BRAGG SCATTERING

I. INTRODUCTION

When Wilhelm Röntgen discovered X-rays in 1895, he was not sure whether they were particles or waves, hence, the name "X-rays". The wave nature of some radiation can be determined by using a diffraction grating with the appropriate grating spacing. If the radiation has a wave nature, then interference will occur, and a diffraction pattern will result behind the grating. This was observed for the light from the mercury vapor lamp passing through the grating in Experiment #1. If X-rays were made of waves, as was thought, however, they had to be waves of very short wavelength, because no diffraction pattern could be produced using manufactured gratings.

Max Von Laue had the idea to use the known, regular structure of atomic crystals as diffraction gratings for X-rays, since the spacing in such crystals was known to be very small, of the order of a few Angstroms. If one could shine X rays on a crystal whose structure was already known by some other means and produce a diffraction pattern, then one could demonstrate the wave nature of the radiation and calculate its wavelength. Von Laue successfully demonstrated that this idea was correct, and for this he won the Nobel prize in 1914. W.L. Bragg and W. H. Bragg developed the use of crystals as diffraction gratings for X-rays. They designed an X-ray spectrometer from a crystal of known structure in order to analyze the spectral content of X-rays and to use monochromatic X-rays for probing the structure of unknown crystals. They won the Nobel prize for their work in 1915.

Unlike diffraction through diffraction gratings, X-ray diffraction in crystals involves interference between waves reflected off of planes of atoms in the crystal. Reflection off of a single plane of atoms, shown in Fig. 1a, is similar to reflection of visible light off of a smooth surface, like a mirror. Although each atom scatters the incoming radiation in a spherical pattern, when the scattered light off of all the atoms in the plane is combined, cancellation due to interference constrains the reflected wave to leave the surface at the same angle at which the incident wave struck the surface. This is essentially the law of specular reflection; $\theta_i = \theta_r$.

In a regular atomic crystal, planes of atoms are layered one on top of the other, separated by a distance, d. Not all of the radiation is reflected by the upper plane, but much of it passes through to the next plane, where some percentage of that radiation is reflected back according to the law of specular reflection. The reflected waves from the adjacent planes will have a path difference, Δ, determined by the distance of separation between the planes d and the angle of incidence θ_i. This can be seen in Fig. 1b.

Q.1. From Figure 1b, prove that the path difference between the waves scattered from adjacent planes is given by the formula,

$$\Delta = 2d\sin\theta_i. \quad \textbf{(1)}$$

(Hint: the wavefronts represent the points of equal phase in the incoming and outgoing waves and are perpendicular to the direction of motion.)

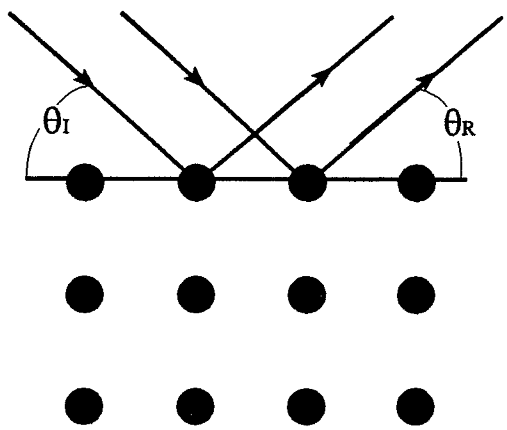

Figure 1a. Bragg scattering from centers in a single plane.

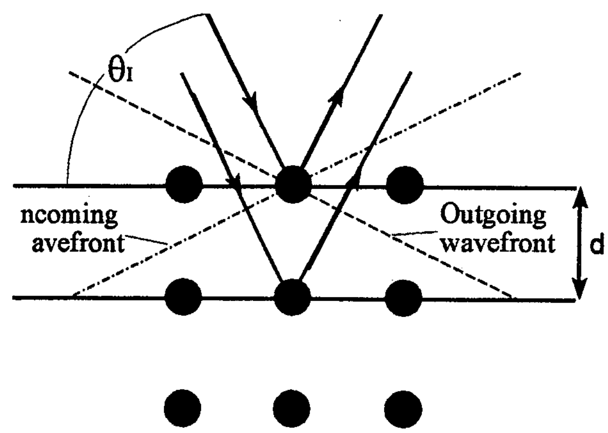

Figure 1b. Bragg scattering from centers in adjacent planes.

