1) $$\frac{\partial \mathcal{L}}{\partial \partial_\mu \phi^*} = (\partial^\mu + ieA^\mu)\phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi^*} = -m^2\phi - ieA_\mu (\partial^\mu + ieA^\mu)\phi$$

$$\partial_\mu (\partial^\mu + ieA^\mu)\phi + m^2 + ieA_\mu (\partial^\mu + ieA^\mu)\phi = 0$$

We can put terms together in an explicit gauge invariant form,

$$\underline{\underline{(\partial_\mu + ieA_\mu)(\partial^\mu + ieA^\mu)\phi + m^2\phi = 0}}$$

$$\Delta\phi = i\phi \; ; \quad \Delta\phi^* = -i\phi^*$$

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\Delta\phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^*)}\Delta\phi^*$$

$$= i\left[(\partial^\mu + ieA^\mu)\phi\right]^*\phi - i\phi^*(\partial^\mu + ieA^\mu)\phi$$

(2) $\mathcal{M}(H^0 \longrightarrow W^+W^-) = g_2 M_W \varepsilon_+ \cdot \varepsilon_-$

$$\sum |\mathcal{M}|^2 = g_2^2 M_W^2 \sum \left(\varepsilon_+^\mu \varepsilon_+^\nu \ \varepsilon_{-\mu} \ \varepsilon_{-\nu} \right)$$

$$= g_2^2 M_W^2 \left(-g^{\mu\nu} + \frac{W_+^\mu W_+^\nu}{M_W^2} \right) \left(-g_{\mu\nu} + \frac{W_{-\mu} W_{-\nu}}{M_W^2} \right)$$

$$= g_2^2 M_W^2 \left(4 - \frac{(W_+ \cdot W_+)}{M_W} - \frac{W_- \cdot W_-}{M_W^2} + \frac{(W_+ \cdot W_-)^2}{M_W^4} \right)$$

$$= g_2^2 M_W^2 \left(2 + \frac{(W_+ \cdot W_-)^2}{M_W^4} \right)$$

$$= \frac{g_2^2}{M_W^2} \left(2M_W^4 + \frac{1}{4} M_H^4 + M_W^4 - M_W^2 M_H^2 \right)$$

$$(W_+ + W_-)^2 = H^2$$

$$2m_W^2 + 2W_+ \cdot W_- = m_H^2$$

$$W_+ \cdot W_- = \frac{m_H^2 - 2m_W^2}{2}$$

$$= \frac{g_2^2 M_H^4}{4M_W^2} \left(1 - 4\frac{m_W^2}{m_H^2} + 12 \frac{m_W^4}{m_H^4} \right)$$

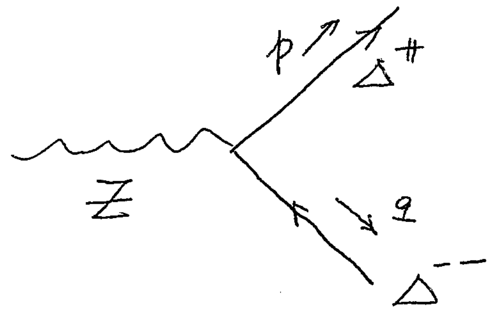
$$\Gamma = \frac{1}{2M_H} \frac{g_2^2 M_H^4}{4M_W^2} \left(1 - 4\frac{m_W^2}{m_H^2} + 12 \frac{m_W^4}{m_H^4} \right) \frac{1}{8\pi} \sqrt{1 - \frac{4m_W^2}{m_H^2}}$$

(3) a) $Q = T_3 + \frac{Y}{2}$

T_3	Q
$+1$	2
0	1
-1	0

$$\begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix} \text{ triplet.}$$

b)



$$\mathcal{M} = -\frac{e}{\cos\theta_W \sin\theta_W}\left(T_3 - Q\sin^2\theta_W\right)(p-q)^\mu .$$

$$= -\frac{e}{\cos\theta_W \sin\theta_W}\left(\underbrace{1 - 2\sin^2\theta_W}\right)(p-q)^\mu$$

$$= -\frac{e}{\frac{1}{2}\sin 2\theta_W}\cos 2\theta_W \,(p-q)^\mu$$

$$= -2e\cot 2\theta_W\,(p-q)^\mu .$$

c) $$\mathcal{L}_{\text{Neutral}} \rightarrow (\Delta T_3)^\dagger (T_3 \Delta)\, Z_\mu Z^\mu \frac{g_2^2}{\cos^2\theta_W}$$

$$= \left(-1\,\frac{u}{\sqrt{2}}\right)^2 Z_\mu Z^\mu\, g_2^2/\cos^2\theta_W$$

$$\cancel{\tfrac{1}{2}} m_Z^2 = \cancel{\tfrac{1}{2}}\,\frac{g_2^2 u^2}{\cos^2\theta_W}$$

$$\mathcal{L}_{\text{charge}} = \left|(T_3 \Delta)\right|^2 W_\mu W^\mu \left(\frac{g_2}{\sqrt{2}}\right)^2 = \left|\cancel{\sqrt{2}}\,\frac{u}{\cancel{\sqrt{2}}}\right|^2 W_\mu W^\mu \frac{g_2^2}{2}$$

$$m_W^2 = \frac{g_2^2 u^2}{2} \qquad \boxed{\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{1}{2}}$$