BETA DECAY OF NUCLEI

The purpose of this experiment is to investigate the beta decay of nuclei by studying the beta particles emitted by $^{204}_{81}T\ell$. The experiment consists of measuring the momentum distribution of the emitted betas using a simple magnetic spectrometer. Analysis of the data will indicate that beta decay involves three particles in the final state, and that the third particle - the electron neutrino - has essentially zero rest mass. In addition, analysis of the data will show the need for relativistic kinematics.

INTRODUCTION

I. THE NATURE OF THE BETA-DECAY PROCESS

Soon after Henri Becquerel's discovery of natural radioactivity in 1896, it was found that three different types of particles can be emitted in this process: alpha (α) rays were found to be helium nuclei; beta (β) rays were found to be negative or positive electrons; and gamma (γ) rays were found to be electromagnetic radiation.

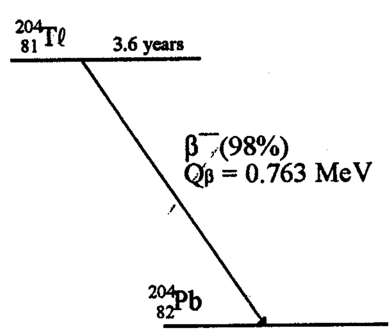

Figure 1. Energy-level diagram for the beta decay of $^{204}_{81}T\ell$ to $^{204}_{82}Pb$.

In beta decay, the charge of the nucleus changes by one unit, but the total number of nucleons does not change. Originally, it was thought that the negative beta decay of a parent (P) nucleus to a daughter (D) nucleus consisted of the reaction

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e, \tag{1}$$

in which the upper indices denote nucleon number and the lower denote charge. A schematic representation of such a decay, the beta decay of $^{204}_{81}T\ell$ to $^{204}_{82}Pb$, is shown in Fig. 1.

In a decay such as indicated in Eqn. (1), in which a single particle initially at rest decays into two particles in the final state, each of the two particles emerges with a unique value of energy and momentum. That is, the requirements of energy conservation and momentum conservation, together with the kinematic relation connecting energy and momentum, provide two equations in two unknowns (the momentum of each of the two final-state particles, for example). These equations have, of course, a single solution for each of the unknowns. Thus, an experiment measuring the momentum of the betas (electrons) emitted in the process described by Eq. (1) should produce the results shown in Fig. 2. All betas should have the same momentum, except for a small smearing due to the finite precision of the experimental apparatus.

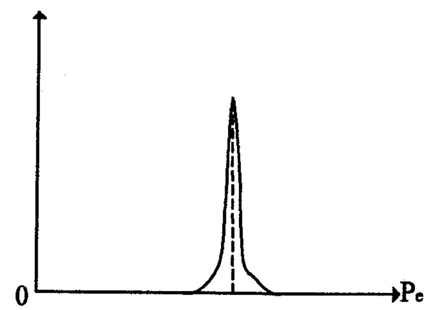

Figure 2. Beta momentum spectrum for a two-body final state.

However, when such experiments were performed, the distribution of beta momentum was found to be as shown in Fig. 3. Instead of all emitted betas having the same momentum, it was discovered that the betas exhibited a <u>continuous spectrum</u> over a range of momentum from zero up to a maximum value, $P_{e\,max}$, that was characteristic of the particular decay under investigation.

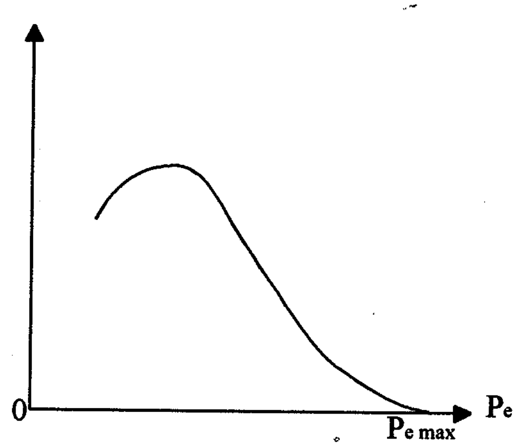

Figure 3. Experimentally determined electron momentum spectrum in beta decay.

