1. The final exam scores of a math class is as follows (out of 200 point): 129, 115, 195, 139, 114, 197, 198, 165, 105, 153, 139, 112, 143, 155, 181

   Do a stem plot, find the 5-number summary and draw boxplot(all by hand). Find the mean and sample standard deviation (by any method but write down the formula).

   Answer:
   The order of this series of data is 105, 112, 114, 115, 129, 139, 139, 143, 153, 155, 165, 181, 195, 197, 198
   The five number summary is:
   minimum: 105; first quartile: 115; median: 143; third quartile: 181; maximum: 198

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = 149.33 \quad S = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}} = 32.15$$

2. If A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.2$. Find the following:

   (a) $P(A \bigcup B)$

$$\begin{aligned} P(A \bigcup B) &= P(A) + P(B) - P(A \bigcap B) \\ &= P(A) + P(B) - P(A)P(B) \\ &= 0.4 + 0.2 - 0.4 * 0.2 = 0.52 \end{aligned}$$

   (b) $P(A' \cap B)$

$$\begin{aligned} P(A' \bigcap B) &= P(A')P(B) \\ &= (1 - P(A))P(B) \\ &= (1 - 0.4) * 0.2 \\ &= 0.12 \end{aligned}$$

   (c) $P(A' \cup B')$

$$\begin{aligned} P(A' \bigcup B') &= P((A \bigcap B)') \\ &= 1 - P(A \bigcap B) \\ &= 1 - P(A)P(B) \\ &= 1 - 0.4 * 0.2 \\ &= 0.92 \end{aligned}$$

(d) $P(A|B)$

$$\begin{aligned} P(A|B) &= \frac{P(A\bigcap B)}{P(B)} \\ &= \frac{P(A)P(B)}{P(B)} \\ &= P(A) = 0.4 \end{aligned}$$

3. If A and B are mutually exclusive events with $P(A) = 0.4$ and $P(B) = 0.2$. Find the following:

(a) $P(A \cup B)$

$$\begin{aligned} P(A\bigcup B) &= P(A) + P(B) - P(A\bigcap B) \\ &= P(A) + P(B) \\ &= 0.4 + 0.2 \\ &= 0.6 \end{aligned}$$

(b) $P(A' \cap B)$

$$\begin{aligned} P(A'\bigcap B) &= P(B) \\ &= 0.2 \end{aligned}$$

(c) $P(A' \cup B')$

$$P(A'\bigcup B') = P((A\bigcap B)') = 1$$

(d) $P(A|B)$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = 0$$

4. A jury of 6 people is to be selected from an available pool of 8 women and four men. Let X denote the number of men in the selected jury.

(a) Determine the p.d.f. (probability distribution) of X.

Answer:

$$f(x) = P(X = x) = \frac{\binom{4}{x}\binom{8}{6-x}}{\binom{12}{6}}, \quad x = 0, 1, 2, 3, 4$$

(b) Find the mean and the variance of X.

Answer:

$$E(x) = \sum_{x=0}^{4} x \frac{\binom{4}{x}\binom{8}{6-x}}{\binom{12}{6}}$$
$$= 0 * 0.03030 + 1 * 0.2424 + 2 * 0.4545 + 3 * 0.2424 + 4 * 0.03030 = 1.9998$$

$$E(x^2) = \sum_{x=0}^{4} x^2 \frac{\binom{4}{x}\binom{8}{6-x}}{\binom{12}{6}}$$
$$= 0 * 0.03030 + 1 * 0.2424 + 4 * 0.4545 + 9 * 0.2424 + 16 * 0.03030 = 4.7268$$

$$\begin{aligned} V(x) &= E(x^2) - [E(x)]^2 \\ &= 4.7268 - (1.9998)^2 = 0.7276 \end{aligned}$$

5. Let X be a random variable. What value of a is necessary in the chart below to make it a probability distribution?

k	Pr(X=k)
0	0.5
1	a
2	0.35

Answer:
a = 1-(0.5+0.35) = 0.15

6. A single die is tossed 10 times. What is the probability that a 2 appears at least 4 times?

Answer:

$$1 - \binom{10}{0}(\frac{5}{6})^{10} - \binom{10}{1}(\frac{1}{6})^1(\frac{5}{6})^9 - \binom{10}{2}(\frac{1}{6})^2(\frac{5}{6})^8 - \binom{10}{3}(\frac{1}{6})^3(\frac{5}{6})^7$$

=1-0.1615-0.3230-0.2907-0.1550=0.0698

7. P(3rd "6" happened on the 5th toss) $= C(4,2)(\frac{1}{6})^3(\frac{5}{6})^2 = 0.0193$

8. Two people play a game. A single die is rolled. If the outcome is a 3, then A pays B \$3. If the outcome is a 5, then no money change hands. How much should B pay A when a 1, 2, 4, or 6 is thrown so that A and B break even, on average,

over many repetitions of the game?