If the path difference is equal to an integral number of wavelengths, n = ±1, ±2, ±3, ···, then the reflected radiation will add constructively, and a peak in the intensity of the reflected radiation will be observed. Thus, the Bragg formula for the maxima in the intensity of the light reflected off of an atomic crystal is given by

$$\Delta = n\lambda = 2d\sin\theta_i. \quad \textbf{(2)}$$

Atomic crystals continue to be used to investigate the wave nature of short wavelength probing radiation.

In addition, radiation of a known wavelength can be used to investigate the cystalline structure of unknown materials. The purpose of this experiment is to introduce and familiarize the student with this important technique, and understand the principles behind it. In practice, however, the control of the X-ray source in order to produce monochromatic radiation (radiation having a single wavelength) and the preparation of the crystalline sample is not easy, nor is the interpretation of the scattered signal, since there are more than one scattering planes in a real crystal.

Therefore, we will investigate Bragg scattering using a scale model of a crystal constructed out of metal spheres arranged in a cubic pattern in a styrofoam block. Instead of X-rays, we will use microwaves of wavelength 3 cm. The physical mechanism, however, is the same as for the X-ray scattering in atoms.

> **Q.2** How is the scattering of electromagnetic radiation by individual metal spheres equivalent to scattering by single atoms in a crystal? Hint: For both scatterers, what is responding to the electric field of the radiation?

The large scale enables us to measure the crystalline structure directly and to use electromagnetic radiation from a more controllable source. All the same, this is not an easy experiment to perform, especially because the microwave amplitude is very sensitive to the laboratory setup. However, by making careful measurements, you should be able to observe maxima in the scattered radiation at the angles predicted by the Bragg scattering formula for scattering off of several planes in the "crystal".

The steps in the experiment are as follows: You will first measure the spacing between spheres in the "crystal". Then you will calculate the spacing between planes for three different orientations of the "crystal". Then you will take measurements of the amplitude of the scattered signal for various angles of the incoming radiation relative to each of the scattering planes, and compare the location of each of the experimentally determined maxima with the theoretical values. Finally, you will use your experimentally determined values of θ_{max} in order to calculate the wavelength of the microwave radiation, and compare it with the given value of 3 cm.

II. APPARATUS

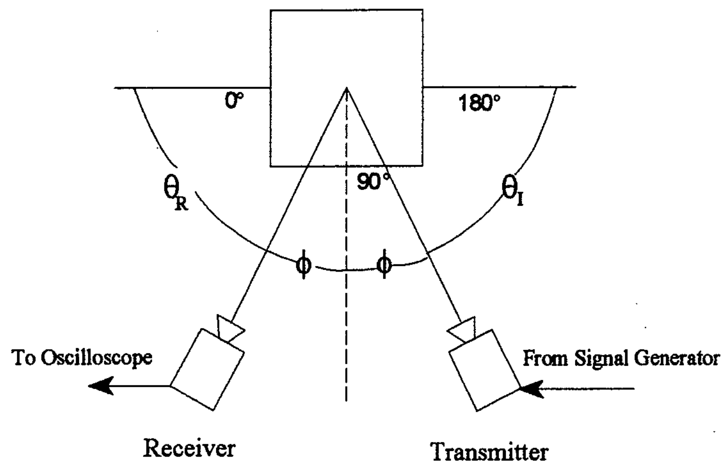

Figure 2. Experimental apparatus layout.

The microwave spectrometer, shown in Fig. 2., consists of a microwave source, receiver, frame to enable angular measurements and a 100 element crystal of metal spheres mounted in a polyethylene cube. The amplitude of the received microwave is observed on an oscilloscope. The wavelength of the generated microwaves is 3cm.

Measure the distance between the centers of two of the ball bearings to the nearest half millimeter. Be sure to include some estimate of your error in gauging the centers of the balls by eye. Record the measurement as $a \pm \Delta a$.