This unexpected result could be explained in one of two ways: either 1) energy and/or momentum conservation was invalid in beta decay, or 2) beta decay was not a two-body process as indicated in Eqn. (1). Additional work eventually showed that the second conclusion was correct. It is now known that beta decay produces a <u>three-body final state</u>, and that negative beta decay consists of the reaction

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e + \bar{\nu}_e. \qquad (2)$$

The third body is a member of a family of particles known as <u>neutrinos</u>. (The particle denoted by the symbol $\bar{\nu}_e$ is called the electron anti-neutrino; the electron neutrino, ν_e, is emitted in positive beta decay; other types of neutrinos are emitted in the weak decays of certain elementary particles.) The fact that beta decay was thought to be a two-body process resulted from the elusive nature of neutrinos: they have zero charge and zero or near zero rest mass, yet carry energy and momentum, and they interact so weakly with matter that they were not detected directly until 1953.

The continuous nature of the beta spectrum, as shown in Fig. 3, is thus understood in terms of the three-body final state. The total energy available for the decay is shared between the three particles, and all three together serve to conserve momentum. As a consequence, there is no unique solution for the momentum and energy of any of the three bodies; a range from zero up to some maximum value is possible, and the values for any particular decay depend upon the angles at which the three particles are emitted.

Three of the infinite number of possibilities are shown in Fig. 4. The general case, in which the three particles are emitted in arbitrary directions, is shown in Fig. 4(a); for this situation the electron momentum is somewhere in the middle region of the spectrum shown in Fig. 3, and all three momentum vectors are needed to satisfy momentum conservation from rest, $0 = \vec{p}_D + \vec{p}_e + \vec{p}_\nu$.

The special case in which the daughter and neutrino momenta are almost equal and opposite is shown in Fig. 4(b). Here, $\vec{p}_D$ and $\vec{p}_\nu$ are almost sufficient to satisfy momentum conservation, and so $\vec{p}_e$ is quite small in magnitude; in this situation, the electron momentum falls near the low end of the spectrum, and the electron has very little kinetic energy. The opposite extreme is shown in Fig. 4(c), where $\vec{p}_D$ and $\vec{p}_e$ are almost sufficient to satisfy momentum conservation and so $\vec{p}_\nu$ is quite small. Here, the electron has essentially its maximum possible value of momentum, $|\vec{p}_e| \approx p_{e\,max}$ (and almost its maximum kinetic energy), which corresponds to the upper end of the momentum spectrum.

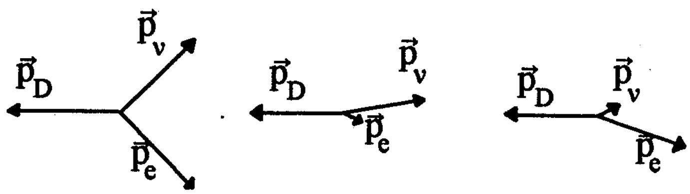

Figure 4. Three possible final states in beta decay.

II. ENERGY CONSIDERATIONS: THE NEUTRINO MASS

The fact that neutrinos have zero rest mass is determined, of course, from experiment. If the neutrino is assumed to have a rest mass m_v, then application of total energy conservation to the beta-decay process of Eqn. (2) gives

$$[M_P c^2 - Zm_e c^2] = [M_D c^2 - (Z+1)m_e c^2 + K_D] + [m_e c^2 + K_e] + [m_v c^2 + K_v]. \quad (3)$$

Here, M_P and M_D are the atomic (not nuclear) rest masses of the parent and daughter, respectively, m_e is the electron rest mass, and K_D, K_e and K_v are the daughter, electron and neutrino kinetic energies.

The total energy release, or Q-value, for this process is then

$$Q = K_D + K_e + K_v = [M_P c^2 - (M_D c^2 + m_v c^2)] \ . \quad (4)$$

Now, m_v is known to be very small, at best, in comparison with M_P and M_D, and so Q is well-approximated by the difference, $M_P c^2 - M_d c^2$. Typically, this rest-mass energy difference is of the order of one MeV, which is shared between the three final-state particles. Since the rest-mass energy of the daughter is very much greater than 1 MeV, the daughter always moves with speeds very much smaller than c, even if the entire energy released were acquired by the daughter; thus, the daughter can be treated using non-relativistic relations. On the other hand, the rest-mass energies of the beta and the neutrino are not large compared with 1 MeV, and so they must be treated relativistically.

