton. According to this account, the *Scholium* is not analysis of absolute space, but an argument for it, of a familiar scientific form. In particular, the bucket (the other arguments are sidelined in this way of looking at things) attempts to show that Descartes' mechanics cannot *explain* the effects of rotation, because they are not correlated with motion relative to the bucket: there cannot be an explanation of the form 'the water curves because it moves relative to the bucket'. On the other hand Newtonian mechanics can explain the curvature of the water's surface, in terms of motion with respect to absolute space (of course invoking the laws). So Newtonian mechanics and absolute space can be justified by the respectable scientific argument form of 'inference to the best explanation'. Often the argument is even extended using the example of the 'spheres' (which we will discuss below); a pair of spheres connected by a taut cord in an otherwise empty universe will have some quantity of tension in the string between them (perhaps zero). Newtonian mechanics can explain the value by attributing a suitable absolute rotation to the spheres about their centre of mass. But supposing that the relations between the bodies involved are the same whatever the tension, if all motion is the motion of bodies relative to others, then the difference cannot be explained (since there is no relevant difference to get an explanation started).[23] Again, we infer to the best explanation – Newtonian mechanics in absolute space.

We'll discuss and criticise the logic of such 'abductive' arguments concerning space at length in Chapter Five; for now the issue is one of Newtonian scholarship. I hope that it's clear why this interpretation – though sympathetic to Newton – sits very uncomfortably with the texts. None of the arguments, not even that of the bucket, are naturally read as being abductive. For instance, Newton does not talk about *explaining* the phenomena at all in the *Scholium*; moreover, as we shall soon see, the spheres example is clearly not intended as an argument for absolute space at all. Furthermore, in an important regard Newton simply didn't have to argue that his mechanics offered a better explanation than its competitors, the Cartesians, because his mechanics is in many ways a full articulation of the mechanics that Descartes gestured towards. It's not clear that Descartes and Newton would have been in any substantial disagreement about the explanation of the globes or bucket in terms of the inertial tendencies of the parts of water (no doubt Descartes' explanations are confused, but idea is much the same). Instead of a better explanation, Newton shows what it takes to make sense of the consensus explanation. In short, we do not have to take Newton to be arguing abductively to see him as making a perfectly reasonable scientific argument.[24]

[23]In fact, Newton does not say that the system of spheres is rigid, and so does not make any explicit assumption that the distance between the spheres or the shape of the spheres is independent of their motions. Indeed, it is perfectly compatible with the text that he imagined determining the tension in the cord by measuring its length and applying Hooke's law. In that case the supposed argument would have to be that since such differences are to be explained, they cannot very well enter into the explanans.

[24]We should acknowledge that the abductive argument is a more powerful argument for

And so we see Newton, in historical context, responding to his contemporaries in terms of their common assumptions about mechanics, showing how the spatial conceptions provided by Descartes are incoherent, and considering what kind of thing space could be if it were to have the properties it must according to mechanics.

4 Dynamical Relativity

The issue of kinematic relativity is quite simple in Newton; there is none at all. Every body has a unique absolute velocity, since it has a unique motion relative to the parts of absolute space. In this section then we turn to the question of dynamical relativity; to what extent is Newtons mechanics dynamically relativistic – i.e., to what extent can absolute motions be empirically determined – and especially, what did Newton acknowledge in this regard?

At the end of the *Scholium*, having distinguished TM – now identified as AM – from relative motions, Newton turns to the issue of how one can figure out what the TM of a body is. The problem of course is that we cannot visually (or otherwise sensibly) track the parts of absolute space to see how fast bodies change absolute places, and thereby measure AM directly. Instead, Newton shows how his mechanics also us to infer absolute quantities from observable quantities in special cases.

