I. The "bouncing light clock" has been well described in the class and in the book. The clock ticks when the photon leaves the source, and tacs when it reaches the opposite mirror, L_0 away in the clock's rest frame, and then get reflected instantaneously, and finally tocs as the photon arrives at the beginning position. In the rest frame of this clock, the round trip time of the photon is t_0. Express L_0 in terms of t_0 and c

$$\boxed{L_0 = ct_0/2}$$

Now the clock is flying at a speed $v/c = 0.28$ in the **lab** frame with the photon bouncing path along the clock motion direction.

Determine the instantaneous spatial separation in the lab frame between the mirror and the source in terms of t_0, c.

$$\boxed{L = \sqrt{1 - v^2/c^2}ct_0/2 = \tfrac{12}{25}ct_0}$$

How much time, in terms of t_0, does it take the photon to travel from the source to the mirror in the lab frame?

$$\boxed{\tfrac{L}{c-v} = \tfrac{2}{3}t_0}$$

How much time, in terms of t_0, does it take the photon to travel from the mirror to the source in the lab frame

$$\boxed{\tfrac{L}{c+v} = \tfrac{3}{8}t_0}$$

Determine the total round trip time in the lab frame.

$$\boxed{\tfrac{25}{24}t_0}$$

IIA. An X-ray tube is operated under a high voltage $8,000$ volts. Bragg's diffraction on NaCl crystal is used to select a particular wavelength. The angle between the incident and the reflected direction is $120°$. The spacing of relevant planes is $d = 0.28$ nm.

Find the wavelength of the X-ray (allowed by available energy) after the Bragg's diffraction.

$$\lambda_{\text{min}} = hc/(eV_0) = 1240/8000 = 0.155 \text{ nm} \ ,$$

$$n\lambda = 2d \sin\theta = 2 \times 0.28 \text{ nm} \ \ \sin 30° = 0.28 \text{ nm} \quad .$$

For $n = 2$, the corresponding $\lambda = 0.14$ nm is smaller than λ_{min} calculated above, so it is not allowed. The only acceptable wavelength is $\lambda = 0.28$ nm for $n = 1$.

IIB. Two massless photon, with $E_1 = 4$ GeV and $E_2 = 5$ GeV, collide head-on and form a moving subnuclear particle A. Find the energy E (in GeV), the momentum p (in GeV/c), and the rest mass m (in GeV/c^2), of this particle A. (1 GeV=10^9 eV)

$$\boxed{E_A = E_1 + E_2 = 9 \text{ GeV}, \ (p_A)_x = -1 \text{ GeV}/c.}$$

$$\boxed{m_A c^2 = \sqrt{9^2 - 1^2} = 8.9 \text{ GeV}.}$$

III. In a photoelectric experiment it is found that a stopping potential of V_0 is need to to stop all the electrons from the metal surface of workfunction ϕ when the incident light of wavelength 260 nm is used. The voltmeter has not been calibrated, and only relative voltage is reliable. A higher stopping potential $2.5V_0$ is needed for the light of wavelength 207 nm for the same metal.

Determine the workfunction of the metal and the value of V_0.

$$\begin{cases} eV_0 = \frac{1240}{260} - \phi \\ 2.5eV_0 = \frac{1240}{207} - \phi \end{cases}, \quad \begin{cases} \phi = 4.0 \text{ eV} \\ V_0 = 0.81 \text{ volt} \end{cases}$$

Without a stopping potential, what is the minimum wavelength for the incident light to eject the photoelectron?

$$\lambda_{\text{threshold}} = hc/\phi = 1240/4.0 = 313 \text{ nm} \quad .$$

IV. An energetic photon can transfer its energy and momentum to a proton which is initially at rest. The process is like the usual Compton scattering except that the electron is replaced by the proton. The incident photon has a wavelength 2×10^{-6} nm. The scattered photon is observed at an angle 60°.

(1) What is the wavelength of the scattered photon at $\theta = 60°$?

The Compton length for the proton is $h/(m_p c) = 1240/938$ fm$=1.32 \times 10^{-6}$ nm.

$$\lambda = 2 \times 10^{-6} \text{ nm } + 1.32 \times 10^{-6} \text{ nm } (1 - \cos 60°) = 2.66 \times 10^{-6} \text{ nm} \quad .$$

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(2) At $\theta = 60°$, what is the kinetic energy of the recoiling proton?

$$E = 1240/(2 \times 10^{-6}) \text{ eV } = 620 \text{ MeV} \quad , \quad E = 1240/(2.66 \times 10^{-6}) \text{ eV } = 466 \text{ MeV} \quad ,$$

$$K(\text{proton}) = E_0 - E = 620 - 466 = 154 \text{ MeV} \quad .$$

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(3) At which angle θ, the proton picks up the maximum speed? Find the maximum value v/c of the proton.

The maximum value v/c of the proton occurs when $\theta = 180°$

$$\frac{1}{E} - \frac{1}{E_0} = \frac{2}{m_p c^2} \ , \ \frac{1}{E} = \frac{2}{938} + \frac{1}{620} \ , \ E = 267 \text{ MeV} \quad .$$

$$K(\text{proton}) = E_0 - E = 620 - 267 = 353 \text{ MeV} \quad ,$$

$$E(\text{proton}) = K + m_p c^2 = 353 + 938 = 1291 \text{ MeV} \quad .$$

$$pc = \sqrt{E^2 - m_p c^2} = \sqrt{1291^2 - 938^2} = 887 \text{ MeV} \quad .$$

$$v/c = pc/E = 887/1291 = 0.68 \ .$$

V. The hydrogen atom is set at its second excited state ($n = 3$).

(1) Using the Bohr model, determine the potential energy of the electron. What are the radius of this excited atom?

$$\boxed{r = n^2 a_B/Z = 9 \times 0.0529 = 0.476 \text{ nm}}$$

$$\boxed{E_3 = -13.6/3^2 = -1.511 \text{ eV},\ U = -2E = -3.02 \text{ eV}.}$$

(2) List all available wavelength(s) of photon(s) emitted by the de-excitation of this state?

$$E_1 = -13.6/1^2 = -13.6 \text{ eV}\ , \quad E_2 = -13.6/2^2 = -3.4 \text{ eV}$$

$$\lambda_{3\to1} = 1240/(13.6-1.511) = \boxed{102.6 \text{ nm},} \quad \lambda_{3\to2} = 1240/(3.4-1.511) = \boxed{659.6 \text{ nm}.}$$

There is also the cascade transition, first from $3 \to 2$, and then $2 \to 1$,

$$\lambda_{2\to1} = 1240/(13.6 - 3.4) = \boxed{121.6 \text{ nm}.}$$

(3) The second excited state ($n = 3$) can only absorb certain wavelengths of incoming photons. Calculate the two longest wavelengths of absorption.

$$E_4 = -13.6/4^2 = -0.85 \text{ eV}\ , \quad E_5 = -13.6/5^2 = -0.544 \text{ eV}$$

$$\lambda_{3\to4} = 1240/(1.511-0.85) = \boxed{1876 \text{ nm},} \quad \lambda_{3\to5} = 1240/(1.511-0.544) = \boxed{1282 \text{ nm}.}$$