Furthermore, it is easy to show that, because of the large rest-mass energy of the daughter compared to both the electron rest-mass energy and the energy released, the energy acquired by the daughter is negligible. Consider a decay of the type shown in Fig. 4(c), where the decay energy is shared almost entirely between the daughter and the beta. Here, momentum conservation takes the form

$$O \approx \vec{p}_D + \vec{p}_e \, .$$

or

$$p_D \approx p_e \, . \tag{5}$$

Combining this result with the relation

$$K_D = p_D^2/2M_D \tag{6}$$

for the daughter, and with

$$(K_e + m_e c^2)^2 = p_e^2 c^2 + (m_e c^2)^2 \tag{7}$$

for the beta, leads to the result

$$K_D \approx \frac{K_e(K_e + 2m_e c^2)}{2M_D c^2} \, . \tag{8}$$

Since the largest possible value of K_e is Q (see Eq. (4)), then

$$K_D^{max} \approx \frac{Q(Q + 2m_e c^2)}{2M_D c^2} \ll Q. \tag{9}$$

This result may be used to determine an upper limit for the neutrino rest mass. Since the daughter's kinetic energy is negligible, Eq. (4) takes the form

$$K_e + K_\nu \approx [M_P c^2 - (M_D c^2 + m_\nu c^2)] . \qquad (10)$$

Now, in decays of the type shown in Fig. 4(c), essentially all of the decay energy is carried away by the beta which has a momentum near $p_{e\,max}$ and has essentially its maximum kinetic energy, $K_{e\,max}$; the neutrino energy, K_ν, is essentially zero. Hence, Eq. (10) yields

$$m_\nu c^2 \leq (M_P c^2 - M_D c^2) - K_{e\,max} \qquad (11)$$

Thus, in a given beta decay, a measurement of $K_{e\,max}$ serves to place an upper limit on the neutrino rest mass.

III. MEASURING THE BETA MOMENTUM SPECTRUM

The momentum of charged particles is usually determined by measuring their deflections in a known uniform magnetic field. The special device using this principle to determine the momentum spectrum of the electrons emitted in beta decay is called a beta-ray spectrometer, and its basic features are shown in Fig. 5. A beam of electrons from a beta source enters a region of uniform magnetic field. Electrons of different momenta follow circles of different radii, and the counting rate at various distances D = 2R is measured. This determines the momentum spectrum since, from Newton's second law and the Lorentz force law, it follows that

$$p_e = e|\vec{B}|R, \qquad (12)$$

a result valid under relativistic, as well as non-relativistic, conditions. Thus, the data of counting rate as a function of distance is easily converted to the momentum spectrum, $\Delta N/\Delta P_e$ vs. p_e.

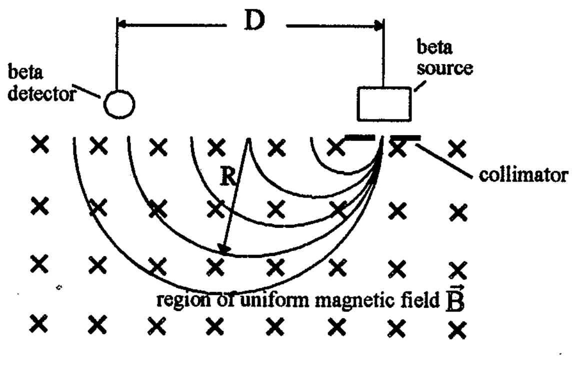

Figure 5. Beta-ray spectrometer.

THE EXPERIMENT

I. WARNING

A very strong permanent magnet is used in this experiment. If you wear a watch, be sure to keep it far away from the magnet; otherwise, it may become magnetized and cease functioning properly. In addition, avoid disturbing the steel pole pieces - the rectangular blocks attached to the magnet, and used to provide a uniform field (see Fig. 6); if handled improperly they may snap shut violently, perhaps catching a finger between them. For similar reasons, keep ferromagnetic materials away from the magnet.