Make eight more measurements of the spacing between different spheres and record the results in a table. Find the average value of a, $\bar{a}$, and the standard deviation, σ, of your data and compare the standard deviation with your previous error estimate. Assuming that the spacing is actually the same between all the spheres, σ should be about $\frac{1}{\sqrt{9}}$ (or ⅓) Δa.

III. BRAGG PEAKS

Mount the source and receiver at 0° and 180° respectively, i.e. opposite to each other; verify that without the cube a signal is observed. Note: the frequency of the signal seen on the oscilloscope is not the frequency of the microwave signal itself, but of the signal imposed by the signal generator. This signal is introduced only for the sake of being able to observe the radiation with the oscilloscope. We will use the oscilloscope readings only for the sake of determining the relative amplitude of the received signal.

Adjust the amplitude of the signal by turning the amplitude knob on the signal generator and see if the pattern on the oscilloscope responds accordingly. Then set the amplitude to the highest setting that retains a good sinusoidal shape and DO NOT ADJUST THE AMPLITUDE FOR THE REST OF THE EXPERIMENT. Now mount the cube so that one face of the cube is perpendicular to the 90° line on the protractor, as shown in Fig. 3. The center of the cube should be over the pivot point of the protractor.

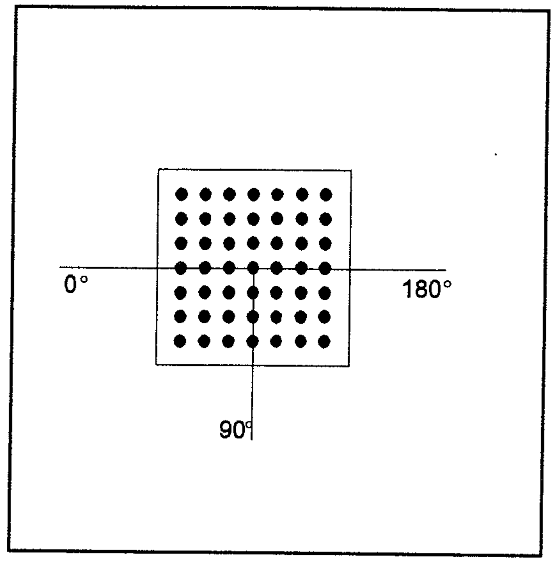

Figure 3. Orientation of the cube for normal incidence.

Q.3. Using Eqn. (2) and your measured value of a, predict the values of θ_i for the peaks in amplitude for this orientation of the cube when n = 1, 2, 3,... What does it mean if $n\lambda > 1$?

When taking measurements, BE SURE TO KEEP $\theta_i = \theta_r$. Starting with $\theta_i = \theta_r = 10°$ (See Fig. 2), record the observed amplitude of the scattered signal as a function of θ in increments of 5° up to 75°. MAKE SURE THAT THE ORIENTATION OF THE CUBE DOES NOT CHANGE AS YOU MOVE THE SOURCE AND RECEIVER. Make a plot of the relative amplitude in arbitrary units as a function of θ. You should notice peaks near the values you calculated for Q.3. Take additional readings for smaller increments of the angle on both sides of the peaks, in order to determine more precisely the experimental value of each maximum. Make sure that you record errors in the values of θ as well as in the amplitude. Use your plot to determine your experimental values of θ_{max}, including an estimate of $\Delta\theta_{max}$ for each peak. Do your experimental values agree within experimental error to those calculated in Q.3?

IV. CRYSTAL STRUCTURE

Rotate the cube by 45° about the vertical axis as shown in Fig. 4.