Answer:
Let x be the money that B should give to A,

$$\frac{1}{6} * 3 = \frac{4}{6}x$$

$$x = 0.75$$

9. It is estimated that 1% of all items coming off an assembly line are defective. Let X be the number of defective items in a random sample of 1000 items from the assembly line. Compute the mean and variance of X.

Answer:
Binomial with $n = 1000, p = 0.01$

$$E(X) = np = 1000 * 0.01 = 10$$

$$V(X) = np(1-p) = 1000 * 0.01 * (1 - 0.01) = 9.9$$

10. (a) Determine c so that the function can serve as the p.d.f of a random variable: $f(x) = c(\frac{2}{5})^x, x = 1, 2, 3, \ldots$

Answer:

$$\sum_{x=1}^{\infty} c(\frac{2}{5})^x = c\frac{2/5}{1-2/5} = c\frac{2}{5} * \frac{5}{3} = \frac{2}{3}c = 1$$

$$c = 3/2$$

(b) Suppose that X is a r.v with the p.d.f. of part (a). Find $P(X < 4)$.

Answer:

$$P(x < 4) = f(1) + f(2) + f(3)$$

$$= \frac{3}{2}(\frac{2}{5})^1 + \frac{3}{2}(\frac{2}{5})^2 + \frac{3}{2}(\frac{2}{5})^3 = \frac{117}{125}$$

11. Each day, a weather forecaster predicts whether or not it will rain. For 80% of rainy days, she correctly predicts that it will rain. For 90% of non-rainy days, she correctly predicts that it will not rain. Suppose that 10% of days are rainy and 90% are non-rainy.

Draw a tree diagram: the first layer of the branches should be rain or no-rain; the 2nd layer are predicted rain or predicted rain.

(a) Given that her prediction for tomorrow is rainy, what is the probability that there is no rain?

Using the tree diagram,

$P(\text{no rain}|\text{ predicted rain}) = \frac{P(\text{no rain})P(\text{ predicted rain}|\text{no rain})}{P(\text{ predicted rain})} = \frac{0.9*0.1}{0.9*0.1+0.1*0.8} = \frac{9}{17}$ or 53%.

(b) Given that her prediction for tomorrow is non-rainy, what is the probability that it is going to rain? (Keep 3 decimal places.)

Similarly,

$P(\text{rain}|\text{ predicted no rain}) = \frac{P(\text{rain})P(\text{ predicted no rain}|\text{ rain})}{P(\text{ predicted no rain})} = \frac{0.2*0.1}{0.2*0.1+0.9*0.9} = \frac{2}{83}$ or 2.4%.

12. A certain delivery service offers both express and standard delivery. 75% of parcels are sent by standard delivery, and 25% are sent by express. Of those sent standard, 80% arrive by the next day, and of those sent express, 95% arrive by the next day. A record of a parcel delivery is chosen at random from the company's files.

    Tree diagram: 1st layer of the tree has 2 branches: standard or express; 2nd layer are "by next day" or "more than two days".

    (a) What is the probability that it arrived the next day?

    By the law of total probability:

    P(next day)= P(standard)*P(next day, given standard)+P(express)*P(next day, given express)=$.75 * .80 + .25 * .95 = .8375$

    (b) Given that the package arrived by the next day, what is the probability that it was sent express?

    By Bayes' theorem:

    $P(\text{express}|\text{next day}) = \frac{P(\text{next day}|\text{express})P(\text{express})}{P(\text{next day})} = 28.36\%$

13. Electric circuit boards are rated excellent, acceptable, or unacceptable. Suppose that 30% of boards are excellent, 60% are acceptable, and 10% are unacceptable. Further, suppose that 10% of excellent boards fail, 20% acceptable boards fail, and 100% of unacceptable boards fail(unacceptable boards are discarded without being used).

    (a) What is the probability that a board is rated excellent **and** fails?

    Product rule:

    P(Excellent and fails)=P(excellent)*P(fails given excellent)=$0.3 * 0.1 = 3\%$

    (b) What is the probability that a board fails?

    Total Probability:

    P(fails)=P(excellent)*P(fails given excellent)+P(acceptable)*P(fails given acceptable)+P(unacceptable)*P(fails given unacceptable)=$0.3*0.1+0.6*0.2+0.1*1 = 25\%$

    (c) Given a board fails, what is the probability that it was rated excellent?

Bayes' Th:

P(excellent given fail)= P(excellent)*P(fails given excellent)/P(fails)=0.03/0.25 = 12%.

14. A fair die is rolled 5 times.

    (a) What is the probability that the die shows 6 exactly twice?

    $C(5,2)(1/6)^2(5/6)^3$

    (b) What is the probability that the die shows 6 all 5 times?

    $(1/6)^5$

    (c) What is the probability that the die didn't show 6 all 5 times?

    $(5/6)^5$