II. APPARATUS

The rudimentary beta-ray spectrometer used in this experiment is shown in Fig. 6. It consists of a strong permanent magnet, two pole pieces to provide a region of uniform field, a $^{204}_{81}T\ell$ source mounted on a movable aluminum bracket containing a small hole which acts as a collimator, a Geiger-Müller-tube electron detector, and an aluminum bracket with a small slot which is located under the detector window in order to collimate the entering betas.

III. PROCEDURE

Check to see that the counting circuit is connected correctly, and that the counter controls are set properly. Check out a $^{204}_{81}T\ell$ source. Place it 5~10cm from the Geiger-Müller tube, and measure and tabulate the counting rate as a function of high voltage. Plot the data, including error bars, and determine the proper operating voltage.

With the source removed, determine the background rate by counting for 2 min. Calculate the background rate and its uncertainty per 30 sec.

Use a small compass (whose poles have been checked using the known North direction) to determine the polarity of your spectrometer magnet. Measure the strength of the B field between the pole pieces using a Gaussmeter. Record this information. The field strength should be between .1 and .135 Tesla in order for the full momentum spectrum to be seen.

Assemble the spectrometer and source as shown in Fig. 6. Determine the sign of the emitted beta particles. Record your information together with an explanation.

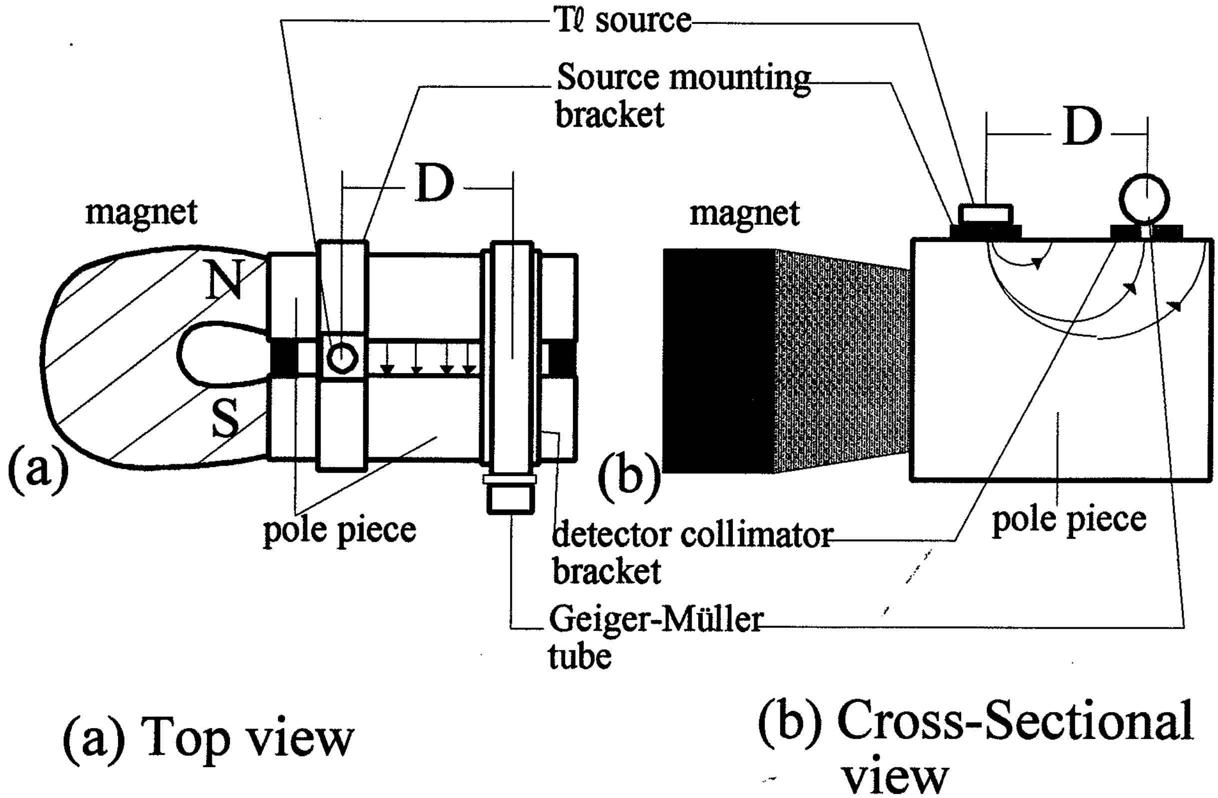

Figure 6. Experimental apparatus. Note: the orientation of the magnet may not be the same as in the sketch

IV. THE BETA MOMENTUM SPECTRUM

Use the spectrometer to determine the spectrum of the emitted betas. It is generally more convenient to leave the Geiger-Müller tube and its collimator fixed near the ends of the pole pieces, and to move the source bracket to vary the center-of-source to center-of-counter distance, D.