As an example, he imagines two spheres in vast region of otherwise empty space, connected by a taut rope, and considers what we can determine about their AM. (As I mentioned in footnote 23, it is often asserted that the system is rigid – i.e., that the relations of the parts are motion-independent – but Newton does not say so.) First, we could see whether there was any tension in the rope, which, like the concave surface of the water in Newton's bucket, would indicated a tendency to recede from an axis of rotation, and hence absolute rotation. Moreover, just as the height of the water measured the rate of rotation, so a measurement of the tension (and knowledge of the rope's length and the spheres' masses) would enable us to calculate the rate of rotation, using Huygens' formula (and Newton's third law). And, Newton points out, we can even determine the direction of rotation. Apply a force to the back of one sphere and the front of the other, and see whether the tension goes up (meaning that the spheres rotate faster) or down (slower), thus determining in which direction

absolute space than that given by Newton, for Newtonian mechanics in absolute space remains the best explanation until a better mechanics without absolute space is developed. The actual argument, based on a disjunctive syllogism, is only as strong as the premises and especially the completeness of the disjunction. Since there are logically possible accounts of motion other than AM, PM and RM, Newton's argument does not *prove* that TM is AM. Of course, he never intended it to, just to show that of the contemporary candidates, TM could only be AM. Similarly, all through the *Principia*, when he makes any inference from the phenomena Newton makes careful selections about what possibilities are worth considering. Such a methodology seems perfectly reasonable, and is in line with Newton's general attitude – codified in the fourth *Rule of Reasoning* of *Book III* – that 'hypotheses' – in this case speculative accounts of TM – are to be discounted unless there are concrete physical (and preferably experimental) reasons to adopt them.

the spheres are rotating. In other words, according to his theory it is possible to determine experimentally the exact state of absolute rotation of the balls; dynamical relativity does not cover different states of rotation.[25]

As Newton says explicitly, the project of the *Principia* is one of determining AMs from phenomena, particularly from the observed motions of the planets and moons. Indeed, after discussing the case of the balls in deep space, Newton imagines an identical system placed within a collection of bodies like the fixed stars, in rotation relative to them. The very same empirical method described above would allow us to determine the rate of rotation of the balls and hence the contributions to the relative rotation of the balls and 'fixed stars'. But this problem has the same form as that of determining whether the Copernican or Tychonic hypotheses is correct: do the other planets rotate about the Sun with the Earth, or – with the very same relative motions – do the other planets rotate around the Sun, which itself orbits the Earth? As the problem of the spheres and fixed stars can be answered, so Newton answers (in Book III) the problem of the planets.

The phenomena available for determining the absolute motions of the planets are considerably thinner than in the case of the balls, because we only have direct empirical access to relative motions, in the form of astronomical data, not to the forces involved. Indeed, the relative motions of bodies necessarily underdetermine the problem, because providing suitable forces are acting, any motions whatsoever are compatible with any frame whatsoever being at absolute rest, and so any absolute motion of the bodies. Hence Newton was required in Book III of the *Principia* to: (i) simultaneously infer the forces involved (the law of gravitation) and some 'inertial' frame in which the laws hold (that of the fixed stars); and (ii) use a powerful non-deductive inference to do so – perhaps on the grounds that $1/r^2$ and the frame of the fixed stars is inertial is the *simplest* possibility.

The flip side of the denial of relativity regarding rotation is Newton's discussion of the dynamical relativity of his mechanics. After the *Scholium* come the three Laws (which I assume are known – at least in their modern formulation – to the reader) and after the Laws a set of Corollaries. The first four have to do with the composition of forces and (what we would call) conservation of momentum, but the fifth and sixth describe the relativity of the Newtonian laws.

> Corollary 5: *When bodies are enclosed in a given space, their motions in relation to one another are the same whether the space is at rest or whether it is moving uniformly straight forward without circular motion.*

[25]It's irrelevant to point out that other effects could produce the same phenomena – some mysterious force source behind two stationary spheres, for instance. Newton's point is not that nothing else could explain the tension in such a cord; he is describing an idealized situation in which there would be no such forces by stipulation, and showing that one could experimentally determine the AM.

