Keung's class February 10, 2006

Homework 1 Elementary Particle Physics (PHYSICS 552 UIC)

In the homework of the first semester, we work out the matrix element for the process $e^-e^+ \to \tilde{\mu}^-\tilde{\mu}^+$,

$$\sum |\mathcal{M}|^2 = 16e^4(e^- \cdot \tilde{\mu}^+)(e^- \cdot \tilde{\mu}^-)/(e^- \cdot e^+) = 8e^4 ut/s^2 = 2e^4(1-\cos\theta_{\rm CM}) \ .$$

First review this formula from your returned homework, and verify that

$$d\sigma(e^-e^+ \to \tilde{\mu}^-\tilde{\mu}^+) = \frac{1}{8s}2e^4(1-\cos\theta_{\rm CM})\frac{1}{8\pi}\frac{d\cos\theta_{\rm CM}}{2} \ .$$

After getting familiar with above algebra, we switch our attention to a hypothetical structure that the charged parton in the nucleon is spin-0 scalar quarks $\widetilde{q}$. Convince yourself that

$$\sum |\mathcal{M}|^2(e\widetilde{q} \to e'\widetilde{q}') = -8e^4 e_{\widetilde{q}}^2 \hat{s}\hat{u}/\hat{t}^2.$$

Work out the x, y dependence as follows,

$$\frac{d\sigma}{dQ^2dx} = \sum_{\widetilde{q}} \frac{e^4 e_{\widetilde{q}}^2}{8\pi(Q^2)^2}2(1-y)\widetilde{q}(x) \ ,$$

where $s = (e+P)^2 = 2Em_N$, $y = (E-E')/E$, $Q^2 = -(e-e')^2$, $x = Q^2/[2m_N(E-E')]$, and the function $\widetilde{q}(x)$ gives the probability density of the scalar quark at the fractional momentum x. Experimental measurements agrees with the spin-$\frac{1}{2}$ pattern $1+(1-y)^2$, disagrees totally with this scalar-quark form of y dependence $2(1-y)$ and hence rule out the scalar quarks inside nucleon.

$$\sum_{\rm spin} |\mathcal{M}|^2 = (e^4/s^2)4(e^{-\alpha}e^{+\beta} + e^{-\beta}e^{+\alpha} - e^-\cdot e^+ g^{\alpha\beta})(\tilde{\mu}-\bar{\tilde{\mu}})^\alpha(\tilde{\mu}-\bar{\tilde{\mu}})^\beta$$

$$= 4(e^4/s^2)[2(\tilde{\mu}-\bar{\tilde{\mu}})\cdot e^-(\tilde{\mu}-\bar{\tilde{\mu}})\cdot e^+ - e^-\cdot e^+(\tilde{\mu}-\bar{\tilde{\mu}})^2] = 2(e^4/s^2)[-(t-u)^2+s^2]$$

$$= 2(e^4/s^2)(s-t+u)(s+u-t) = 8e^4ut/s^2$$

In the process $e\widetilde{q} \to e'\widetilde{q}'$, we just make the replacement $s \leftrightarrow -t$,

$$\sum |\mathcal{M}|^2(e\widetilde{q} \to e'\widetilde{q}') = -8e^4 e_{\widetilde{q}}^2 \hat{s}\hat{u}/\hat{t}^2.$$

With careful substitutions, the result can be obtained.