NUCLEAR LIFETIME

In Experiment #4 we emphasized the fact that radioactive decays are by nature a random process, and that this is a result of the quantum mechanical nature of the motion of the particles that make up the nucleus. However, while we cannot predict when a given nucleus will decay, or even by what process, we can measure and sometimes predict (using quantum mechanics) the probability that a certain decay process will occur within a given time period. The time interval over which the nucleus has a 50% chance of undergoing decay is called the half-life, and a measurement of the half-life is a good way of characterizing the instability of a nucleus. The purpose of this lab is to determine the half-life of a certain excited and unstable nucleus, ^{137}Ba.

I. INTRODUCTION

Unstable nuclei decay to other nuclear states with a characteristic time after their formation. The decay does not occur a fixed time after formation, but is distributed over a wide range of times. Quantum mechanics allows knowledge only of the probability that a state will decay, not the exact time of the state's demise.

The probability that any state will decay in a certain unit of time is described by the decay constant, r. If there are N(t) unstable nuclei at time t, then r represents the fraction of existing nuclei that will decay within a short time interval Δt after t.. The decay constant is independent of when the nucleus was formed, i.e. the nucleus does not know about its past. If we have N(t) nuclei at time t, then the rate at which nuclei will decay, R(t), is

$$R(t) = r\,N(t) \quad (1)$$

R is the rate of decrease of N:

$$R = -\,dN/dt. \quad (2)$$

So,

$$dN/dt = -\,r\,N. \quad (3)$$

The solution is an exponential since the rate of decay of N is proportional to N:

$$N(t) = N_o e^{-rt} \quad (4)$$

There are two common ways to describe the lifetime of the nucleus. The mean life τ is defined as the time in which N drops to 1/e its original value. It is inversely proportional to the decay rate r.

$$\tau = 1/r. \qquad (5)$$

The half-life, $t_{1/2}$, is the time in which the number of nuclei decreases to half its original value:

$$N(t_{1/2}) = N_0 e^{-t_{½}} = ½ N_0 \qquad (6)$$

$$-t/\tau = \ln(1/2), \text{ or} \qquad (7)$$

$$t_{1/2} = \tau \ln 2. \qquad (8)$$

An important conclusion is that an absolute t = 0 time is not defined. Half of the existing nuclei at time t will decay during a period t to $t + t_{1/2}$, independent of when the measurement was started.

Graphs of Eqn. (4) are shown in Fig. 1. Figure 1a. is a plot on linear graph paper. The same data is shown plotted on semi-$\log_{10}$ graph paper in Fig. 1b. The points in the second graph fall on a straight line whose intercept is N_0 and whose slope is

$$-\frac{\log_{10}e}{\tau} = \frac{\log_{10}N(t_2) - \log_{10}N(t_1)}{(t_2 - t_1)}. \qquad (9)$$

The equation of the line is then:

$$\log_{10}N(t) = \log_{10}N_0 - \frac{t}{\tau}\log_{10}e. \qquad (10)$$

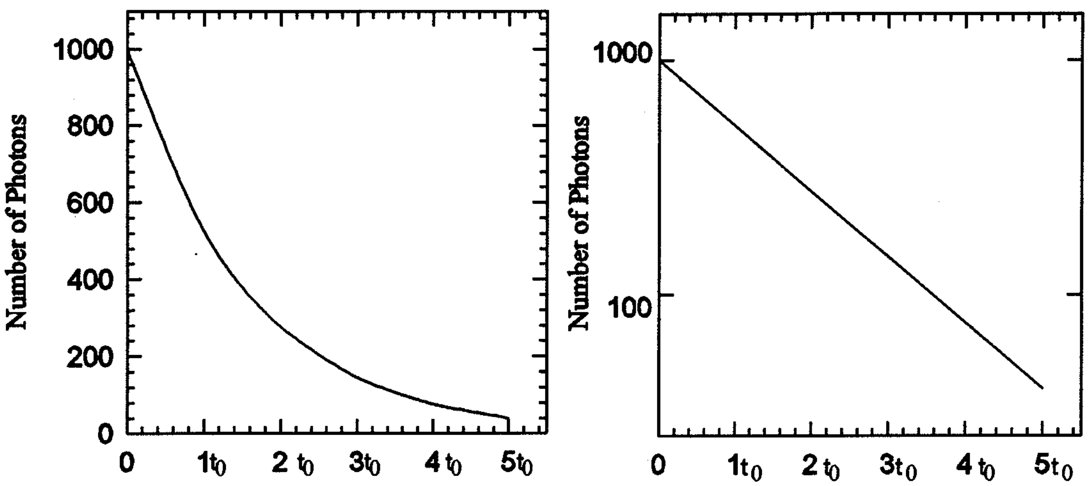

Figure 1. Number of nuclear decays versus time. (a) Plotted on linear and (b) plotted on semi-log graph paper.