> Corollary 6: *If bodies are moving in any way whatsoever with respect to one another and are urged by equal accelerative forces along parallel lines, they will all continue to move with respect to one another in the same way as they would if they were not acted on by those forces.*

Taking 'spaces' to be, as we discussed, relative frames of reference, Newton intends Corollary 5 to imply that identical closed systems with the same frame-relative positions and velocities will evolve in the same way in their respective frames, whether those frames are at true (i.e., absolute) rest or in constant linear true motion. At least, that is the situation treated in the proof of the corollary and the sense in which it is invoked in Book I of the *Principia*. Newton invokes the corollary (in Propositions I.2, 6, 63 and 66) to conclude that some solution – the geometry of some curve in a frame – is independent of the common true velocity of the bodies involved: if the solution holds in a frame at rest, then it also holds for an identical moving system in a frame with the common motion of the bodies.[26]

Newton's proof in fact makes Corollary 5, logically speaking, a generalisation of Huygens' principle of relativity (recall that Huygens' *On the Motions of Colliding Bodies* was not published until 16 years after the *Principia*, though the results were published in 1669). For the proof aims to show that the results of *collisions* are independent of the common motions of bodies (it is a generalisation in the sense that there is no restriction to elastic collisions). In the context of Newton's work it is interesting that the proof refers only to collisions, for it is certainly invoked in the propositions mentioned for cases of centripetal forces. Of course, Newton famously *claimed* that he left open the possibility that forces such as gravity had a mechanical explanation, as many of his readers would suppose (see the *General Scholium* to Book III, especially p.943). But he also explicitly left open the possibility that gravity was not the result of collision, in which case the proof given is not sufficient for his use – though of course the corollary is also true for the case of central forces.

The proof runs as follows: suppose that some closed system of bodies with certain initial positions and velocities in a frame undergo some sequence of interactions. Now consider an identical system of bodies with corresponding positions and velocities in a frame in constant linear relative motion: a system identical except for a common velocity of the bodies. Then, Newton argues, since the bodies will all initially have the same positions and velocities relative to one another, 'by law 2, the effects of the collisions will be equal in both cases'. That is, filling in for Newton, suppose that the forces exerted in collisions depend only on the relative velocity of the bodies involved (an idea that Leibniz also had, and which motivated his thinking on relativity, as we shall see in the next chapter), not for instance on their absolute velocities. Then, by the 'Galilean' relativity of the Second Law – if it holds in some frame, for some set of forces, then it holds in any frame in constant linear relative motion – since the forces

[26] Newton's use of the corollary in II.37 does not refer to a closed system, but one in a gravitational field.

are the same, the outcomes in each frame must be too (assuming the velocity-independence of mass). The proof uses the same form of reasoning as Huygens in his proof of the rule for the collisions of equal bodies at unequal speeds; if the law for equal bodies and speeds holds in one frame then it holds in one in constant relative linear motion. (The explicit restriction to frames in constant linear motion of course was not made by Huygens, but it is so trivial – freely moving bodies obviously change their motions in accelerating frames – that he must have been aware of it.)

Of course the principle of linear relativity is not Galileo's principle of geocentric circular relativity, but Newton does give Galileo's example of the undetectability of a ship's smooth motion to illustrate the corollary. That is, it follows immediately, unavoidably, obviously from the corollary, its proof and its applications, that two closed systems which differ only by a common absolute velocity of their constituent bodies cannot be distinguished by any mechanical experiment. (For this reason Newton can make the hypothesis – not demonstrable proposition – in Book III [p.816] that the centre of mass of the solar system is at rest; the determination of the centre of the orbits of the planets is independent of any common motion.) This dynamical relativity is known as the principle of 'Galilean relativity' for closed systems (despite the mismatch with Galileo's precise views).

