

I. (20%) Two fixed nozzles A and B on the ground, 0.8 m apart, spray paint-marks simultaneously (as measured by the ground observer  $O$ ) onto a long rod which passes by as shown in the figure at a relativistic speed  $v = 0.6c$ .

- An observer  $O'$ , who co-moves with the rod, observes that the rod is stationary. What is the instantaneous separation between the two nozzles as observed by  $O'$ ?

$$0.8 \sqrt{1-0.6^2} = 0.8 \times 0.8 = 0.64 \text{ m.}$$

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- Determine separation between the two paint-marks measured by the observer  $O'$ .

$$0.8 \text{ m} = \sqrt{1-0.6^2} l_0 \quad ; \quad l_0 = 1 \text{ m.}$$

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- According to the observer  $O'$ , which nozzle sprays the paint-mark first? How many ns later does the other nozzle spray?

$$\begin{aligned} \text{A first B later by } \frac{Lv/c^2}{\sqrt{1-0.6^2}} &= \frac{0.8 \text{ m} \times \frac{0.6}{c}}{\sqrt{1-0.6^2}} \\ &= \frac{0.6 \text{ m}}{c} = \frac{0.6}{300} \mu\text{s} = 2 \times 10^{-3} \mu\text{s} \\ &\quad \text{or } 2 \text{ ns} \end{aligned}$$

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- During that time delay, how far do the nozzles move according to observer  $O'$ ? (Check your answer to show you have consistent results)

$$\Delta L = v \Delta t = 0.6 \times 300 \times 2 \times 10^{-3} \text{ m} = 0.36 \text{ m.}$$

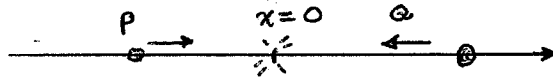
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$$\Delta L + 0.64 \text{ m} = 1 \text{ m} \quad (\text{consistent check})$$

- Each paint-mark only lasts for  $100 \mu\text{s}$  as measured by  $O'$ . Find the time span of the paint-mark as measured by the ground observer  $O$ .

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$$\frac{100 \mu\text{s}}{\sqrt{1-0.6^2}} = \frac{100}{0.8} = 125 \mu\text{s}$$



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II. (26%) In the ground frame  $O$ , particle  $P$  passes a detector at  $t = 0$ ,  $x = -72$  m, Simultaneously particle  $Q$  passes another detector at  $x = +96$  m. Both  $P$  and  $Q$  have the same rest mass  $105 \text{ MeV}/c^2$  and move toward the origin. They collide at the origin  $x = 0$  when  $t = 0.4 \mu\text{s}$ , and becomes a composite object  $X$ .

Find the velocities (in  $c$ ), momenta (in  $\text{MeV}/c$ ), and energies in (MeV) for both  $P$  and  $Q$  before the collision in the ground frame.

$$u_P = \frac{72}{0.4} = 180 \text{ m}/\mu\text{s} = \underline{0.6c}; \quad E_P = \frac{105 \text{ MeV}}{0.8} = \underline{131.25 \text{ MeV}}$$

$$P_P = E_P u_P / c^2 = 131.25 \times 0.6 \text{ MeV}/c = \underline{78.75 \text{ MeV}/c}$$

$$u_Q = -\frac{96 \text{ m}}{0.4 \mu\text{s}} = -240 \text{ m}/\mu\text{s} = \underline{-0.8c}; \quad E_Q = \frac{105 \text{ MeV}}{0.6} = \underline{175 \text{ MeV}}$$

$$P_Q = -175 \times 0.8 \text{ MeV}/c = \underline{-140 \text{ MeV}/c}$$

Determine the rest mass (in  $\text{MeV}/c^2$ ) of  $X$ .

$$M_X c^2 = \sqrt{(131.25 + 175)^2 - (140 - 78.75)^2} = 300 \text{ MeV}$$

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Physics can be independently studied by the the observer  $O'$  in the rest frame of  $Q$ . Determine the velocity  $u'_P$  of  $P$  before the collision as measured by  $O'$

$$u'_P = \frac{0.6 + 0.8}{1 + 0.6 \cdot 0.8} c = \frac{1.4}{1.48} = 0.945945c.$$

$$\gamma = \frac{70}{74} c = \frac{35}{37} c$$

$$\sqrt{1 - u'^2/c^2} = \frac{12}{37}$$

Find the momentum and energy of  $P$  before the collision, measured by  $O'$ .

$$P'_P = \frac{E'_P}{c^2} u'_P = \left(\frac{35 \cdot 37}{4}\right) \frac{35}{37} = \frac{35^2}{4} = \frac{1225}{4} = 306.25 \text{ MeV}/c$$

$$E'_P = \frac{105}{\sqrt{1 - (\frac{35}{37})^2}} = \frac{105 \cdot 37}{12} = 323.75 \text{ MeV}$$

Find the momentum and energy of  $X$  after the collision, measured by  $O'$ .