In order to measure the lifetime of an excited nucleus, a significant percentage of the nuclei must decay during the time of observation: our lab period. From Eqn. (11) below, we can show that a nucleus with a long lifetime, e.g. 10 years = 8.8×10^5 hours, would only have 1.1×10^{-4}% decay in 1 hour.

$$\frac{N_0 - N(t)}{N_0} = 1 - e^{-\frac{t}{\tau}}. \qquad (11)$$

This would be exceedingly difficult to measure, so a nucleus with a short half-life is used in this lab. A parent nucleus, ^{137}Cs, is used to generate the daughter nucleus ^{137}Ba, which is in an excited and unstable state. The decay of the unstable ^{137}Ba nucleus releases a 0.66 MeV

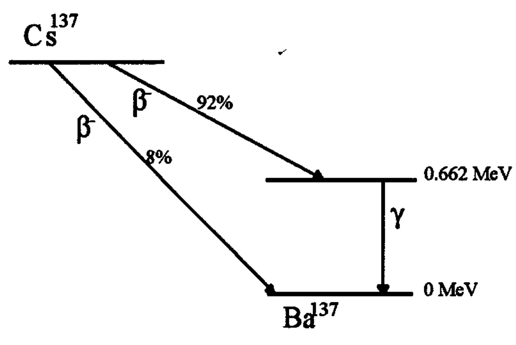

Figure 2. Decay scheme of ^{137}Cs to ^{137}Ba.

photon as shown in Fig. 2. The half-life of ^{137}Cs is 30 years whereas the half-life of ^{137}Ba is 2.6 minutes.

II. WARNING

The primary radiative source of ^{137}Cs is contained in a bottle and is not sealed as in other experiments. Cesium decays to Barium inside a source bottle. The Barium is then chemically extracted with a HCI and NaCI solution and stored in a separate container. DO NOT OPEN ANY CONTAINERS to avoid contamination. THE LAB INSTRUCTOR WILL HANDLE THE Ba GENERATOR KIT and provide you with a sealed Ba sample. Because these are not permanently sealed samples, you must wear plastic gloves and aprons.

Unfortunately, the extraction process is not 100% efficient and your Ba sample will most likely contain some additional Cs nuclei. You will have to include the Cs decay as part of the background radiation count in your data analysis. In addition, the extraction process requires a sufficiently long period of time that it may be necessary to work with larger groups. You may have to wait some time for your sample. But you must be prepared to take data as soon as the sample is ready.

III. DETERMINATION OF ^{137}Ba HALF-LIFE

You will use the Geiger counter as in Experiment #4 in order to count the number of photons emitted from the Ba^{137} nucleus as it de-excites to the ground state. First, you must set up the Geiger tube and plateau the counter, using a $T\ell^{204}$ source. Next, determine the background count with no radioactive source nearby for 2 minutes, and divide by four. This is your background rate for a 30 second time interval. Be sure to determine the statistical error of your background rate. Next, obtain a Ba^{137} sample from the lab instructor and IMMEDIATELY begin taking <u>consecutive</u> readings of the number of counts emitted from the sample <u>every 30 seconds</u>. You should take counts for at least 7 minutes, and continue recording the number of decays until the number becomes essentially constant for 4 consecutive time intervals. This is your actual background rate which may be different from the background rate which you measured with no sources present. Estimate the error in your actual background rate.

In a table, calculate the counts per time interval and the statistical error for each count, assuming a normal distribution. Then determine the corrected number of counts by subtracting the actual background rate. The error in the corrected number of counts should be calculated using the formula in Appendix A for the difference between two quantities.

Plot the number of decays per time interval on a semi-$\log_{10}$ plot. Include your calculated error bars; if you use regular graph paper, use the formula for error in logarithms given in Appendix B. Determine the mean life and half-life from your data and record them using the proper units. Estimate a range of lifetimes which could fit the data points and report this as an estimate of the statistical error.

Q.1 Is your result for the half-life of ^{137}Ba consistent with the accepted value?

Q.2 Discuss any systematic errors which could affect your result.

PRELAB QUESTIONS

Q.1 Show that Eqn. (4) satisfies Eqn. (1).

Q.2 How would Eqns. (9) and (10) change if we used $\log_e$ instead of $\log_{10}$?

Q.3 If 40% of an unstable substance remained after 10 hours, what is the half-life of that substance?

Q.4 What is the probability that a single nucleus which still remains in the sample from Q.3 will decay within the next 1 hour?

Q.5 There are two decay processes in a sample, a primary one with a half-life of 5 minutes and a residual one of 30 counts per 30 sec which is essentially constant over the lab period. The initial number of counts is 300. What is the apparent half life of the primary decay process if we fail to correct for the contamination?