Corollary 6 enunciates a principle of relativity for the even more general case of systems in relative acceleration of a certain type. By 'accelerative forces' Newton just means the (true) acceleration of a body, so read literally the principle seems to be a straight-forward truth concerning Newtonian kinematics: if two systems of bodies at a time have the same relative positions and velocities, then if they differ only by a common acceleration at each time, then they will have the same relative positions and velocities at all later times. A common acceleration – even one changing over time – will not change the relations.[27] This point

[27] This point is a very simple one, as a more formal example makes clear. Suppose there is a system of N bodies with initial positions and velocities $\{\vec{q}_i(0); \vec{v}_i(0)\}$ $(i = 1, 2, \dots N)$. Then consider a second system that differs only by a constant common acceleration, $\vec{a}$ from the first (as I said, the acceleration need not be constant, but it simplifies the example). Then at some later time τ, suppose that the bodies in the first system have positions and velocities $\{\vec{q}_i(\tau); \vec{v}_i(\tau)\}$; it follows that the positions and velocities of the bodies in the second system are $\vec{q'}_i(\tau) = \vec{q}_i(\tau) + 1/2\vec{a}\tau^2$ and $\vec{v'}_i(\tau) = \vec{v}_i(\tau) + \vec{a}\tau$. (This is simply a matter of integrating the acceleration once or twice over time, which is why the example of constant acceleration simplifies things a little.)

Now, let the distance and relative speed between the i^{th} and j^{th} body of the first system at τ be q_{ij} and v_{ij}; then $q_{ij} = |\vec{q}_i - \vec{q}_j|$ and $v_{ij} = |\vec{v}_i - \vec{v}_j|$. Similarly, let the distance and relative speed between the i^{th} and j^{th} body of the second system at τ be q'_{ij} and v'_{ij}; then $q'_{ij} = |\vec{q'}_i - \vec{q'}_j|$ and $v'_{ij} = |\vec{v'}_i - \vec{v'}_j|$. It immediately follows from the relation between the $q(\tau)_i$s and the $q'(\tau)_i$s, and from the relation between the $v(\tau)_i$s and the $v'(\tau)_i$s that $q'_{ij} = q_{ij}$ and $v'_{ij} = v_{ij}$, and so the two systems still share their relative positions and motions. The whole result simply turns on the simple fact that because the accelerations are common, when they are integrated over time they make the same contributions to the positions and velocities of all the bodies in a system, and so cancel out in relative quantities. Note carefully that no assumption about dynamics are made whatsoever. Of course, there is nothing special about accelerations in this regard: the same is true for derivatives of any order (including zero) of position.

is not dynamical because it makes no use of the laws of motion, or indeed of any assumption about the cause of the accelerations. However, this interpretation is neither sufficient for the uses that Newton makes of the corollary, nor all that is established by its proof. The proof first.

> For those forces, by acting equally (in proportion to the quantities of the bodies to be moved) and along parallel lines, will (by law 2) move all the bodies equally (with respect to velocity), and so will never change their positions and motions with respect to one another.

The proof obviously does concern dynamics, specifically the case in which parallel forces – in the familiar sense of the Second Law – act on bodies in proportion to their masses; or as Newton might put it, the case in which 'motive forces' act on bodies in proportion to the 'quantity of matter' of a body along parallel lines (see the *Definitions*, 403-8). Then, by the Second Law, the accelerations of the bodies will be the same at any time (assuming that the forces between the bodies are independent of absolute motions) and the kinematic argument shows that the relations of the bodies are the same as if they were not accelerating. Notice, however, that the corollary does not depend on the relativity of the Second Law; indeed, the Second Law is not relativistic in this way, for if it holds in one frame then it is false in frames in relative acceleration. Instead it does the work of showing that a certain kind of force will produce a certain kind of (unobservable) acceleration.