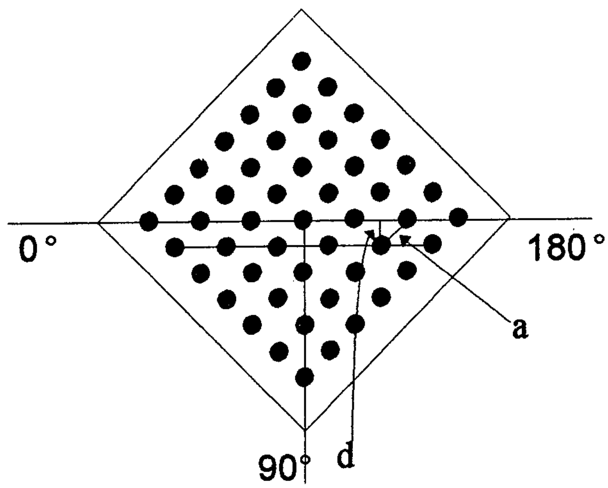

Figure 4. Orientation of the cube for part IV.

Q.4. What is the spacing, d, between the scattering planes for this orientation of the cube?

Q.5. Calculate the angle(s) where a peak should be found for this orientation. Make sure to check for $n > 1$.

As in part III, record the amplitude of the received signal as a function of the incident and reflected angle. Vary the angle from 20° to 50° in increments of 5°, plot your points, and then take more data points at smaller increments of θ near the peak. Use your plot to determine the experimental value of θ_{max} and include your estimate of the error. Does your experimental value agree within experimental error to the value you calculated in Q.5?

V. MORE CRYSTAL STRUCTURE

Orient the cube as in Fig. 5.

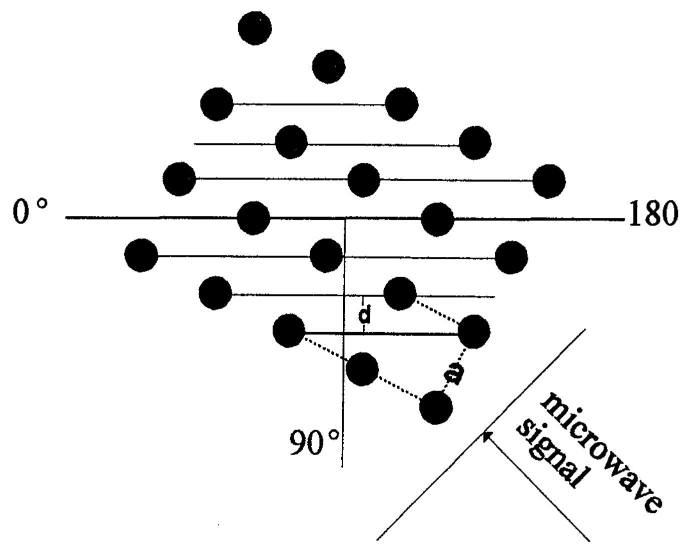

Figure 5. Orientation of the cube for part V.

Q.6. Calculate the distance between scattering planes for this new orientation of the cube.

Q.7. Predict the angle where a peak in the scattered radiation should be observed for this orientation of the cube.

As in the two previous sections, record the amplitude of the scattered radiation as a function of the incident angle. Vary the angle from 40° to 70° using increments of 5°. Plot the points, locate the probable position of the peak and then take more data points near the peak. Use your plot to determine the experimental value of θ_{max} and include your estimate of the error. Does your experimental value agree within experimental error to the value you calculated in Q.7?

VI. MEASUREMENT OF λ.

As was mentioned in the introduction, Bragg scattering can also be used to determine the wavelength of the probing radiation. Using your experimentally determined values for d and θ_{max} from part III, calculate the wavelength of the microwave radiation. Make sure that you perform an error analysis. Does your result agree, within experimental error, with the accepted value of 3 cm?

PRELAB QUESTIONS

Q.1. Lab Questions 1-2.

Q.2. If $d = 4.5$ cm and $\lambda = 1.5$ cm, how many maxima will be observed and at what angles?

Q.3. If the spacing between spheres in a simple cubic lattice is $a = 4.5$ cm, what is the spacing between scattering planes, d, if the cube is rotated by 45°, as in Fig. 4?

Q.4. What is the angle of rotation for the orientation of the cube in Fig. 5 (That is, by how much was the cube rotated from the position in Fig. 3 to obtain the orientation in Fig. 5)?

Q.5. Describe briefly how diffraction through a crystal lattice could be used to isolate a single wavelength from X-rays of many wavelengths. (This is the principle behind an X-ray spectrometer.)