

**FIGURE 1.11**

As seen in the boy's frame  $S$ , the two hatchets bounce simultaneously (at  $t = 0$ ) 100 cm apart. Since the snake is 80 cm long, it escapes injury.

As observed in frame  $S$ , the two hatchets bounce simultaneously at  $t = 0$ . At this time the snake's tail is at  $x = 0$  and his head must therefore be at  $x = 80$  cm. [You can check this using the transformation  $x' = \gamma(x - vt)$ ; with  $x = 80$  cm and  $t = 0$ , you will find that  $x' = 100$  cm, as required.] Thus, as observed in  $S$ , the experiment is as shown in Fig. 1.11. In particular, the boy's prediction is correct and the snake is unharmed. Therefore, the snake's argument must be wrong.

To understand what is wrong with the snake's argument, we must examine the coordinates, especially the times, at which the two hatchets bounce, as observed in the frame  $S'$ . The left hatchet falls at  $t_L = 0$  and  $x_L = 0$ . According to the Lorentz transformation (1.37), the coordinates of this event, as seen in  $S'$ , are

$$t'_L = \gamma \left( t_L - \frac{vx_L}{c^2} \right) = 0$$

and

$$x'_L = \gamma(x_L - vt_L) = 0$$

As expected, the left hatchet falls immediately beside the snake's tail, at time  $t'_L = 0$ , as shown in Fig. 1.12(a).

On the other hand, the right hatchet falls at  $t_R = 0$  and  $x_R = 100$  cm. Thus, as seen in  $S'$ , it falls at a time given by the Lorentz transformation as

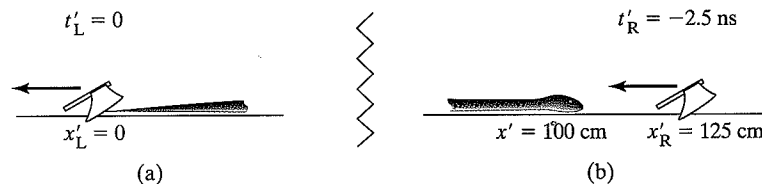
$$t'_R = \gamma \left( t_R - \frac{vx_R}{c^2} \right) = \frac{5}{4} \left( 0 - \frac{(0.6c) \times (100 \text{ cm})}{c^2} \right) = -2.5 \text{ ns}$$

We see that, as measured in  $S'$ , the two hatchets *do not fall simultaneously*. Since the right hatchet falls before the left one, it does not necessarily have to hit the snake, even though they were only 80 cm apart (in this frame). In fact, the position at which the right hatchet falls is given by the Lorentz transformation as

$$x'_R = \gamma(x_R - vt_R) = \frac{5}{4}(100 \text{ cm} - 0) = 125 \text{ cm}$$

and, indeed, the hatchet misses the snake, as shown in Fig. 1.12(b).

The resolution of this paradox and many similar paradoxes is seen to be that two events which are simultaneous as observed in one frame are not necessarily simultaneous when observed in a different frame. As soon as one recognizes that the two hatchets fall at different times in the snake's rest frame, there is no longer any difficulty understanding how they can both miss the snake.



**FIGURE 1.12**

As observed in  $S'$ , both hatchets are moving to the left. The right hatchet falls before the left one, and even though the hatchets are only 80 cm apart, this lets them fall at positions that are 125 cm apart.