$$P'_X = 306.25 \text{ MeV}/c + 0$$

$$E'_X = 323.75 + 105 = 428.75 \text{ MeV}$$

Base on the last answer, use the Pythagorean relation to find the rest mass of  $X$ .

$$M_X c^2 = \sqrt{428.75^2 - 306.25^2} = 300 \text{ MeV} \checkmark$$

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III. (27%) In a photo-electric experiment, a stopping potential of  $V_0$  is needed to quench the current due to electrons from the metal surface of work-function  $\phi$  with the incident red light of wavelength 620 nm. The voltmeter has not been calibrated, and only relative voltage is reliable. A higher stopping potential  $3.2V_0$  is needed for the blue light of wavelength 400 nm for the same metal.

Determine the work-function  $\phi$  of the metal and the value of  $V_0$ .

$$\lambda_{\text{red}} = 620 \text{ nm} \quad E_{\text{red}} = \frac{1240 \text{ nm} \cdot \text{eV}}{620 \text{ nm}} = 2 \text{ eV}$$

$$\lambda_{\text{blue}} = 400 \text{ nm} \quad E_{\text{blue}} = \frac{1240 \text{ nm} \cdot \text{eV}}{400 \text{ nm}} = 3.1 \text{ eV}$$

$$\left. \begin{aligned} V_0 &= 2 - \phi \\ 3.2V_0 &= 3.1 - \phi \end{aligned} \right\}$$

$$2.2V_0 = 1.1$$

$$V_0 = \frac{1}{2} \text{ (volt)}$$

$$\phi = 1.5 \text{ eV}$$

What is the maximum velocity of the photo-electron just ejected from the metal by the blue light?

$$K.E. = 3.1 - 1.5 = 1.6 \text{ eV}$$

$$\left(\frac{v}{c}\right)^2 = \frac{2K}{mc^2} = \frac{3.2 \text{ eV}}{511000}; \quad \frac{v}{c} = 0.0025; \quad v = 0.75 \text{ m}/\mu\text{s}$$

Without a stopping potential, what is the maximum wavelength for the incident light to induce a current due to the photo-electric effect?

$$\frac{hc}{\phi} = \frac{1240 \text{ nm} \cdot \text{eV}}{1.5 \text{ eV}} = 826.7 \text{ nm}$$

An X-ray of the photon energy 124 keV scatters with a free electron. Find the maximum kinetic energy given to the electron.

$$\lambda = \frac{1240 \text{ nm} \cdot \text{eV}}{124,000 \text{ eV}} = \frac{10}{1000} \text{ nm} = 0.01 \text{ nm}$$

$$\lambda' = \lambda + 0.00243 \text{ nm} (1 - \cos 180^\circ) = 0.01486 \text{ nm}$$

$$E' = \frac{1240 \text{ nm} \cdot \text{eV}}{0.01486 \text{ nm}} = 83,445 \text{ eV}$$

$$E - E' = 124 - 83,445 = 40.554 \text{ keV}$$

$$K.E. = 40.554 \text{ keV}$$

IV. (27%) The negative muon is a subatomic particle with the same charge as the electron but a mass that is about 207 times greater:  $m_\mu = 207m_e$ . A bare Helium nucleus  ${}^4\text{He}$  has all electrons removed. It carries a bare positive charge of two units. A single muon is arranged to be captured by this nucleus to form a hydrogen-like system.

- (1) List energies and radii of the various Bohr orbit ( $n = 1, n = 2, n = 3$ )?

$$E_1 = -13.6 \text{ eV} \cdot 4 \times 207 = -11260.8 \text{ eV}$$

$$E_2 = -\frac{11260.8}{4} = -2815.2 \text{ eV}$$

$$E_3 = -\frac{11260.8}{9} = -1251.2 \text{ eV}$$

$$r_1 = \frac{0.0529 \text{ nm}}{207 \times 2} = 0.000128 \text{ nm}$$

$$r_2 = 4r_1 = 0.000511 \text{ nm}$$

$$r_3 = 9r_1 = 0.00115 \text{ nm}$$

- (2) From which orbit to which orbit, does the transition emit a photon of a wavelength 0.1239 nm? Your claim will be credited only when calculation is provided.

$$E_3 - E_1 = 10009.6 \text{ eV}$$

$$\lambda_{(3 \rightarrow 1)} = \frac{1240 \text{ nm} \cdot \text{eV}}{10009.6 \text{ eV}} = 0.1239 \text{ nm}$$

- (3) This photon is diffracted by a crystal. The second order constructive interference occurs at an angle  $37^\circ$  between the incoming beam and the surface normal direction. Determine the unknown lattice separation of the crystal.

$$2d \sin \theta = n \lambda$$

$$2d (\sin 37^\circ) = 2 (0.1239 \text{ nm})$$

$$d = \frac{0.1239}{0.8} = 0.1548 \text{ nm}$$