I am not suggesting that Newton made a mistake by formulating the corollary in terms of accelerations not forces, since the proof makes it quite clear what dynamical situation is captured by stipulating equal accelerations. However, I do want to emphasise the dynamical nature of the corollary, for Newton uses the relativity stated in dynamical situations.[28] For of course the crucial situation in which bodies in a system are subjected to parallel forces proportional to their masses, and so possess a common acceleration, is when that system is in a constant gravitational field (within the boundary of the system) – in addition to the gravitational field arising from the bodies themselves. And so the corollary allows Newton to treat a system in such a field – Jupiter and its moons for instance, or to a first approximation the Earth and its moon and oceans, or even perhaps the whole solar system – as if it were not accelerating at all. And this is exactly what he does (for instance, I.3, 58, 66). And so, Newton's Corollary Six, is in fact one part of what is known as the 'principle of equivalence': the indistinguishability of free-fall from inertial motion.[29]

[28]In Newton's theory the distinction between the dynamical and kinematical points is not very sharp, because force is *defined* to be the rate of change of momentum. Then the unobservability of common accelerations entails by definition the existence of some set of unobservable forces (those proportional to mass). But one could envision a theory of mechanics in which no dynamical situation will produce equal acceleration; the unobservability of common acceleration would be true of kinematically possible situations, and yet vacuous for dynamically possible situations.

[29]The other part is the indistinguishability of accelerated motion from rest in a gravitational field.

So, the status of dynamical relativity in the *Prinicipia* is that different states of rotation of a system are in principle observable because of the central (and hence non-parallel) forces involved. That things are that way means that the problem of the planets can be solved. But otherwise closed systems that differ only by a common velocity or acceleration are indistinguishable, the former because of the relativity of the Second Law, the latter by the possibility of constant gravitational fields. That things are so is crucial for Newton's project, but as we shall soon see in the next chapter cause foundational problems, for it means that he is committed to real but unmeasureable quantities.

5 Huygens' Response

In an exchange of 1694 concerning motion, Leibniz asked Huygens if his views on rotation hadn't once agreed with Newton's. Huygens replied that indeed he had been, but that he was no more, having recently (after he read the *Principia*) found the 'truer' understanding (see Stein, 1977, 7-10). Recall from the previous chapter that Huygens had claimed that circular motion had its 'criterion' (centrifugal forces); after the *Principia*, he explicitly denied this claim in his note books.

This change of heart is rather surprising, since one would think that anyone who understood Newton's arguments would see that they underscored the connection between rotation and centrifugal effects – how could one come to deny that a tendency to recede indicates rotation? Huygens doesn't explain, but the simplest explanation is surely that he accepted the validity of Newton's arguments concerning the effects of absolute motion, but took them *modus tolens*. That is, one might gloss Newton as saying that if centrifugal effects are not correlated with relative motions, then the rotations of which they are criterial are with respect to absolute space. One who denies absolute space must then conclude, despite Newton's arguments, that rotations are after all a species of relative motion. That is, that centrifugal forces are not, as Huygens held earlier, a criterion of non-relative motions.

And indeed, Huygens says as much in his notes:

> Two bodies are known to be at rest relative to each other when they maintain the same distance without being held together by any bond .... If, however, they move with respect to each other while held together by a bond, then the velocity of that motion is discovered from ... their centrifugal force. ... [In] the motion of two bodies bound together, [the] change of distance is zero, and there is only relative velocity. (Fragment I)[30]

Huygens could hardly be more explicit that he counts bodies in rigid rotation – Newton's spheres, for instance – as being in 'relative motion' despite the fact

[30] I am hugely indebted to Marius Stan for making an English translation of Huygens' *Oeuvre* available to me, from which all the quotations in this section are taken, and for discussions of Huygens' views. It is quite fair to say that without his help, this section would not exist.

