STAT 381 **L. Liu**

Extra Material on counting

- **The Multiplication Principle : If a task consists of 2 steps, and there are m ways to perform the first step; n ways to perform the second step. Then there are $m \cdot n$ ways to complete the task**

  Examples:

  1. **Auto plate number: One state's plate number requires 3 letters followed by 3 digits. How many different plate numbers can they issue?**
  2. **Auto plate number: One state's plate number requires 6 positions with letters or digits at each position. How many different plate numbers can they issue?**
  3. **More complex cases: There must be 3 letters and 3 digits, but can be in any position.**
  4. **Still 3 letter first followed by 3 digits, but AAA is not allowed.**
  5. **How many different two-letter words(including nonsense words) can be formed when repetition of letters is allowed?**
  6. **How many different two-letter words(including nonsense words) can be formed if the 2 letters must be distinct?**

- **PERMUTATION: How many ways to take r objects from n distinct objects to arrange in specific order? (In short: with order, without repetition)**

  Example: From a group of 5 people, we elect a president, a vice-president, and a secretary?

  5*4*3=60

  the number of Permutations of n objects taken r at a time
  $= P(n,r) = n \cdot (n-1) \cdot (n-2) ... \cdot [n-(r-1)]$
  $= \dfrac{n!}{(n-r)!}$

- **COMBINATION: How many ways to take r objects from n distinct objects without caring about order?**

  (In short: without order, without repetition)

  Example: From a group of 5 people, we elect three to form a committee?

  the number of combinations of n objects taken r at a time
  $= C(n,r) = \dfrac{n!}{(n-r)!r!}$

  1. Pick r objects from n with order and replacement (In short: with order, with repetition)

  Ex: Toss a coin 3 times, record the sequency of heads and tails. How many possible outcomes?

  Ex: An exam consists of 10 questions, each a multiple-choice problem with 5 choices of answers. How many different ways to answer the questions?

- **The Binomial Theorem**

  1. **Binomial expansion:**
     $(x+y)^n = \Sigma_{k=0}^{n} C(n,k)x^{n-k}y^k =$
     $C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + ... + C(n,r)x^r y^{n-r} + ... + C(n,n-1)xy^{n-1} + C(n,n)y^n$
  2. **Binomial coefficients:** $C(n,k)$
     1) By Yang's triangle
     2) By combination: $C(n,k) = \dfrac{P(n,k)}{k!} = \dfrac{n!}{k!(n-k)!}$
  3. **Example:**
     - $C(n,0) = C(n,n) = 1$;
     - $C(n,1) = C(n,n-1) = n$;
     - **Symmetry:** $C(n,k) = C(n,n-k)$
  4. **Let** $x = y = 1$**, we get** $2^n = (1+1)^n =$
     $C(n,0) + C(n,1) + C(n,2) + ... + C(n,n-1) + C(n,n)$
     Connection: How many subsets does a set of n **elements have?**
     Toss a coin 3 times, how many heads and tail sequencies has exactly 2 heads?
  5. **Let** $x = 1, y = -1$**, we get** $0 = (1-1)^n =$
     $C(n,0) - C(n,1) + C(n,2) - ... + (-1)^n C(n,n)$