Begin counting at the smallest possible value of D. Measure and record the counts per 60 sec twice. Determine the spectrum by moving the source away in 0.5 cm steps, and taking two 60-sec readings at each position. Continue data taking until the counting rate has fallen to the background level. SINCE THE END-POINT (i.e., MAXIMUM BETA MOMENTUM) OF THE SPECTRUM IS IMPORTANT, go back and take additional data at closely spaced D settings (say, 1mm or 2mm apart) in the region just before and just after the counting rate diminishes to the background value.

Record your data in a table containing the following columns: 1) D; 2) R; 3) raw counts for each of the two trials; 4) background; 5) corrected counts for each of the two trials; 6) average corrected counts per 60 sec. Be sure to include the uncertainties of all quantities. Calculate the uncertainty in the corrected number of counts as in Experiment #5. Don't forget to take into account the fact that you made two trials!

Plot a graph of the average corrected counting rate vs. R, including error bars. Since R is directly proportional to p_e (see Eqn. (12)), this graph has the same shape as the beta momentum spectrum, dN/dp_e vs. p_e. From your graph, determine the maximum value of the radius, R_{max}, which corresponds to the end-point of the beta spectrum. Estimate the uncertainty of this quantity.

Use the value of R_{max} to calculate the maximum beta momentum, $p_{e\,max}$, (Eqn. (12)). Include the uncertainty of this quantity. Using the value of $p_{e\,max}$, determine the maximum beta kinetic energy, $K_{e\,max}$, and its uncertainty (see Q.6).

V. THE UPPER-LIMIT OF THE NEUTRINO REST-MASS

Finally, use your data to set an upper limit on the rest-mass energy of the neutrino (see Eq. (11)). Express your result as a multiple of the electron rest-mass energy.

PRELAB STUDY AND QUESTIONS

This experiment involves the beta decay of ${}^{204}_{81}T\ell$ to ${}^{204}_{82}Pb$ via the reaction

$$ {}^{204}_{81}T\ell \rightarrow {}^{204}_{82}Pb + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}_e . \qquad (13)$$

A table of rest masses and equivalent energies for particles related to this decay is included below.

Particle	Rest Mass (μ)	Rest Energy (MeV)
${}^{0}_{\pm1}e$	0.000549	0.511
${}^{1}_{0}n$	1.008665	939.550
${}^{1}_{1}p$	1.007276	938.256
${}^{204}_{81}T\ell$ (atom)	203.97386	189 997.166
${}^{204}_{82}Pb$ (atom)	203.973044	189 996.403

Q.1. Determine the energy released in the decay. Where does the energy come from?

Q.2. Derive Eqn. (4) from Eqn. (3). Where did the electron rest mass go?

Q.3. Verify by numerical calculation the statement that the ${}^{204}_{82}Pb$ daughter can be treated non-relativistically. Use the criterion that $\beta = v/c = pc/E_{tot} < .001$ is non-relativistic.

Q.4. Derive Eqn. (8), beginning from Eqn. (5) and including all required steps. What are the assumptions in Eqns. (5&6)?

Q.5. Find the largest possible value for the kinetic energy of the ${}^{204}_{82}Pb$ nucleus, and verify that this quantity is negligible in comparison with the beta and neutrino kinetic energies.

Q.6. Determine an expression for the maximum beta kinetic energy, $K_{e\ max}$, in terms of the maximum momentum, $p_{e\ max}$, and the rest-mass energy, $m_e c^2$. Did you use the relativistic energy equation in your calculation?

Q.7. Starting from Newton's second law, the criterion for uniform circular motion and the Lorentz force law, derive Eqn. (12).

Q.8. Why will the device shown in Fig. 6 work for beta rays, but not for gamma rays (photons)?