that they do not change their mutual relations, just as expected according to the proposed understanding of his response to Newton. Of course, given the usual meaning of 'relative motion', the globes aren't in relative motion at all, but Huygens is free to define words as he chooses, and his definition here is intended to distinguish rotation from Newton's *absolute* rotation. From a logical point of view, Huygens is quite justified in calling the motion 'relative', in the sense that it has no single subject. Presumably what he has in mind is that the *pair* of Newtonian spheres stand in a relation of mutual rotation (and in general the parts of a rotating body stand in that relation). He intends to deny that rotating is a monadic predicate, which a body may satisfy, as of course it is according to Newton. As we noted, although a body has AM iff it moves with respect to absolute space, the property of possessing absolute motion is itself monadic: a body either moves absolutely or not. So in that important sense, Huygens is entitled to call rotation *relative*.

To summarize, we should understand Huygens as saying that he came to realise that rotation was, after all, a kind of relative motion, one which does not require the moving bodies to change their relative positions: centrifugal forces are just a criterion of that kind of relative motion, not of any non-relative motion.

Now, Huygens' use of 'relative' may be reasonable, but what about 'motion'? Motion was, after all, generally understood to be 'change of place', but what, in Newton's spheres for instance, is changing at all – if not position with respect to absolute space? Huygens seems to feel the pull of this question, for he offers the following analysis of rotation: 'Circular motion is a relative motion along parallel straight lines, with the direction changing continuously, while the distance remains the same because of the bond' (Fragment VI). For example, at any instant, when rotating, the velocities of Newton's spheres are in opposite directions along parallel lines orthogonal to the cord joining them; but the direction rotates with the spheres. The problem with this account is precisely to say what it means for the 'direction of the parallel lines' (equivalently the orientation of the cord) to change. The natural understanding, in the light of Newton's *Scholium* is that the velocity is along different lines in absolute space as different times, but clearly that is not an interpretation that Huygens can offer.

As a matter of fact, Huygens seems to have had an answer available to him.[31] Consider the following:

> ...those bodies are relatively at rest that, without being kept together by some bond or obstacle so as to prevent them from freely

[31] Not only does the following rely on Stan's translation, it is also based on a suggestion that he made and elaborated to me in discussion. On the other hand, Stan is also rightly concerned with passages in Huygens' notes that do not sit that well with what is proposed here: I am content them to put them aside, for now at least, on the grounds that (a) interpretations of personal notebooks have a lower standard of consistency, and (b) the following ideas play a prominent role in Huygens' later thought. I have no doubt that work from Stan will clarify matters further. Note that the proposal here is quite different from that of Stein (1977) and Earman (1989, 66-9).

> receding from each other, they however maintain the same place and distance relative to each other.... From this motion, referred to bodies at rest relative to each other, one can at last understand and define what it is to move freely and uniformly in a straight line.... Because, assuming some bodies to be at rest with regard to each other, many other bodies can move relative to them along various straight lines, and then these lines will be called the various directions of motion.... And we can determine the number and velocity of rotations by observing other bodies at rest relative to each other. (Fragment VII)

What these passages suggest is that Huygens had the idea of setting up an inertial frame by reference to free bodies: the parallel but changing lines referred to in his account of rotation, are defined with respect to such bodies. However, some discussion is required to establish that reading.

Most importantly, there is the fact that when Huygens constructs his frames, by picking out reference bodies, he chooses unconnected 'bodies at rest relative to each other'. But clearly such bodies need not be force-free, and so need not be at rest in an inertial frame; one of Newton's arguments from the causes of motion precisely made the point that if suitable forces are applied, bodies at mutual rest may accelerate – they may even rotate. So did Huygens simply overlook this rather obvious point, and after all fail to properly define inertial frames?

