Short course on *Symmetry Studies*
TU Eindhoven, Maart 2005
Marlos Viana

In this lab we will discuss a symmetry study to explore data indexed by the symmetries of a regular triangle (S_3). Note that this is the case in which the structure is in itself a group. All calculations are simple enough and can be done by pencil and paper. If you prefer, however, you may utilize the MAPLE worksheet indicated below. In that case, first save the worksheet under a different file name before modifying it.

Veel plezier!

(1) Choosing a group of symmetries: Identify and interpret the symmetries of S_3 and its multiplication table (p.37) - Page and reference numbers refer to the lecture notes;
(2) Choosing a group action: Let S_3 act on itself by conjugacy, that is,
$$\varphi(\tau, \sigma) = \tau\sigma\tau^{-1}.$$
(3) Show that φ is a group action (p. 43);
(4) Construct a table similar to those of Example 2.5.1 p. 44, describing the group action defined above; see also Exercise 2.1 p. 108. See also Matrix 2.2 p. 34;
(5) Identify and interpret the orbits;
(6) Illustrate the proof of Burnside Lemma by counting the number of fixed points and the size of the isotropy groups (stabilizers);
(7) Use Routine 2.1 (p. 113- available in your computer folder perm-reps-cayley-tables, worksheet perm-rep-lab1a) to evaluate the linear representations $\rho(\tau)$, $\tau \in S_3$;
(8) Evaluate the character table of ρ;
(9) Identify the character table of S_3, from Example 1.21.1 p. 68;
(10) Construct the canonical projections ($\mathcal{P}_i$) following Example 2.14.1;
(11) Determine the dimension of the subspaces in the decomposition $\mathcal{V} = \mathbb{R}^6 = \mathcal{V}_1 \oplus \mathcal{V}_2 \oplus \mathcal{V}_3$;
(12) Determine the multiplicities with which the irreducible representations (1, sgn and β) appear in ρ, that is, the multiplicities in the decomposition of $\mathcal{V}_i$- use Proposition 2.12.1 p. 68;
(13) Verify the properties of a canonical decomposition;
(14) Define a data vector indexed by S_3. An example of these data is on p. 89;
(15) Identify the invariants associated with each projection e.g., Example 2.14.2;
(16) Interpret these invariants;
(17) Decompose the sum of squares associated with the data vector;
(18) Interpret each component;
(19) Identify the parametric hypotheses associated with these invariants;
(20) Identify the non-centrality parameters and degrees of freedom for normally distributed data. This item completes the first part of the lab.
(21) Now consider another group action of the same group on the same set, namely, the multiplicative action $\varphi(\tau, \sigma) = \sigma\tau$ generated by the Cayley table of S_3 of p.37. Now repeat all steps above, compare and interpret the results. This item completes the second part of the lab;
(22) Now consider the multiplicative action of C_3 (the rotations only) on the same set. This is simply the multiplicative action generated by the Cayley table of p. 37 restricted to rows a, e and f (defining C_3). Now repeat all steps above, compare and interpret the results;
(23) To conclude this lab, write a summary underlying the effect that choosing different group actions and symmetries has on the analysis of the data indexed by the structure of interest.