→ 5-JAN-92

PASSAGE OF PARTICLES THROUGH MATTER

Nikos Varelas
Notes
3/8/04 (Phy482)

1. The PHOTOELECTRIC EFFECT

a) Non Relativistic QFT (Born Approx.)

Cross section per atom for photo e⁻ from K shell

$$\sigma^{T_K} = f(\xi)\sqrt{32}\,\sigma_{Th}\,\alpha^4 Z^5 \left[\frac{m_e c^2}{E_\gamma}\right]^{7/2}$$

$$= f(\xi)\,1.02\,10^{-33}\,Z^5 E_\gamma^{-7/2}\ \mathrm{cm}^2/\mathrm{atom}$$

where

$$f(\xi) = 2\pi\sqrt{\frac{(Be)_i}{E_\gamma}}\ \frac{\exp(-4\xi\cot^{-1}\xi)}{1-\exp(-2\pi\xi)}$$

$$\xi = \sqrt{\frac{(Be)_i}{E_\gamma-(Be)_i}}\ ,\quad (Be)_i = \frac{1}{s_i}R_y(Z-S_i)^2$$

$$R_y = \frac{m_e e^4}{2\hbar^3} \simeq 13.61\ \mathrm{eV}$$

$$\sigma_{Th} = \frac{8}{3}\pi r_e^2$$

$$r_e = \frac{e^2}{m_e c^2}$$

$S_K = 1$, $s_K = 1$
$S_L = 5$, $s_L = 4$
$S_M = 13$, $s_M = 9$

b) Relativistic QFT for Z<< or E_γ>>

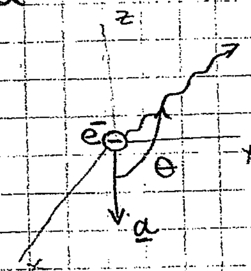
$$\sigma^{T_K} = \frac{3}{2}\sigma_{Th}\,\alpha^4 Z^5 \frac{m_e c^2}{E_\gamma}$$

$$= 1.45\,10^{-33}\,Z^5 E_\gamma^{-1}\ \mathrm{cm}^2/\mathrm{atom}$$

2. THOMSON SCATTERING

Power emitted from the e^-

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\underline{a}|^2 \sin^2\theta$$

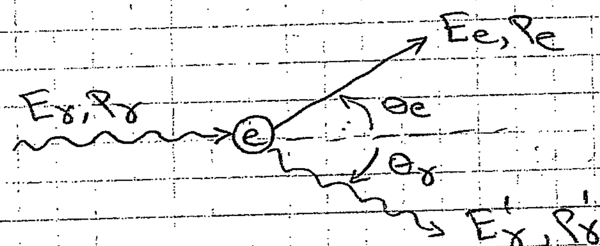


$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{c}{8\pi} \frac{e^4}{m_e^2 c^4} E_0^2 \sin^2\theta$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{m_e c^2} \right)^2 \sin^2\theta$$

$$\sigma_{Th} = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 0.665\ 10^{-24}\ cm^2$$

3. COMPTON SCATTERING



α) $\Delta\lambda \equiv \lambda' - \lambda = \dfrac{1}{m_e c}(1 - \cos\theta_\gamma)$

β) $E'_\gamma = E_\gamma \dfrac{1}{1 + \varepsilon(1 - \cos\theta_\gamma)}$

$$\varepsilon = \frac{E_\gamma}{m_e c^2}$$

γ) $T_e = E_\gamma - E'_\gamma$

$$T_e = E_\gamma \frac{\varepsilon(1 - \cos\theta_\gamma)}{1 + \varepsilon(1 - \cos\theta_\gamma)}$$

δ) $\cot\theta_e = (1+\varepsilon)\tan\frac{\theta_\gamma}{2}$

per e $\leftarrow$ $$\frac{d_e\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left(\frac{E'_\gamma}{E_\gamma}\right)^2 \left(\frac{E_\gamma}{E'_\gamma} + \frac{E'_\gamma}{E_\gamma} - \sin^2\theta_\gamma\right)$$

Klein - Nishina Cross Section

$$_e\sigma = 2\pi r_e^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2}\left[\frac{2(1+\varepsilon)}{1+2\varepsilon} - \frac{1}{\varepsilon}\ln(1+2\varepsilon)\right] + \frac{1}{2\varepsilon}\ln(1+2\varepsilon) - \frac{1+3\varepsilon}{(1+2\varepsilon)^2}\right\}$$

4. PAIR PRODUCTION

Bethe - Heitler (Born Appr.)

$$\gamma = 100\,\frac{m_e c^2 E_\gamma}{E_+ E_- Z^{1/3}}$$

for $15 > \gamma > 2$ (low e^- E)

$$\frac{d_e\kappa}{dE_+} = \frac{4\bar{\Phi}}{E_\gamma^3}\left(E_+^2 + E_-^2 + \frac{1}{3}E_+E_-\right) \cdot \left[\ln\frac{2E_+E_-}{E_\gamma m_e c^2} - \frac{1}{2} - c(\gamma)\right]$$

for $2 > \gamma > 0$ (for higher e^- E)

$$\frac{d_e\kappa}{dE_+} = \frac{\bar{\Phi}}{E_\gamma^2}\left\{(E_+^2 + E_-^2)\left[\phi_1(\gamma) - \frac{4}{3}\ln Z\right] + \frac{2}{3}E_+E_-\left[\phi_2(\gamma) - \frac{4}{3}\ln Z\right]\right\}$$

for $\gamma \sim 0$ ($\bar{e}$ E >>)

$$\frac{d_e k}{dE_+} = \frac{4\bar{\Phi}}{E_\gamma^3}\left[\left(E_+^2 + E_-^2 + \frac{2}{3}E_+E_-\right)\ln\left(\frac{183}{Z^{1/3}}\right) - \frac{1}{9}E_+E_-\right]$$

where $\bar{\Phi} = \frac{Z^2 r_e^2}{137}$, $\phi_1(\gamma)$, $\phi_2(\gamma)$ and $c(\gamma)$ functions of γ