I don't think so. In the first place, when Huygens introduces the reference bodies (in the first sentence) he indicates that he is talking about unconstrained bodies – those that are free, as is required. Now it is true that he later drops that qualification and talks about 'bodies at relative rest' not '*free* bodies at relative rest'; hence the worry that he thought relative rest was sufficient for inertial motion. But there is another explanation. In the first place, it's reasonable to assume that, having stated it once, he took it for granted that the reference bodies were free. In the second place, the specification of bodies at relative rest arises from his realisation that free bodies do not in general have uniform relative motions: only those at mutual rest or that collide are.[32] This is a point that he discusses several times (for instance in the ellipses of the quoted passage). In this context one can read the restriction to bodies at mutual rest as reminding the reader that in other situations there is no general characterisation of the relative motions of free reference bodies. The only general kind of relative motion compatible with free motions is mutual rest, so free bodies at mutual rest form natural reference frames.[33]

[32]For example, consider two bodies moving in opposite directions along $y = \pm 1$ at a speed v: $x(t) = \pm v \cdot t$. The distance between them is $2 \cdot \sqrt{1 + v^2 t^2}$, so the relative speed is $2v^2 t/\sqrt{1 + v^2 t^2}$: the relative speed is thus 0 when they pass at $t = 0$, and $2v$ as $t \to \infty$.

[33]One might have lingering worries that the distinction drawn here indicates that Huygens (incorrectly) thought that mutual rest did characterise free motion, because he emphasised that uniform motion in general did not. But there is nothing explicit that I can see in the text to support this suggestion.

Supposing that Huygens did indeed define motion relative to free bodies then he had a rather deep understanding of the situation, one that is a serious competitor to absolute space (one, that is, that cannot be simply shrugged off as confused, or as overladen with baroque metaphysics). For it offers an account of motion that serves the needs of mechanics: for example, the laws are to hold in these frames, so the effects and causes of motion will be properly correlated with motions in them. Moreover, as would have been clear from Corollary Five of the Laws, no special frame is picked out by the prescription: different sets of free bodies at mutual rest can move with respect to each other, and relative to each a body will have different motions. So a body has no true, preferred motion.

Huygens' notes are, however, rather frustrating on the topic. By modern lights he doesn't seem to have fully appreciated the power of his suggestion, and in particular, his notes were never written up into the book that they were supposed to become. So while Huygens seems to have seen a way to explicate motion in classical mechanics without appeal to absolute space, his influence on these matters was indirect, and very hard to pin down. There seems to be no tradition of appeal to inertial frames coming from his work. That he seems to be the first to have such a notion is worth emphasising, because it will become an important idea as we continue.

6 Conclusion

Like Descartes, Newton addressed the problem of interpreting the concept of motion contain in mechanics. Compared to Descartes, however, he had infinitely greater understanding of the practical applicability of mechanical laws and tackled the problem with a far clearer philosophical vision. Descartes gave laws that he knew could not hold in the real world as he believed it to be, but only *in vacuua*, while Newton used his laws to solve the problem of the planets. It's hardly surprising given their wildly different applications of mechanics that Newton's account pays far closer attention to what the laws actually require. That said, Newton's account makes absolute velocity well-defined, although differences in such a quantity are not observable according to his mechanics – it is hard to doubt that he was aware of the mismatch between the lack of kinematic relativity implicit in absolute velocity and the dynamical Galilean relativity of the laws. Further, while the role absolute space plays in the *Principia* is limited to what Newton took to be necessary to ground the laws, *De Grav* shows that it to be a metaphysically complex entity, and connected to strands in Newton's thought outside the immediate vicinity of the application of mechanics.

We've also seen that Newton's famous arguments for absolute space make perfect sense as responses to Descartes' proposals, showing that they are simply incompatible with reasonable assumptions and Descartes' professed views concerning true motion – motion properly understood, especially as it features in the laws of motion. In particular, they share the assumption that true motion is not a kinematically relativistic notion, and so cannot be RM. Motion with respect to absolute space was the only alternative apparent to Newton, and so to be accepted until there were empirical grounds to adopt an alternative hypothesis. In the next chapter we shall see Leibniz's very different approach to the problem of defining motion within a theory of mechanics, in response to both Descartes and Newton.