

STAT 381

Review for Exam 2

Topics: Exam 2 covers 2.5-4.1. You are allowed to take one page of formula sheet (regular 8.5 in by 11 in) to the midterm 2. Be sure you know the following concepts:

1. One variable continuous random variable, pdf, mean, variance.
2. Two variables: discrete or continuous, Joint pdf, marginal pdf, conditional pdfs of 2 discrete or continuous r.vs.
3. How to compute probability, $E(X)$, $E(XY)$, $E(X|Y)$, $V(X)$, $Cov(X, Y)$, ρ given joint pdf?
4. Independence, $E(\sum_{i=1}^n a_i X_i)$, in particular, if $a_i = 1$ or $a_1 = 1/n$. Compute $V(\sum_{i=1}^n a_i X_i)$. What if X_i 's are independent? What is $V(\sum_{i=1}^n a_i X_i)$ if X_i 's are iid with mean μ and variance σ^2 ? In particular, if $a_i = 1$ or $a_1 = 1/n$.
5. Special distributions, their mean, variances and pdfs: Especially Poisson, Exponential, Normal.
6. Taylor expansion of e^x , $\frac{1}{1-x}$, and its term-by-term differentiation.
7. The Central Limit Theorem.

Review Problems:

1. Suppose we draw 2 balls out of an urn with 8 red, 6 blue, and 4 green balls. Let X be the number of red balls we get and Y the number of blue balls.
 - (a) Find the joint pmf of X and Y .
 - (b) Find the marginals of X and Y .
 - (c) Find the conditional pmf of $f_{Y|x}(y|x = 1)$
 - (d) Find $P(Y = 1|X = 1)$
2. Suppose $P(X = x, Y = y) = c(x + y)$ for $x, y = 0, 1, 2, 3$.
 - (a) What value of c will make this a joint pmf?
 - (b) What is $P(X > Y)$?
 - (c) Find $E(X)$, $V(X)$, $cov(X, Y)$
3. Suppose X and Y has joint density $f(x, y) = c(x + y)$ for $0 < x, y < 1$.
 - (a) What is c ?
 - (b) What is $P(X < 1/2)$?
 - (c) What is $P(X + Y > 1/2)$?
 - (d) Find $E(X)$, $V(X)$, $cov(X, Y)$

4. A chromosome mutation believed to be linked with color blindness is known to occur, on the average, once in every 10,000 births. If 20,000 babies are born this year in a certain city, what is the probability that at least one will develop color blindness? What is the exact probability model that applies here?
5. In a certain published book of 520 pages, 390 typographical errors occur. What is the probability that one page, selected randomly by printer as a sample of her work, will be free from errors?(Hint: Poisson distribution)
6. The p.d.f of a random variable X is given by $f(x) = e^x$, for $x < 0$
 - (a) Find $E(X)$
 - (b) Find $E(e^{3X/2})$
7. The p.d.f of a random variable X is given by $f(x) = kxe^{-x^2}$, for $x > 0$
 - (a) Find the constant k .
 - (b) Find the distribution function $F(x)$.
 - (c) Find $P(X > 4)$.
8. The p.d.f of a random variable X is given by $f(x) = \frac{c}{\sqrt{x}}$, for $0 < x < 4$
 - (a) Find the constant c .
 - (b) Find the distribution function $F(x)$.
 - (c) Find $P(X < \frac{1}{4})$ and $P(X > 1)$.
 - (d) Find the mean $E(X)$ and the variance $V(X)$.
9. The distribution function of a r.v X is given by

$$F(x) = \begin{cases} 1 - (1+x)e^{-x} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \quad \text{Find}$$
 - (a) $P(X < 2)$
 - (b) $P(-1 < X < 3)$
 - (c) $P(X > 4)$
 - (d) the p.d.f of X
10. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are normally distributed with standard deviation equal to 0.3 ounces, give the setting for μ so that 8-ounce cups will overflow only 1% of the time.
11. Show that the normal density with mean μ and a standard deviation σ obtains maximum at $x = \mu$. What is the maximum value? Show that it has inflection points at $x = \mu \pm \sigma$.

12. If the length of life Y (in unit of years) for a battery has a Weibull distribution with p.d.f

$$f(y) = \frac{3y^2}{4}e^{-y^3/4}, \quad y \geq 0.$$

Find the probability that the battery lasts less than 4 years given that it is now 2 years old.

13. Let X be the number of customers login on a web site per minute. Assume X has a Poisson distribution with a mean of 6 login requests per minute.

(a) What is the probability that no one requested to log on this site in the next minute?

(b) Let W be the time in minutes between the 2nd and 3rd requests. What is the distribution name of W ? What is the expected value of W ?

14. If a r.v X satisfies a normal distribution with mean 60 and standard deviation 20. Find the following results using the Empirical rule (68-95-99.7 rule):

(a) $P(X \geq 80)$

(b) $P(|X - 60| < 40)$

(c) $P(|X - 60| \geq 20)$

(d) $P(X < 20)$

15. Let \bar{X} be the mean of a random sample of size $n = 36$ from an Exponential distribution with mean 6. Use the Central Limit Theorem to approximate $P(4 < \bar{X} \leq 8)$

16. Let \bar{X} be the mean of a random sample of size $n = 36$ from a Poisson distribution with mean 6. Use the Central Limit Theorem to approximate $P(4 < \bar{X} \leq 8)$

17. Toss a coin 120 times. Suppose $P(H) = p = 1/3$. Let X be the number of heads. Use the Central Limit Theorem to approximate $P(20 < X \leq 50)$

18. If a r.v. X satisfies a normal distribution with mean 2, variance 9 and a r.v. Y satisfies a normal distribution with mean 3, variance 4. Suppose X and Y are independent.

(a) What is the mean and variance of $X + Y$?

(b) What is the mean and variance of $X - Y$?

(c) What is the mean and variance of $2X - 3Y$?

19. If a r.v. X satisfies a Poisson distribution with mean 2 and a r.v. Y satisfies a Poisson distribution with mean 3. Suppose X and Y are independent.

(a) What are the variances of X and Y ?

(b) What is the mean and variance of $X + Y$?