For $E_\pm$ >> $\langle\Theta_{e^\pm}\rangle \sim \frac{m_e c^2}{E_\pm}$

If we ignore the screening effect from the atomic $\bar{e}$s we get

$$\alpha^k = \bar{\Phi}\left[\frac{28}{9}\ln\left(\frac{2E_\gamma}{m_e c^2} - \frac{218}{27}\right)\right] \quad E_\gamma >>$$

For full nucleus screening ($\gamma \sim 0$) we get

$$\alpha^k = \bar{\Phi}\left[\frac{28}{9}\ln\left(183 Z^{-1/3}\right) - \frac{2}{27}\right] \quad E_\gamma >>>$$

5. IONIZATION $\frac{dE}{dx}$

e^-s through water Bethe-Bloch

$$-\left\langle \frac{dE}{dx} \right\rangle = 0.153 \frac{\rho}{\beta^2} \frac{Z}{A} \left[\ln \frac{E(E+m_ec^2)\beta^2}{2I^2 m_e c^2} \right.$$

$$+ (1-\beta^2) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)\ln 2 +$$

$$\left. \frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2 - \Delta_{pol} \right]$$

For charged particles ze $m > m_e$

$$-\frac{dE}{dx} = \frac{4\pi r_e^2 m_e c^2 N_0 Z z^2}{A\beta^2}\left[\ln\left(\frac{2m_ec^2\beta^2}{(1-\beta^2)I}\right) - \beta^2\right]$$

6. BREMSSTRAHLUNG

$$P_{total} \equiv \frac{dE}{dt} = \int \frac{dP}{d\Omega} d\Omega = \frac{2}{3} \frac{Z_1^2 e^2}{c^3} |\dot{v}|^2$$

$$\frac{dE}{dt} \propto \frac{Z_1^4 Z_2^2}{M_1^2}$$ M_1, Z_1 particle

Z_2 material

$$\frac{d\sigma_B}{dE_\gamma d\Omega_e d\Omega_\gamma} = \frac{V^3}{(2\pi)^5 (\hbar c)^7} \frac{p_f}{p_i} E_i E_f E_\gamma^2 |\langle f|V_B|i\rangle|^2$$

γ angular distribution for high energies

$$\Theta \sim \frac{m_e c^2}{E_i}$$

$$\frac{d\sigma_B}{dE_\gamma} = \int \frac{d\sigma_B}{dE_\gamma d\Omega_e d\Omega_\gamma} d\Omega_e d\Omega_\gamma$$

Bethe-Heitler

$$\frac{d\sigma_B}{dE_\gamma} = \frac{\alpha Z_2^2 r_e^2}{E_\gamma}\left(1 + \frac{E_f^2}{E_i^2}\right)\left\{\left[\phi_1(\xi) - 4\ln(Z_2^{-1/3})\right] - \frac{2}{3}\frac{E_f}{E_i}\left[\phi_2(\xi) - 4\ln(Z_2^{-1/3})\right]\right\}$$

where $\xi = \frac{m_e c^2 E_\gamma}{E_i E_f} Z_2^{-1/3}$

$$\phi_1(0) = 4\ln(183) \quad , \quad \phi_2(0) = 4\ln(183) - \frac{2}{3}$$

For $m_ec^2 \ll T_i \ll 137 m_ec^2 Z_2^{-1/3}$ and ignoring the screening effects

$$\sigma_{rad} \equiv \int_0^{T_i} \frac{E_\gamma}{E_i} \frac{d\sigma_B}{dE_\gamma} dE_\gamma$$

$$= 8\alpha Z_2^2 r_e^2 \left[\ln\left(\frac{E_i}{m_ec^2}\right) - \frac{1}{6} \right]$$

For $T_i \gg 137 m_ec^2 Z_2^{-1/3}$ and with screening

$$\sigma_{rad} = 4\alpha Z_2^2 r_e^2 \left[\ln\left(183 Z_2^{-1/3}\right) + \frac{1}{18} \right]$$

$$-\frac{dE_i}{dx} = N \int_0^{E_{\gamma max}} E_\gamma \frac{d\sigma_B}{dE_\gamma} dE_\gamma$$

For $T_i \gg 137 m_ec^2 Z_2^{-1/3}$

$$\frac{dE_i}{dx} = E_i 4\alpha \frac{N_0}{A} Z_2^2 r_e^2 \left[\ln\left(183 Z_2^{-1/3}\right) + \frac{1}{18} \right] \equiv \frac{E_i}{X_0}$$

$$R = \left.\frac{\left(-\frac{dE_i}{dx}\right)_{Br}}{\left(-\frac{dE_i}{dx}\right)_{ion}}\right|_{e^-} \sim \frac{Z_2 \overset{\text{MeV}}{E_i}}{550}$$

$$E_c \sim \frac{550}{Z} \text{ MeV}$$

For thick target the Brem spectrum is given by:

$$\frac{d\sigma_B}{dE_\gamma} = K\left[Z_2 (E_{\gamma max} - E_\gamma) + 0.0025 Z_2^2 \right]$$